LECTURES
ON
SELECT SUBJECTS
IN

MECHANICS,	PNEUMATICS,
HYDROSTATICS,	AND OPTICS:

WITH

The USE of the GLOBES,

The ART of DIALING,

AND

The Calculation of the Mean Times of NEW and FULL MOONS and ECLIPSES.

By JAMES FERGUSON.

Philosophia mater omnium bonarum artium est. CICERO. 1 Tusc.

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MDCCLX.

TO HIS

ROYAL HIGHNESS

Prince EDWARD.

SIR,

AS heaven has inſpired your ROYAL HIGHNESS with ſuch love of ingenious and uſeful arts, that you not only ſtudy their theory, but have often condeſcended to honour the profeſſors of mechanical and experimental philoſophy with your preſence and particular favour; I am thereby encouraged to lay myſelf and the following work at

your Royal Highneſs's feet: and at the ſame time beg leave to expreſs that veneration with which I am,

SIR,

Your Royal Highness's

Moſt obliged,

And moſt obedient,

Humble Servant,

James Ferguſon.

PREFACE.

EVER since the days of the LORD CHANCELLOR BACON, *natural philosophy hath been more and more cultivated in England.* THAT *great genius first set out with taking a general survey of all the natural sciences, dividing them into distinct branches, which he enumerated with great exactness; he enquired scrupulously into the degree of knowledge already attained to in each, and drew up a list of what still remained to be discovered: this was the scope of his first undertaking. Afterward he carried his views much farther, and shewed the necessity of an experimental philosophy, a thing never before thought of. As he was a professed enemy to systems, he considered philosophy no otherwise than as that part of knowledge which contributes to make men better and happier: he seems to limit it to the knowledge of things useful, recommending above all the study of nature, and shewing that no progress can be made therein, but by collecting facts, and comparing experiments, of which he points out a great number proper to be made.*

But notwithstanding the true path to science was thus exactly marked out, the old notions of the schools so strongly possessed people's minds at that time, as not to be eradicated by any new opinions, how rationally soever advanced, until the illustrious Mr. BOYLE, *the first who pursued* LORD BACON'*s plan, began to put experiments in practice with an assiduity equal to his great talents. Next, the* ROYAL SOCIETY

being

being established, the true philosophy began to be the reigning taste of the age, and continues so to this day.

The immortal SIR ISAAC NEWTON *insisted, even in his early years, that it was high time to banish vague conjectures and hypotheses from natural philosophy, and to bring that science under an entire subjection to experiments and geometry. He frequently called it* the experimental philosophy, *so as to express significantly the difference between it and the numberless systems which had arisen merely out of the conceits of inventive brains: the one subsisting no longer than the spirit of novelty lasts; the other never failing whilst the nature of things remains unchanged.*

The method of teaching and laying the foundation of physics, by exhibiting public courses of experiments, was first undertaken in this kingdom, I believe, by Dr. JOHN KEILL, *and since improved and enlarged by Mr.* HAUKSBEE, *Dr.* DESAGULIERS, *Mr.* WHISTON, *Mr.* COTES, *Mr.* WHITESIDE, *and Dr.* BRADLEY *our present Regius and Savilian professor of Astronomy. Nor has the same been neglected in other countries: Dr.* JAMES, *and Dr.* DAVID GREGORY, *Sir* ROBERT STEWART, *and after him Mr.* MACLAURIN, *in Scotland; Dr.* HELSHAM *in Ireland, Messieurs* s'GRAVESANDE *and* MUSCHENBROEK *in Holland, and the* ABBE' NOLLET *in France, have acquired just applause thereby.*

The substance of my own attempts in this way of instrumental instruction, the following sheets (exclusive of the astronomical part) will shew: the satisfaction they have generally given, read as lectures

to

to different audiences, affords me ſome hope that they may be favourably received in the ſame form by the public.

I ought to obſerve, that though the four laſt lectures cannot be properly ſaid to concern experimental philoſophy, I conſidered however that they were not of ſo different a claſs, but that they might, without much impropriety, be ſubjoined to the preceding ones.

*My apparatus (part of which is deſcribed here, and the reſt in a * former work) is rather ſimple than magnificent; which is owing to a particular point I had in view at firſt ſetting out, namely, to avoid all ſuperfluity, and to render every thing as plain and intelligible as I could.*

* *Aſtronomy explained upon* SIR ISAAC NEWTON'*s principles, and made eaſy to thoſe who have not ſtudied mathematics.*

ERRATA.

Page 35, line 8, for *wx* read *wz*. Page 42. l. 20, *dele* only. Page 117, l. 15 from the bottom, *for* three *read* ſix. Page 147, l. 3 and 5 from the bottom, *for* 0.785339 *read* 0.785399. Page 200, l. 1. *for* VIII *read* VII. Page 211, l. 19, for *ABE* read *aBE*. Page 212, l. 16, for *eb* read *eB*. Page 213, l. 11 from the bottom, for *IkH* read *IkK*. Page 216, l. 5, for *merciſcus* read *meniſcus*. Page 265, l. 13, *for* plane *read* place.

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CONTENTS.

LEC-

LECTURES

ON

SELECT SUBJECTS.

LECT. I.

Of matter and its properties.

AS the defign of the firft part of this courfe is to explain and demonftrate thofe laws by which the material univerfe is governed, regulated, and continued; and by which the various appearances in nature are accounted for; it is requifite to begin with explaining the properties of matter.

By the word *matter* is here meant every thing that has length, breadth, and thicknefs, and refifts the touch. Matter, what?

The inherent properties of matter are folidity, inactivity, mobility, and divifibility. Its properties.

The *folidity* of matter arifes from its having length, breadth, thicknefs, and being impenetrable. Hence it is that all bodies are comprehended under fome fhape or other, and that every particular body hinders all others from occupying the fame part of fpace which it poffeffeth. Thus, if a piece of wood or metal be fqueez'd ever fo hard between two plates, they cannot be brought into Solidity.

 contact.

contact. And even water or air has this property; for if a ſmall quantity of it be fixed between any other bodies, they cannot be brought to touch one another.

Inactivity. A ſecond property of matter is *inactivity*, or *paſſiveneſs*; by which it always endeavours to continue in the ſtate that it is in, whether of reſt or motion. And therefore, if one body conſiſt of twice or thrice as much matter as another body, it will have twice or thrice as much inactivity; that is, it will require twice or thrice as much force to give it an equal degree of motion, or to ſtop it after it hath been put into ſuch a motion.

That matter can never put itſelf into motion is allowed by all men. For they ſee that a ſtone, lying on the plane ſurface of the earth, never removes itſelf from that place, nor does any one imagine it ever can. But moſt people are apt to believe that all matter has a propenſity to fall from a ſtate of motion into a ſtate of reſt; becauſe they ſee that if a ſtone or a cannon-ball be put into ever ſo violent a motion, it ſoon ſtops; not conſidering that this ſtoppage is cauſed 1. by the gravity or weight of the body, which ſinks it to the ground in ſpite of the impulſe; and 2. by the reſiſtance of the air through which it moves, and by which its velocity is retarded every moment till it falls.

A bowl moves but a ſhort way upon a bowling-green; becauſe the roughneſs and unevenneſs of the graſſy ſurface ſoon creates friction enough to ſtop it. But if the green were perfectly level, and covered with poliſhed glaſs; and the bowl perfectly hard,

hard, round, and ſmooth, it would go a great way further; as it would have nothing but the air to reſiſt it: if then the air were taken away, the bowl would go on without any friction, and conſequently without any diminution of the velocity it had at ſetting out: and therefore, if the green were extended quite round the earth, the bowl would go on for ever.

If the bowl were carried ſeveral miles above the earth, and there projected in a horizontal direction, with ſuch a velocity as would make it move more than a ſemidiameter of the earth, in the time it would take to fall to the earth by gravity; in that caſe, and if there were no reſiſting medium in the way, the bowl would not fall to the earth at all; but would continue to circulate round it, keeping always in the ſame tract, and returning to the ſame point from which it was projected, with the ſame velocity as at firſt. In this manner the moon moves round the earth, although ſhe be as unactive and dead as any ſtone upon it.

Mobility. The third property of matter is *mobility*; for we find that all matter is capable of being moved, if a ſufficient degree of force be applied to overcome its reſiſtance.

Diviſibility. The fourth property of matter is *diviſibility*, of which there can be no end. For, ſince matter can never be annihilated by cutting or breaking, we can never imagine it to be cut into ſuch ſmall particles, but that if one of them be laid on a table, the uppermoſt ſide of it will be further from the table than the undermoſt ſide. Moreover it would be abſurd

to ſay that the greateſt mountain on earth has more halves, quarters, or tenth parts, than the ſmalleſt particle of matter has.

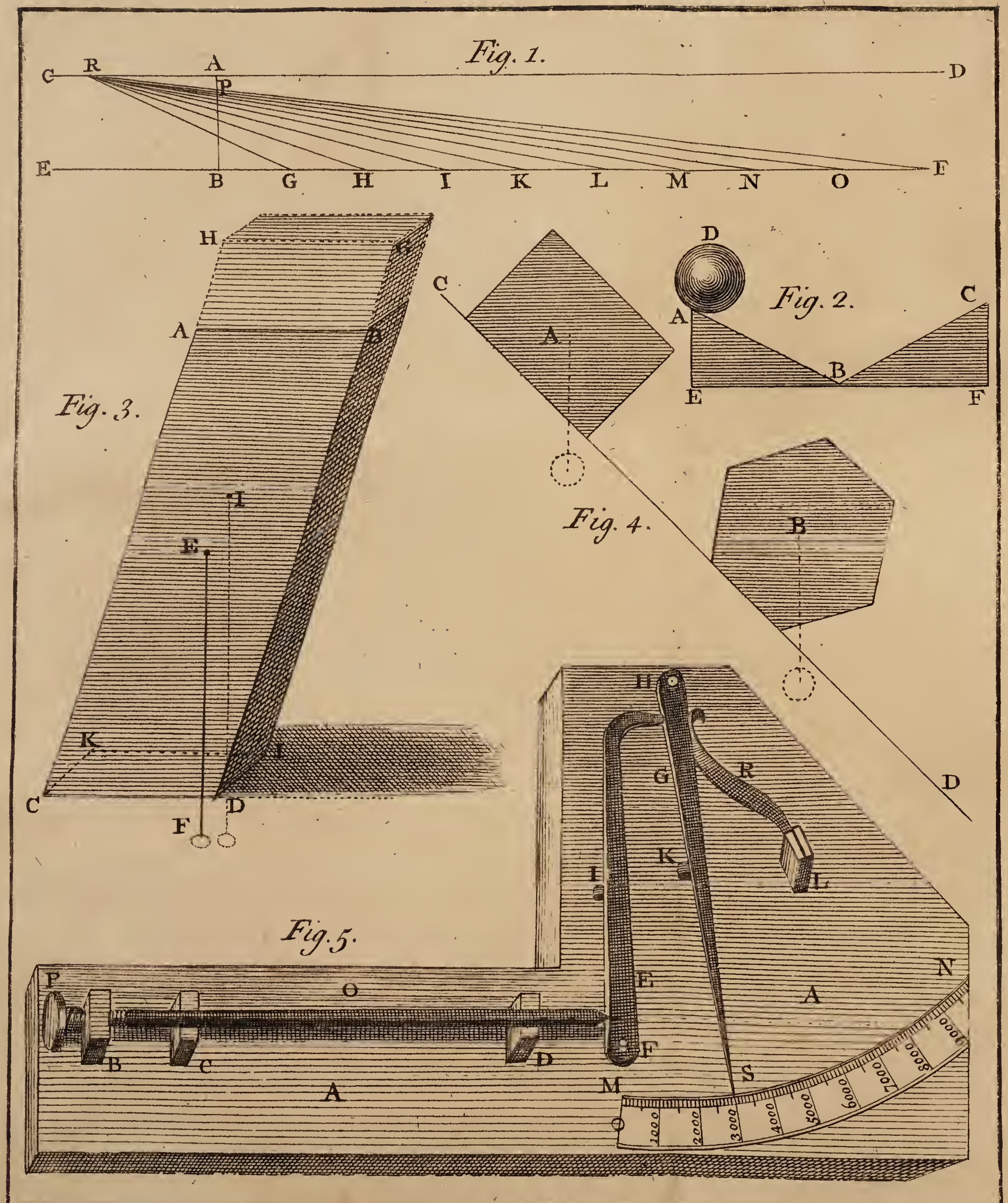
We have many ſurpriſing inſtances of the ſmallneſs to which matter can be divided by art: of which the two following are very remarkable.

1. If a pound of ſilver be fuſed with a ſingle grain of gold, the gold will be equally diffuſed through the whole ſilver, ſo that taking one grain from any part of the maſs (in which there can be no more than the 5760th part of a grain of gold) and diſſolving it in *aquafortis*, that quantity of gold will fall to the bottom.

2. The gold-beaters can extend a grain of gold into a leaf containing 50 ſquare inches; and this leaf may be divided into 500000 parts. For an inch in length can be divided into 100 parts, every one of which will be viſible to the naked eye: conſequently a ſquare inch can be divided into 10000 parts, and 50 ſquare inches into 500000. And if one of theſe parts be viewed with a microſcope that magnifies the diameter of an object only 10 times, it will magnify the area 100 times; and then the 100th part of a 500000th part of a grain (that is the 50 millionth part) will be viſible. Such leaves are commonly uſed in gilding; and they are ſo very thin, that if 124500 of them were laid upon one another, and preſſed together, they would not exceed one inch in thickneſs.

Yet all this is nothing in compariſon of the lengths that nature goes in the diviſion of matter. For Mr. *Leewenhoek* tells us, that there are more animals

PLATE I.



J. Ferguson delin.

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animals in the milt of a ſingle cod-fiſh, than there are men upon the whole earth: and that, by comparing theſe animals in a microſcope with grains of common ſand, it appeared that one ſingle grain is bigger than four millions of them. Now, each animal muſt have a heart, arteries, veins, muſcles, and nerves, otherwiſe they could neither live nor move. How inconceivably ſmall then muſt the particles of their blood be, to circulate through the ſmalleſt ramifications and joinings of their arteries and veins? It has been found by calculation, that a particle of their blood muſt be as much ſmaller than a globe of the tenth part of an inch in diameter, as that globe is ſmaller than the whole earth! and yet, if theſe particles be compared with the particles of light, they will be found to exceed them as much in bulk as mountains do ſingle grains of ſand. For, the force of any body ſtriking againſt an obſtacle is directly in proportion to its quantity of matter multiplied into its velocity: and ſince the velocity of the particles of light is demonſtrated to be at leaſt a million of times greater than the velocity of a cannon-ball, it is plain, that if a million of theſe particles were as big as a ſingle grain of ſand, we durſt no more open our eyes to the light, than we durſt expoſe them to ſand ſhot point-blank from a cannon.

That matter is infinitely diviſible in a mathematical ſenſe, is eaſy to be demonſtrated. For, let *AB* be the length of a particle to be divided; and let it be touched at oppoſite ends by the parallel lines *CD* and *EF*, which, ſuppoſe to be in- Plate I. Fig. 1.

The infinite divisibility of matter proved.

finitely extended beyond D and F. Set off the equal divisions BG, GH, HI, &c. on the line EF, towards the right hand from B; and take a point as at R, any where towards the left hand from A, in the line CD: Then, from this point, draw the right lines RG, RH, RI, &c. each of which will cut off a part from the particle AB. But after any finite number of such lines are drawn, there will still remain a part as AP at the top of the particle, which can never be cut off; because the lines DR and EF being parallel, no line can ever be drawn from the point R to any point of the line EF that will coincide with the line RD. Therefore the particle AB contains more than any finite number of parts.

Attraction.

A fifth property of matter is *attraction*, which seems rather to be infused than inherent. Of this there are four kinds, viz. *cohesion*, *gravitation*, *magnetism*, and *electricity*.

Cohesion.

The *attraction of cohesion* is that by which the small parts of matter are made to stick and cohere together. Of this we have several instances, some of which follow.

1. If a small glass tube, open at both ends, be dipt in water, the water will rise up in the tube to a considerable height above its level in the bason: which must be owing to the attraction of a ring of particles of the glass all around the tube, immediately above those to which the water at any instant rises. And when it has risen so high that the weight of the column balances the attraction of the tube, it rises no higher. This can be no ways

ways owing to the preſſure of the air upon the water in the baſon; for, as the tube is open at top, it is full of air above the water, which will preſs as much upon the water in the tube as the neighbouring air does upon any column of an equal diameter in the baſon. Beſides, if the ſame experiment be made in the exhauſted receiver of an air-pump, there will be found no difference.

2. A piece of loaf ſugar will draw up a fluid, and a ſpunge will ſuck in water: and on the ſame principle ſap aſcends in trees.

3. If two drops of quickſilver be placed near each other, they will run together and become one large drop.

4. If two pieces of lead be ſcraped clean, and preſſed together with a twiſt, they will attract each other ſo ſtrongly as to require a force much greater than their own weight to ſeparate them. And this cannot be owing to the preſſure of the air, for the ſame thing will hold in an exhauſted receiver.

5. If two poliſhed plates of marble or braſs be put together, with a little oil between them to fill up the pores in their ſurfaces, and prevent the lodgment of any air; they will cohere ſo ſtrongly, even if ſuſpended in an exhauſted receiver, that the weight of the lower plate will not be able to ſeparate it from the upper one. In putting theſe plates together, the one ſhould be rubbed upon the other, as a joiner does two pieces of wood when he glues them.

6. If two pieces of cork, equal in weight, be put near each other in a baſon of water, they will

move equally faſt toward each other with an accelerated motion, until they meet: and then, if either of them be moved, it will draw the other after it. If two corks of unequal weights be placed near each other, they will approach with accelerated velocities inverſely proportionate to their weights: that is, the lighter cork will move as much faſter than the heavier, as the heavier exceeds the lighter in weight. This ſhews that the attraction of each cork is in direct proportion to its weight or quantity of matter.

This kind of attraction reaches but to a very ſmall diſtance; for, if two drops of quickſilver be rolled in duſt, they will not run together, becauſe the particles of duſt keeps them out of the ſphere of each other's attraction.

Repulſion. Where the ſphere of attraction ends, a *repulſive force* begins; thus, water repels moſt bodies till they are wet; and hence it is that a ſmall needle, if dry, ſwims upon water; and flies walk upon it without wetting their feet.

The repelling force of the particles of a fluid is but ſmall; and therefore, if a fluid be divided, it eaſily unites again. But if glaſs, or any other hard ſubſtance be broke into ſmall parts, they cannot be made to ſtick together again without being firſt wetted: the repulſion being too great to admit of a re-union.

The repelling force between water and oil is ſo great, that we find it almoſt impoſſible to mix them ſo as not to ſeparate again. If a ball of light

wood

wood be dipt in oil, and then put into water, the water will recede ſo as to form a channel of ſome depth all around the ball.

The repulſive force of the particles of air is ſo great, that they can never be brought ſo near together by condenſation as to make them ſtick or cohere. Hence it is, that when the weight of the incumbent atmoſphere is taken off from any ſmall quantity of air, that quantity will diffuſe itſelf ſo as to occupy (in compariſon) an infinitely greater portion of ſpace than it did before.

Attraction of *gravitation* is that power by which diſtant bodies tend towards one another. Of this we have daily inſtances in the falling of bodies to the earth. By this power in the earth it is, that bodies, on whatever ſide, fall in lines perpendicular to its ſurface; and conſequently, on oppoſite ſides, they fall in oppoſite directions; all towards the center, where the force of gravity is as it were accumulated: and by this power it is, that bodies on the earth's ſurface are kept to it on all ſides, ſo that they cannot fall from it. And as it acts upon all bodies in proportion to their reſpective quantities of matter, without any regard to their bulks or figures, it accordingly conſtitutes their weight. Hence, Gravitation.

If two bodies which contain equal quantities of matter, were placed at ever ſo great a diſtance from one another, and then left at liberty in free ſpace; if there were no other bodies in the univerſe to affect them, they would fall equally ſwift towards one another by the power of gravity, with velocities

cities accelerated as they approached each other; and would meet in a point which was half way between them at firſt. Or, if two bodies containing unequal quantities of matter, were placed at any diſtance, and left in the ſame manner at liberty, they would fall towards one another with velocities which would be in an inverſe proportion to their reſpective quantities of matter; and moving faſter and faſter in their mutual approach, would at laſt meet in a point as much nearer to the place from which the heavier body began to fall, than to the place from which the lighter body began to fall, as the quantity of matter in the former exceeded that in the latter.

All bodies that we know of have gravity or weight. For, that there is no ſuch thing as poſitive levity, even in ſmoke, vapours, and fumes, is demonſtrable by experiments on the air-pump; which ſhews, that although the ſmoke of a candle aſcends to the top of a tall receiver when full of air, yet, upon the air's being exhauſted out of the receiver the ſmoke falls down to the bottom of it. So, if a piece of wood be immerſed in a jar of water, the wood will riſe to the top of the water, becauſe it has a leſs degree of weight than its bulk of water has: but if the jar be emptied of water, the wood falls to the bottom.

Gravity demonſtrated to be as the quantity of matter in bodies.

As every particle of matter has its proper gravity, the effect of the whole muſt be in proportion to the number of attracting particles; that is, as the quantity of matter in the whole body. This is demonſtrable by experiments on pendulums; for,

for, if they are of equal lengths, whatever their weights be, they vibrate in equal times. Now it is plain, that if one be double or triple the weight of another, it muſt require a double or triple power of gravity to make it move with the ſame celerity: juſt as it would require a double or triple force to project a bullet of twenty or thirty pounds weight with the ſame degree of ſwiftneſs that a bullet of ten pounds would require. Hence, it is evident that the power or force of gravity is always proportional to the quantity of matter in bodies, whatever their bulks or figures are.

Gravity alſo, like all other virtues or emanations which proceed or iſſue from a center, decreaſes as the diſtance multiplied by itſelf increaſes. That is, a body at twice the diſtance of another attracts with only a fourth part of the force; at thrice the diſtance, with a ninth part; at four times the diſtance, with a ſixteenth part; and ſo on. This too is confirmed by comparing the diſtance which the moon falls in a minute, from a right line touching her orbit, with the diſtance through which heavy bodies near the earth fall in that time. And alſo by comparing the forces which retain Jupiter's moons in their orbits, with their reſpective diſtances from Jupiter. Theſe forces will be explained in the next lecture.

It decreaſes as the ſquare of the diſtance increaſes.

The velocity which bodies near the earth acquire in deſcending freely by the force of gravity, is proportional to the times of their deſcent. For, as the power of gravity does not conſiſt in a ſingle impulſe, but is always operating in a conſtant and uniform

uniform manner, it muſt produce equal effects in equal times; and conſequently in a double or triple time, a double or triple effect. And ſo, by acting uniformly on the body, muſt accelerate its motion proportionably to the time of its deſcent.

To be a little more particular on this ſubject, let us ſuppoſe that a body begins to move with a celerity conſtantly and gradually increaſing, in ſuch a manner, as would carry it through a mile in a minute; at the end of this ſpace it will have acquired ſuch a degree of celerity as is ſufficient to carry it two miles the next minute, though it ſhould then receive no new impulſe from the cauſe by which its motion had been accelerated: but if the ſame accelerating cauſe continues, it will carry the body a mile farther; on which account, it will have run through four miles at the end of two minutes; and then, it will have acquired ſuch a degree of celerity as is ſufficient to carry it through a double ſpace in as much more time; or eight miles in two minutes, even though the accelerating force ſhould act upon it no more. But this force ſtill continuing to operate in an uniform manner, will again in an equal time produce an equal effect; and ſo, by carrying it a mile further, cauſe it to move through five miles the third minute: for, the celerity already acquired, and the celerity ſtill acquiring, will each have its compleat effect. Hence we learn, that if the body ſhould move one mile the firſt minute, it would move three the ſecond, five the third, ſeven the fourth, nine the fifth, and ſo on in proportion.

And

And thus it appears, that the ſpaces deſcribed in ſucceſſive equal parts of time, by an uniformly accelerated motion, are always as the odd numbers 1, 3, 5, 7, 9, &c. and conſequently, the whole ſpaces are as the ſquares of the times, or as the velocities. For, the continued addition of the odd numbers yields the ſquares of all numbers from unity upward. Thus, 1 is the firſt odd number, and the ſquare of 1 is 1; 3 is the ſecond odd number, and this added to 1 makes 4, the ſquare of 2; 5 is the third odd number, which added to 4 makes 9, the ſquare of 3; and ſo on for ever. Since therefore, the times and velocities proceed evenly and conſtantly as 1, 2, 3, 4, &c. but the ſpaces deſcribed in each equal time are as 1, 3, 5, 7, &c. it is evident that the ſpace deſcribed

In one minute will be	- - -	1 =	ſquare of 1
In 2 minutes - -	1 + 3 =	4 =	ſquare of 2
In 3 minutes -	1 + 3 + 5 =	9 =	ſquare of 3
In 4 minutes	1 + 3 + 5 + 7 =	16 =	ſquare of 4 &c.

N. B. The character + ſignifies *more*, and = *equal*.

As heavy bodies are uniformly accelerated by the power of gravity in their deſcent, it is plain that they muſt be uniformly retarded by the ſame power in their aſcent. Therefore, the velocity which a body acquires by falling, is ſufficient to carry it up again to the ſame height from whence it fell: allowance being made for the reſiſtance of the air, or other medium in which the body is moved. Thus, the body *D* in rolling down the inclined plane

The deſcending velocity will give a power of equal aſcent.

Fig. 2.

plane *AB* will acquire ſuch a velocity by the time it arrives at *B*, as will carry it up the inclined plane *BC*, almoſt to *C*; and would carry it quite up to *C*, if the body and plane were perfectly ſmooth, and the air gave no reſiſtance.—So, if a pendulum were put into motion in a ſpace quite void of air, and all other reſiſtance, and had no friction on the point of ſuſpenſion, it would move for ever: for the velocity it had acquired in falling through the deſcending part of the arc, would be ſtill ſufficient to carry it equally high in the aſcending.

The center of gravity.

The *center of gravity* is that point of a body in which the whole force of its gravity or weight is united. Whatever ſupports that point bears the weight of the whole body: and whilſt it is ſupported, the body cannot fall; becauſe all its parts are in a perfect equilibrium about that point.

And line of direction.

An imaginary line drawn from the center of gravity of any body to the center of the earth, is called the *line of direction*. In this line all heavy bodies deſcend, if not obſtructed.

Since the whole weight of a body is united in its center of gravity, as that center aſcends or deſcends we muſt look upon the whole body to do ſo too. But as it is contrary to the nature of heavy bodies to aſcend of their own accord, or not to deſcend when they are permitted; we may be ſure that, unleſs the center of gravity be ſupported, the whole body will tumble or fall. Hence it is, that bodies ſtand upon their baſes when the line of direction falls within the baſe; for in this caſe the body cannot be made to tumble or fall without firſt raiſing the

the center of gravity higher than it was before. Thus, the inclining body *ABCD*, whofe center of gravity is *E*, ftands firmly on its bafe *CDIK*, becaufe the line of direction *EF* falls within the bafe. But if a weight as *ABGH* be laid upon the top of the body, the center of gravity of the whole body and weight together is raifed up to *I*; and then, as the line of direction *ID* falls without the bafe at *D*, the center of gravity *I* is not fupported; and the whole body and weight tumble down together. Fig. 3.

Hence appears the abfurdity of people's rifing haftily in a coach or boat when it is likely to overfet: for, by that means they raife the center of gravity fo far as to endanger throwing it quite out of the bafe; which if they do, they overfet the vehicle effectually. Whereas, had they clapt down to the bottom, they would have brought the line of direction, and confequently the center of gravity, farther within the bafe, and by that means might have faved themfelves.

The broader the bafe is, and the nearer the line of direction is to the middle or center of it, the more firmly does the body ftand. On the contrary, the narrower the bafe, and the nearer the line of direction is to the fide of it, the more eafily may the body be overthrown: a lefs change of pofition being fufficient to remove the line of direction out of the bafe in the latter cafe than in the former. And hence it is, that a fphere is fo eafily rolled upon a horizontal plane; and that it is fo difficult, if not impoffible, to make things which are

are ſharp pointed to ſtand upright on the point.— From what hath been ſaid, it clearly appears that if the plane be inclined on which the heavy body is placed, the body will ſlide down upon the plane whilſt the line of direction falls within the baſe; but it will tumble or roll down when that line falls without the baſe. Thus, the body *A* will only ſlide down the inclined plane *CD*, whilſt the body *B* rolls down upon it.

Fig. 4.

When the line of direction falls within the baſe of our feet, we ſtand; and moſt firmly when it is in the middle: but when it is out of that baſe, we immediately tumble or fall. And it is not only pleaſing, but even ſurpriſing, to reflect upon the various and unthought of methods and poſtures which we uſe to retain this poſition, or to recover it when it is loſt. For this purpoſe we bend our body forward when we riſe from a chair, or when we go up ſtairs: and for this purpoſe a man leans forward when he carries a burden on his back, and backward when he carries it on his breaſt; and to the right or left ſide as he carries it on the oppoſite ſide. A thouſand more inſtances might be added.

The quantity of matter in all bodies is in exact proportion to their weights, bulk for bulk. Therefore, heavy bodies are as much more denſe or compact than light bodies of the ſame bulk, as they exceed them in weight.

All bodies porous.

All bodies are full of pores, or ſpaces void of matter: and in gold, which is the heavieſt of all known bodies, there is perhaps a much greater quantity

quantity of ſpace than of matter. For the particles of heat and magnetiſm find an eaſy paſſage through the pores of gold; and even water itſelf has been forced through them. Beſides, if we conſider how eaſily the rays of light paſs through ſo ſolid a body as glaſs, in all manner of directions, we ſhall find reaſon to believe that bodies are much more porous than is generally imagined.

All bodies are ſome way or other affected by heat; and all metallic bodies are expanded in length, breadth, and thickneſs thereby.—The proportion of the expanſion of ſeveral metals, according to the beſt experiments I have been able to make with my pyrometer, is nearly thus. Iron and glaſs as 3, ſteel 4, copper 4 and one eighth, braſs 5, tin 6, lead 6 and one eighth. An iron rod 3 feet long is about one 70th part of an inch longer in ſummer than in winter. The expanſion of metals.

The pyrometer here mentioned being (for aught I know) of a new conſtruction, a deſcription of it may perhaps be agreeable to the reader. The pyrometer.

AA is a flat piece of mahogany, in which are fixed four braſs ſtuds *B*,*C*,*D*,*L*; and two pins, one at *F* and the other at *H*. On the pin *F* turns the crooked index *E I*, and upon the pin *H* the ſtraight index *G K*, againſt which a piece of watch-ſpring *R* bears gently, and ſo preſſes it towards the beginning of the ſcale *MN*, over which the point of that index moves. This ſcale is divided into inches and tenth parts of an inch: the firſt inch is marked 1000, the ſecond 2000, and ſo on. A bar of metal *O* is laid into notches in the top of the Fig. 5.

ſtuds *C* and *D*; one end of the bar bearing againſt the adjuſting ſcrew *P*, and the other end againſt the crooked index *E I* at a 20th part of its length from its center of motion *F*.—Now it is plain, that however much the bar *O* lengthens, it will move that part of the index *EI* againſt which it bears juſt as far: but the crooked end of the ſame index, near *H*, being 20 times as far from the center of motion *F* as the point is againſt which the bar bears, it will move 20 times as far as the bar lengthens. And as this crooked end bears againſt the index *GK* at only a 20th part of its whole length *GS* from its center of motion *H*, the point *S* will move through 20 times the ſpace that the point of bearing near *H* does. Hence, as 20 multipiled by 20 produces 400, it is evident that if the bar lengthens but a 400th part of an inch, the point *S* will move a whole inch on the ſcale; and as every inch is divided into 10 equal parts, if the bar lengthens but the 10th part of the 400th part of an inch, which is only the 4000th part of an inch, the point *S* will move the tenth part of an inch, which is very perceptible.

To find how much a bar lengthens by heat, firſt lay it cold into the notches of the ſtuds, and turn the adjuſting ſcrew *P* until the ſpring *R* brings the point *S* of the index *GK* to the beginning of the diviſions of the ſcale at *M*: then, without altering the ſcrew any farther, take off the bar and rub it with a dry woollen cloth till it feels warm; and then, laying it on where it was, obſerve how far it puſhes the point *S* upon the ſcale by means of the

crooked

crooked index *EI*: and the point *S* will ſhew exactly how much the bar has lengthened by the heat of rubbing. As the bar cools, the ſpring *R* bearing againſt the index *KG*, will cauſe its point *S* to move gradually back towards *M* in the ſcale: and when the bar is quite cold, the index will reſt at *M*, where it was before the bar was made warm by rubbing. The indexes have ſmall rollers under them at *I* and *K*; which, by turning round on the ſmooth wood as the indexes move, make their motions the eaſier, by taking off a great part of the friction, which would otherwiſe be on the pins *F* and *H*, and of the points of the indexes themſelves on the wood.

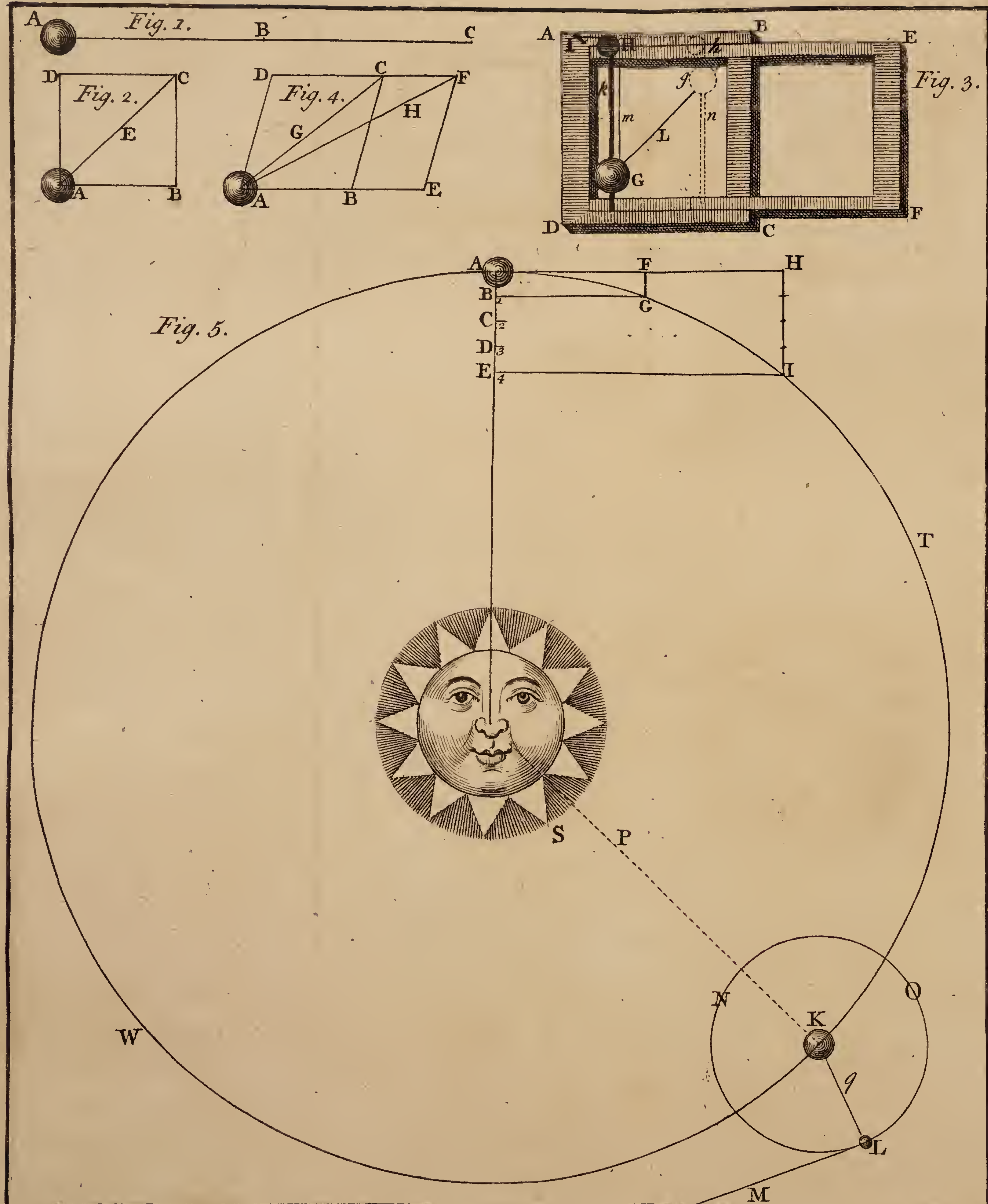
Beſides the univerſal properties above-mentioned, there are bodies which have properties peculiar to themſelves: ſuch as the loadſtone, in which the moſt remarkable are theſe, 1. it attracts iron and ſteel only. 2. It conſtantly turns one of its ſides to the north and another to the ſouth, when ſuſpended by a thread that does not twiſt. 3. It communicates all its properties to a piece of ſteel when rubbed upon it, without loſing any itſelf. Magnetiſm.

According to Dr. *Helſham*'s experiments, the attraction of the loadſtone decreaſes as the ſquare of the diſtance increaſes. Thus, if a loadſtone be ſuſpended at one end of a balance, and counterpoiſed by weights at the other; and a flat piece of iron placed beneath it, at the diſtance of four tenths of an inch, the ſtone will immediately deſcend and adhere to the iron. But if the ſtone be again removed to the ſame diſtance, and as

many grains be put into the ſcale at the other end as will exactly counterbalance the attraction, then, if the iron be brought twice as near the ſtone as before, that is, only two tenth parts of an inch from it, there muſt be four times as many grains put into the ſcale as before, in order to be a juſt counterbalance to the attractive force, or to hinder the ſtone from deſcending and adhering to the iron. So, if four grains will do in the former caſe, there muſt be ſixteen in the latter. But from ſome later experiments, made with the greateſt accuracy, it is found that the force of magnetiſm decreaſes in a ratio between the reciprocal of the ſquare and the reciprocal of the cube of the diſtance; approaching to the one or the other, as the magnitudes of the attracting bodies are varied.

Electricity. Several bodies, particularly amber, glaſs, jet, ſealing-wax, agate, and almoſt all precious ſtones, have a peculiar property of attracting and repelling light bodies when heated by rubbing. This is called *electrical attraction*, in which the chief things to be obſerved are, 1. if a glaſs tube about an inch and an half in diameter, and two or three feet long, be heated by rubbing, it will alternately attract and repel all light bodies at the diſtance of 10 or 15 inches. 2. It does not attract by being heated without rubbing. 3. Any light body being once repelled by the tube, will never be attracted again, till it has touched ſome other body. 4. If the tube be rubbed by a moiſt hand, or any thing that is wet, it totally deſtroys the electricity. 5. Any body except air, being interpoſed, ſtops the

PLATE II.



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the electricity. 6. The tube attracts ſtronger when rubbed over with bees-wax, and then with a dry woollen cloth. 7. When it is well rubbed, if a finger be brought near it, at about the diſtance of half an inch, the effluvia will ſnap againſt the finger, and make a little crackling noiſe; and if this be performed in a dark place, there will appear a little flaſh of light.

LECT. II.

Of central forces.

WE have already mentioned it as a neceſſary conſequence ariſing from the deadneſs or inactivity of matter, that all bodies endeavour to continue in the ſtate they are in, whether of reſt or motion. If the body *A* were placed in any part of free ſpace, where nothing either draws or impels it any way, it would for ever remain in that part of ſpace, becauſe it could have no tendency of itſelf to remove any way from thence. If it receive a ſingle impulſe any way, as ſuppoſe from *A* towards *B*, it will go on in that direction; for, of itſelf it could never ſwerve from a right line, nor ſtop its courſe.—When it has gone through the ſpace *AB*, and met with no reſiſtance, its velocity will be the ſame at *B* as it was at *A*: and this velocity, in as much more time, will carry it thro' as much more ſpace, from *B* to *C*; and ſo on for ever. Therefore, when we ſee a body in motion, we conclude that ſome other ſubſtance muſt have

Motion or reſt equally indifferent to all bodies.

Plate II. Fig. 1.

given it that motion; and when we see a body fall from motion to rest, we conclude that some other body or cause stopt it.

All motion naturally rectilineal.

As all motion is naturally rectilineal, it appears, that a bullet projected by the hand, or shot from a cannon, would for ever continue to move in the same direction it received at first, if no other power diverted its course. Therefore, when we see a body move in a curve of any kind whatever, we conclude it must be acted upon by two powers at least; one to put it in motion, and another drawing it off from the rectilineal course it would otherwise have continued to move in: and whenever that power which bent the motion of the body from a straight line into a curve, ceases to act, the body will again move on in a straight line touching that point of the curve in which it was when the action of that power ceased. For example, a pebble moved round in a sling ever so long a time, will fly off the moment it is set at liberty, by slipping one end of the sling-cord; and will go on in a line touching the circle it described before: which line would actually be a straight one, if the earth's attraction did not affect the pebble, and bring it down to the ground. This shews that the natural tendency of the pebble, when put into motion, is to continue moving in a straight line, although by the force of the sling it be made to revolve in a circle.

The effects of combined forces.

The change of motion produced is in proportion to the force impressed: for the effects of natural causes are always proportionate to the force or power of those causes.

By

By these laws it is easy to prove that a body will describe the diagonal of a square or parallelogram, by two forces conjoined, in the same time that it would describe either of the sides, by one force singly. Thus, suppose the body *A* to represent a ship at sea; and that it is drove by the wind in the right line *AB*, with such a force as would carry it uniformly from *A* to *B* in a minute: then, suppose a stream or current of water running in the direction *AD*, with such a force as would carry the ship through an equal space from *A* to *D* in a minute. By these two forces, acting together at right angles to each other, the ship will describe the line *AEC* in a minute: which line (because the forces are equal) will be the diagonal of an exact square. To confirm this law by an experiment, let there be a wooden square *ABCD* so contrived, as to have the part *BEFC* made to draw out or push into the square at pleasure. To this part let the pulley *H* be joined, so as to turn freely on an axis, which will be at *H* when the piece is pushed in, and at *h* when it is drawn out. To this part let the ends of a straight wire *k* be fixed, so as to move along with it, under the pulley; and let the ball *G* be made to slide easily on the wire. A thread *m* is fixed to this ball, and goes over the pulley to *I*; by this thread the ball may be drawn up on the wire, parallel to the side *AD*, when the part *BEFC* is pushed as far as it will go into the square. But, if this part be drawn out, it will carry the ball along with it, parallel to the bottom of the square *DC*. By this means, the ball *G* may either be drawn

Fig. 2.

Fig. 3.

perpendicularly upward by pulling the thread *m*, or moved horizontally along by pulling out the part *BEFC*, in equal times, and through equal ſpaces; each power acting equably and ſeparately upon it. But if, when the ball is at *G*, the upper end of the thread be tied to the pin *I*, in the corner *A* of the fixed ſquare, and the moveable part *BEFG* be drawn out, the ball will then be acted upon by both the powers together: for it will be drawn up by the thread towards the top of the ſquare, and at the ſame time carried with its wire *k* towards the right hand *BC*, moving all the while in the diagonal line *L*; and will be found at *g* when the ſliding part is drawn out as far as it was before; which then, will have cauſed the thread to draw up the ball to the top of the inſide of the ſquare, juſt as high as it was before, when drawn up ſingly by the thread without moving the ſliding part.

If the acting forces are equal, but at oblique angles to each other, ſo will the ſides of the parallelogram be: and the diagonal run through by the moving body will be longer or ſhorter, according as the obliquity is greater or ſmaller. Thus, if two equal forces act conjointly upon the body *A*, one having a tendency to move it through the ſpace *AB* in the ſame time that the other has a tendency to move it through an equal ſpace *AD*; it will deſcribe the diagonal *AGC* in the ſame time that either of the ſingle forces would have cauſed it to deſcribe either of the ſides. If one of the forces be greater than the other, then one ſide of the parallelogram

Fig. 4.

rallelogram will be ſo much longer than the other. For, if one force ſingly would carry the body through the ſpace *AE*, in the ſame time that the other would have carried it through the ſpace *AD*, the joint action of both will carry it in the ſame time through the ſpace *AHF*, which is the diagonal of the oblique parallelogram *ADEF*.

If both forces act upon the body in ſuch a manner, as to move it uniformly, the diagonal deſcribed will be a ſtraight line; but if one of the forces acts in ſuch a manner as to make the body move faſter and faſter as it goes forward, then the line deſcribed will be a curve. And this is the caſe of all bodies which are projected in rectilineal directions, and at the ſame time acted upon by the power of gravity, which has a conſtant tendency to accelerate their motions in the direction wherein it acts.

The laws of the planetary motions.

From the uniform projectile motion of bodies in ſtraight lines, and the univerſal power of gravity or attraction, ariſes the curvilineal motion of all the heavenly bodies. If the body *A* be projected along the ſtraight line *AFH* in open ſpace, where it meets with no reſiſtance, and is not drawn aſide by any power, it will go on for ever with the ſame velocity, and in the ſame direction. But if, at the ſame moment the projectile force is given it at *A*, the body *S* begins to attract it with a force duly adjuſted *, and perpendicular to its motion at *A*, it

Fig. 5.

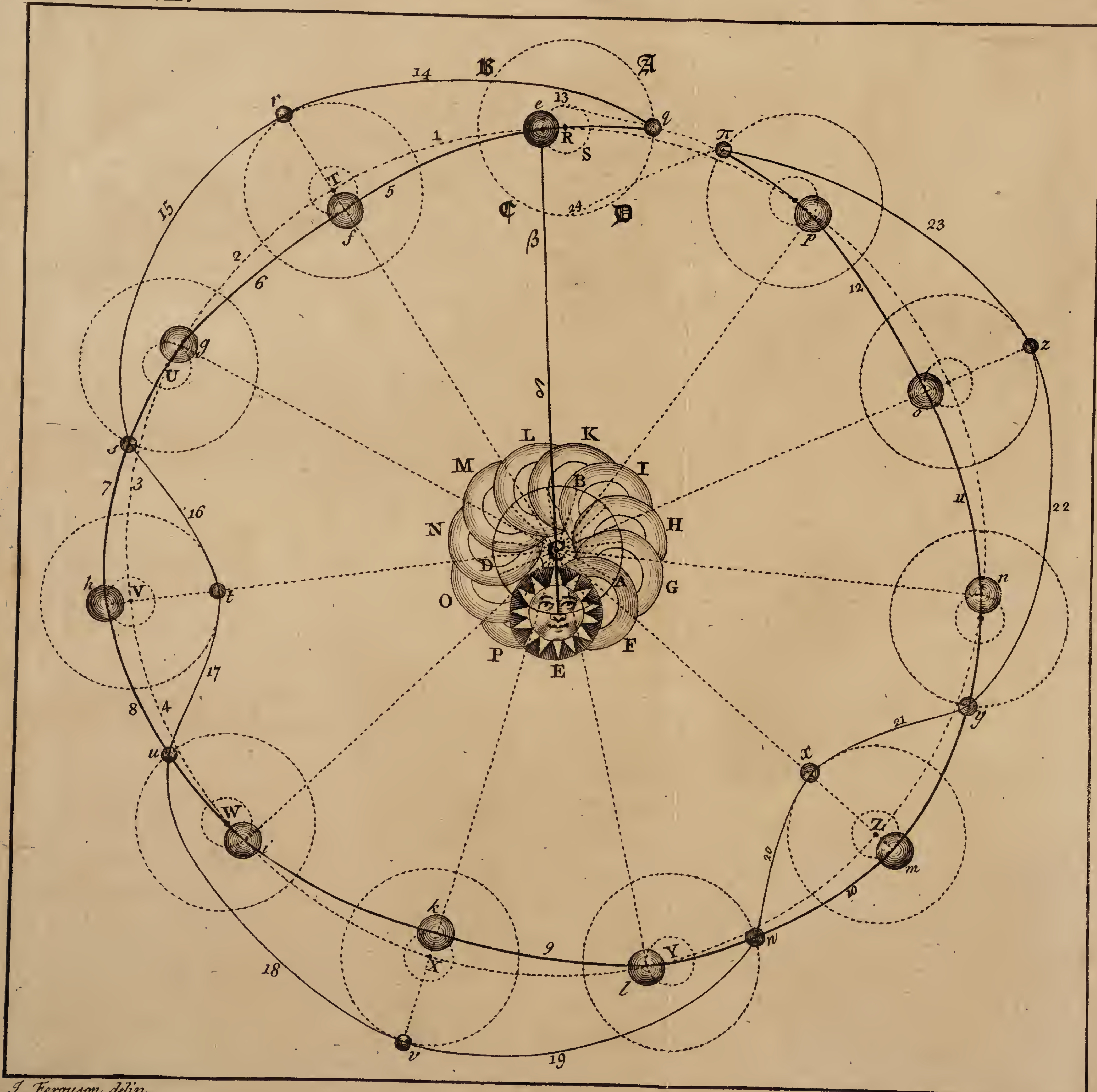
* To make the projectile force a juſt balance to the gravitating power, ſo as to keep the planet moving in a circle, it muſt

it will then be drawn from the ſtraight line *AFH*, and forced to revolve about *S* in the circle *ATW*; in the ſame manner, and by the ſame law, that a pebble is moved round in a ſling. And if, when the body is in any part of its orbit, (as ſuppoſe at *K*) a ſmaller body as *L*, within the ſphere of attraction of the body *K*, be projected in the right line *LM*, with a force duly adjuſted, and perpendicular to the line of attraction *LK*; then, the ſmall body *L* will revolve about the large body *K* in the orbit *NO*, and accompany it in its whole courſe round the yet larger body *S*. But then, the body *K* will no longer move in the circle *ATW*; for that circle will now be deſcribed by the common center of gravity between *K* and *L*. Nay, even the great body *S* will not keep in the center; for it will be the common center of gravity between all the three bodies *S*, *K*, and *L*, that will remain immoveable there. So, if we ſuppoſe *S* and *K* connected by a wire *P* that has no weight, and *K* and *L* connected by a wire *q* that has no weight, the common center of gravity of all theſe three bodies will be a point in the wire *P* near *S*; which point being ſupported, the bodies will be all in *equilibrio* as they move round it. Though indeed, ſtrictly ſpeaking, the common center of gravity of all the three bodies will not be in the wire *P* but when theſe bodies are all in a right line. Here,

muſt give ſuch a velocity as the planet would acquire by gravity when it had fallen through half the ſemidiameter of that circle,

S may

PLATE III.



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S may repreſent the ſun, K the earth, and L the moon.

In order to form a general idea of the curves deſcribed by two bodies revolving about their common center of gravity, whilſt they themſelves with a third body are in motion round the common center of gravity of all the three; let us firſt ſuppoſe E to be the ſun, and e the earth going round him without any moon; and their moving forces regulated as above. In this caſe, whilſt the earth goes round the ſun in the dotted circle $RTUWX$, &c. the ſun will go round the circle ABD, whoſe center C is the common center of gravity between the ſun and earth: the right line $\beta\delta$ repreſenting the mutual attraction between them, by which they are as firmly connected as if they were fixed at the two ends of an iron bar ſtrong enough to hold them. So, when the earth is at e the ſun will be at E; when the earth is at f the ſun will be at F, and when the earth is at g the ſun will be at G, &c.

See Plate III.

The curves deſcribed by bodies revolving about their common center of gravity.

Now, let us take in the moon q (at the top of the figure) and ſuppoſe the earth to have no progreſſive motion about the ſun; in which caſe, whilſt the moon revolves about the earth in her orbit 𝔄𝔅ℭ𝔇, the earth will revolve in the circle S 13, whoſe center R is the common center of gravity of the earth and moon; they being connected by the mutual attraction between them in the ſame manner as the earth and ſun are.

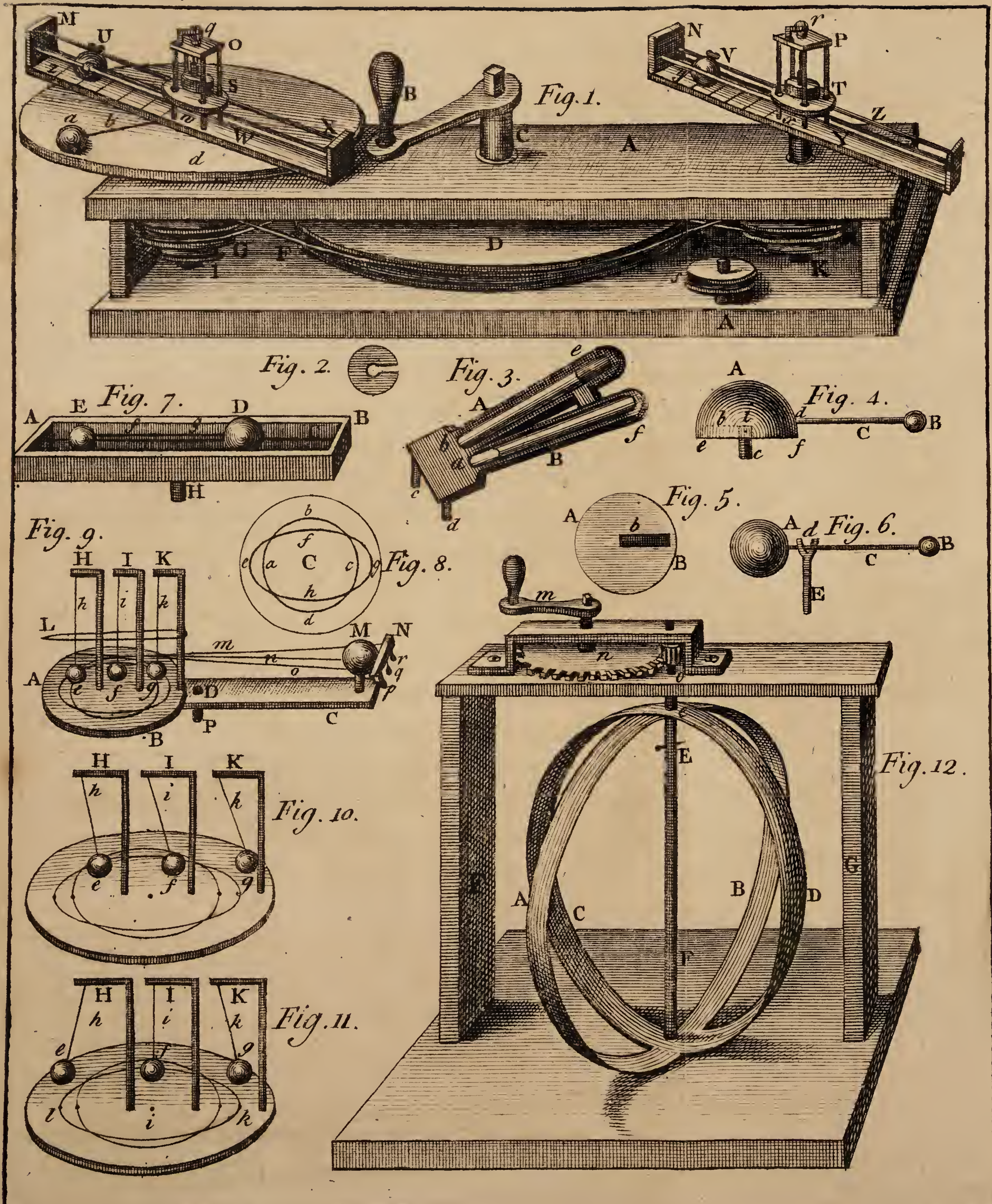
But the truth is, that whilſt the moon revolves about the earth, the earth is in motion about the ſun:

ſun: and now, the moon will cauſe the earth to deſcribe an irregular curve, and not a true circle, round the ſun. For, it is the common center of gravity of the earth and moon that will deſcribe the ſame circle which the earth would have moved in, if it had not been attended by a moon. For, ſuppoſing the moon to deſcribe a quarter of her progreſſive orbit about the earth in the time that the earth moves from *e* to *f*; it is plain, that when the earth comes to *f* the moon will be found at *r*; in which time, their common center of gravity will have deſcribed the dotted arc *R* 1 *T*, the earth the curve *R* 5 *f*, and the moon the curve *q* 14 *r*. In the time that the moon deſcribes another quarter of her orbit, the center of gravity of the earth and moon will deſcribe the dotted arc *T* 2 *U*, the earth the curve *f* 6 *g*, and the moon the curve *r* 15 *s*, and ſo on.—And thus, whilſt the moon goes once round the earth in her progreſſive orbit, their common center of gravity deſcribes the regular portion of a circle *R* 1 *T* 2 *U* 3 *V* 4 *W*, the earth the irregular curve *R* 5 *f* 6 *g* 7 *h* 8 *i*, and the moon the yet more irregular curve *q* 14 *r* 15 *s* 16 *t* 17 *u*; and then, the ſame kind of tracks over again.

The center of gravity of the earth and moon is 6000 miles from the earth's center towards the moon; therefore, the circle *S* 13 which the earth deſcribes round that center of gravity (in every courſe of the moon round her orbit) is 12000 miles in diameter. Conſequently, the earth is 12000 miles nearer the ſun at the time of full moon than at the time of new. [See the earth at *f* and at *h*.]

To

PLATE IV.



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To avoid confufion in fo fmall a figure, we have fuppofed the moon to go only twice and an half round the earth in the time that the earth goes once round the fun: it being impoffible to take in all the revolutions which fhe makes in a year, and to give a true figure of her path, unlefs we fhould make the femidiameter of the earth's orbit at leaft 84 inches; and then, the proportional femidiameter of the moon's orbit would be only a quarter of an inch.—For a true figure of the moon's path I refer the reader to my treatife of aftronomy.

If the moon made any compleat number of revolutions about the earth in the time that the earth makes one revolution about the fun, the paths of the fun and moon would return into themfelves at the end of every year; and fo be the fame over again: but they return not into themfelves in lefs than 19 years nearly; in which time the earth makes nearly 19 revolutions about the fun, and the moon 235 about the earth.

If the planet *A* be attracted towards the fun with fuch a force as would make it fall from *A* to *B* in the time that the projectile impulfe would have carried it from *A* to *F*, it will defcribe the arc *AG* by the combined action of thefe forces, in the fame time that the former would have caufed it to fall from *A* to *B*, or the latter have carried it from *A* to *F*. But, if the projectile force had been twice as great, that is, fuch as would have carried the planet from *A* to *H* in the fame time that now, by the fuppofition, it carries it only from *A* to *F*; the fun's attraction muft then have been four times as

Plate II. Fig. 5.

A double projectile force balances a quadruple power of gravity.

as ſtrong as formerly, to have kept the planet in the circle *ATW*; that is, it muſt have been ſuch as would have cauſed the planet to fall from *A* to *E*, which is four times the diſtance of *A* from *B*, in the time that the projectile force ſingly would have carried it from *A* to *H*, which is only twice the diſtance of *A* from *F* *. Thus, a double projectile force will balance a quadruple power of gravity in the ſame circle; as appears plain by the figure, and ſhall ſoon be confirmed by an experiment.

Plate IV. Fig. 1.

The whirling-table deſcribed.

The whirling-table is a machine contrived for ſhewing experiments of this nature. *AA* is a ſtrong frame of wood, *B* a winch or handle fixed on the axis *C* of the wheel *D*, round which is the catgut ſtring *EF*, which alſo goes round the ſmall wheels *G* and *H*, croſſing between them and the great wheel *D*. On the upper end of the axis of the wheel *G*, above the frame, is fixed the round board *d*, to which the bearer *MSX* may be faſtened occaſionally, and removed when it is not wanted. On the axis of the wheel *H* is fixed the bearer *NTZ*: and it is eaſy to ſee that when the winch is turned, the wheels and bearers are put into a whirling motion.

Each bearer hath two wires *W*, *X*, and *Y*, *Z*, fixed and ſcrewed tight into them at the ends by nuts on the outſide. And when theſe nuts are unſcrewed, the wires may be drawn out, in order

* Here the arcs *AG*, *AI* muſt be ſuppoſed to be very ſmall; otherwiſe *AE*, which is equal to *HI*, will not be quadruple to *AB*, which is equal to *FG*.

to

to change the balls *U* and *V*, which ſlide upon the wires by means of loops fixed into the balls, and keep them up from touching the wood below them. A ſtrong ſilk line goes through each ball, and is fixt to it at any length from the center of the bearer to its end, as occaſion requires, by a nut-ſcrew at the top of the ball; the ſhank of the ſcrew going into the center of the ball and preſſing the line againſt the under ſide of the hole that it goes through.—The line goes from the ball, and under a ſmall pulley fixt in the middle of the bearer; then up through a ſocket in the round plate (ſee *S* and *T*) in the middle of each bearer; then through a ſlit in the middle of the ſquare top (*O* and *P*) of each tower, and going over a ſmall pulley on the top, comes down again the ſame way, and is at laſt faſtened to the upper end of the ſocket fixt in the middle of the above-mentioned round plate. Theſe plates *S* and *T* have each four round holes near their edges for letting them ſlide up and down upon the wires which make the corners of each tower. The balls and plates being thus connected, each by its particular line, it is plain that if the balls be drawn outward, or towards the ends *M* and *N* of their reſpective bearers, the round plates *S* and *T* will be drawn up to the top of their reſpective towers *O* and *P*.

There are ſeveral braſs weights, ſome of two ounces, ſome of three, and ſome of four, to be Fig. 2. occaſionally put within the towers *O* and *P*, upon the round plates *S* and *T*: each weight having a round hole in the middle of it, for going upon the

ſockets

ſockets or axes of the plates, and is ſlit from the edge to the hole, for allowing it to be ſlipt over the foreſaid line which comes from each ball to its reſpective plate.

The experiments to be made by this machine are,

Fig. 1. 1. Take away the bearer *MX*, and take the ivory ball *a* to which the line or ſilk cord *b* is faſtened at one end; and having made a loop on the other end of the cord, put the loop over a pin fixt in the center of the board *d*. Then, turning the winch *B* to give the board a whirling motion, you will ſee that the ball does not immediately begin to move with the board; but, on account of its inactivity, endeavours to continue in the ſtate of reſt which it was in before.—Continue turning, until the board communicates an equal degree of motion with its own to the ball, and then turning on, you will perceive that the ball will remain upon one part of the board, keeping the ſame velocity with it, and having no relative motion upon it: as is the caſe with every thing that lies looſe upon the plane ſurface of the earth, which having the motion of the earth communicated to it, never endeavours to remove from that place. But ſtop the board ſuddenly by hand, and the ball will go on, and continue to revolve upon the board, until the friction thereof ſtops its motion: which ſhews, that matter being once put into motion, would continue to move for ever, if it met with no reſiſtance. In like manner, if a perſon ſtands upright in a boat before it begins to move, he can ſtand firm; but the moment the boat ſets off,

The propenſity of matter to keep the ſtate it is in.

off, he is in danger of falling towards that place which the boat departs from: becauſe, as matter, he has no natural propenſity to move. But, when he acquires the motion of the boat, let it be ever ſo ſwift, if it be ſmooth and uniform, he will ſtand as upright and firm as if he was on the plane ſhore: and if the boat ſtrike againſt any obſtacle, he will fall towards that obſtacle; on account of the propenſity he has, as matter, to keep the motion which the boat has put him into.

2. Take away this ball, and put a longer cord to it, which may be put down through the hollow axis of the bearer *MX* and wheel *G*, and fix a weight to the end of the cord below the machine; which weight, if left at liberty, will draw the ball from the edge of the whirling board to its center.

Bodies moving in orbits have a tendency to fly out of theſe orbits.

Draw off the ball a little from the center, and turn the winch; then the ball will go round and round with the board, and will gradually fly off farther and farther from the center, and raiſe up the weight below the machine: which ſhews that all bodies revolving in circles have a tendency to fly off from theſe circles, and muſt have ſome power acting upon them from the center of motion, to keep them from ſo flying off. Stop the machine, and the ball will continue to revolve for ſome time upon the board; but as the friction gradually ſtops its motion, the weight acting upon it will bring it nearer and nearer to the center in every revolution, until it brings it quite thither. This ſhews, that if the planets met with any reſiſtance in going round the ſun, its attractive power would bring

them nearer and nearer to it in every revolution, until they fell into it.

Bodies move faster in small orbits than in large ones.

3. Take hold of the cord below the machine with one hand, and with the other throw the ball upon the round board as it were at right angles to the cord, by which means it will go round and round upon the board. Then, observing with what velocity it moves, pull the cord below the machine, which will bring the ball nearer to the center of the board, and you will see that the nearer the ball is drawn to the center, the faster it will revolve; as those planets which are nearest the sun revolve faster than those which are more remote; and not only go round sooner, because they describe smaller circles, but even move faster in every part of their respective circles.

Their centrifugal forces shewn.

4. Take away this ball, and apply the bearer *MX*, whose center of motion is in its middle at *w*, directly over the center of the whirling board *d*. Then, put two balls (*U* and *V*) of equal weights upon their bearing wires, and having fixed them at equal distances from their respective centers of motion *w* and *x* upon their silk cords, by the screw nuts, put equal weights in the towers *O* and *P*. Lastly, put the catgut strings *E* and *F* upon the grooves *G* and *H*; which, being of equal diameters, will give equal velocities to the bearers above, when the winch *B* is turned: and the balls *U* and *V* will fly off towards *M* and *N*; and will raise the weights in the towers at the same instant. This shews, that when bodies of equal quantities of

of matter revolve in equal circles with equal velocities, their centrifugal forces are equal.

5. Take away these equal balls, and inftead of them, put a ball of fix ounces into the bearer *MX*, at a fixth part of the diftance *wz* from the center; and put a ball of one ounce into the oppofite bearer, at the whole diftance *xy*, which is equal to *wx*, from the center of the bearer; and fix the balls at thefe diftances on their cords, by the fcrew nuts at top: then the ball *U*, which is fix times as heavy as the ball *V*, will be at only a fixth part of the diftance from its center of motion; and confequently will revolve in a circle of only a fixth part of the circumference of the circle in which *V* revolves. Now, let any equal weights be put into the towers, and the machine turned by the winch; which, (as the catgut ftring is on equal wheels below) will caufe the balls to revolve in equal times; but *V* will move fix times as faft as *U*, becaufe it revolves in a circle of fix times its radius; and both the weights in the towers will rife at once. This fhews, that the centrifugal forces of revolving bodies (or their tendencies to fly off from the circles they defcribe) are in direct proportion to their quantities of matter multiplied into their refpective velocities; or into their diftances from the centers of their refpective circles. For, fuppofing *U*, which weighs 6 ounces, to be two inches from its center of motion *w*; the weight multiplied by the diftance is 12: and fuppofing *V*, which weighs only one ounce, to be 12 inches diftant from its center of motion *x*, the weight 1 ounce multiplied

 by

by the diſtance 12 inches is 12. And as they revolve in equal times, their velocities are as their diſtances from the center, namely, as 1 to 6.

If theſe two balls be fixed at equal diſtances from their reſpective centers of motion, they will move with equal velocities; and if the tower *O* has 6 times as much weight put into it as the tower *P* has; the balls will raiſe their weights exactly at the ſame moment. This ſhews that the ball *U* being ſix times as heavy as the ball *V*, it has ſix times as much centrifugal force, in deſcribing an equal circle with an equal velocity.

A double velocity in the ſame circle, is a balance to a quadruple power of gravity.

6. If bodies of equal weights revolve in equal circles with unequal velocities, their centrifugal forces are as the ſquares of the velocities. To prove this law by an experiment, let two balls *U* and *V* of equal weights be fixed on their cords at equal diſtances from their reſpective centers of motion *w* and *x*; and then, let the catgut ſtring *E* be put round the wheel *K*, (whoſe circumference is only one half of the circumference of the wheel *H* or *G*) and over the pulley *s* to keep it tight: and let four times as much weight be put into the tower *P*, as in the tower *O*. Then, turn the winch *B*, and the ball *V* will revolve twice as faſt as the ball *U*, in a circle of the ſame diameter, becauſe they are equidiſtant from the centers of the circles in which they revolve; and the weights in the towers will both riſe at the ſame inſtant, which ſhews that a double velocity in the ſame circle will exactly balance a quadruple power of attraction at the center of the circle. For the weights in the

towers

towers may be confidered as the attractive forces in the centers, acting upon the revolving balls; which, moving in equal circles, is the fame thing as if they both moved in one and the fame circle. Confequently, if the planets had moved twice as faft round the fun as they do, the fun's attraction muft have been four times as great as it now is, to have retained the planets in their orbits. So that the fun muft have either been four times as big as he now is, or four times as denfe under the fame bulk he now has, or four times as far from the common center of gravity of the whole fyftem as he now is, in order to have preferved a juft balance between the attractive and projectile forces.

7. If bodies of equal weights revolve in unequal circles, in fuch a manner that the fquares of the times of their going round are as the cubes of their diftances from the centers of the circles they defcribe; their centrifugal forces are inverfely as the fquares of their diftances from thofe centers. For, the catgut ftring remaining as in the laft experiment, let the diftance of the ball *V* from the center *x* be made equal to two of the crofs divifions on its bearer; and the diftance of the ball *U* from the center *w* be three and a fixth part; the balls themfelves being of equal weights, and *V* making two revolutions, by turning the winch, in the time that *U* makes one: fo that if we fuppofe the ball *V* to revolve in one moment, the ball *U* will revolve in two moments, the fquares of which are one and four: for the fquare of 1 is only 1, and the fquare of 2 is 4; therefore, the fquare of the *Kepler's* Problem.

 period

period or revolution of the ball *V* is contained four times in the ſquare of the period of the ball *U*. But the diſtance of *V* is 2, the cube of which is 8, and the diſtance of *U* is $3\frac{1}{6}$, the cube of which is 32 very nearly; in which 8 is contained four times: and therefore, the ſquares of the periods of *V* and *U* are to one another as the cubes of their diſtances from *x* and *w*, which are the centers of their reſpective circles. And if the weight in the tower *O* be four ounces, equal to the ſquare of 2, the diſtance of *V* from the center *x*; and the weight in the tower *P* 10 ounces, nearly equal to the ſquare of $3\frac{1}{6}$, the diſtance of *U* from *w*; it will be found upon turning the machine by the winch, that the balls *U* and *V* will raiſe their reſpective weights very nearly at the ſame inſtant of time. Which confirms that famous propoſition of KEPLER, viz. that the ſquares of the periodical times of the planets round the ſun are in proportion to the cubes of their diſtances from him; and that the ſun's attraction is inverſely as the ſquare of the diſtance from his center: that is, at twice the diſtance his attraction is four times leſs; and thrice the diſtance, nine times leſs; at four times the diſtance, ſixteen times leſs; and ſo on, to the remoteſt parts of the ſyſtem.

8. Take off the catgut ſtring *E* from the great wheel *D* and the ſmall wheel *H*, and let the ſtring *F* remain upon the wheels *D* and *G*. Take away alſo the bearer *MX* from the whirling-board *d*, and
Fig. 3. inſtead thereof put the machine *AB* upon it, fixing this machine to the center of the board by the

the pins *c* and *d*, in ſuch a manner, that the end *ef* may riſe above the board to an angle of 30 or 40 degrees. In the upper ſide of this machine there are two glaſs tubes *a* and *b*, cloſe ſtopt at both ends; and each tube is about three quarters full of water. In the tube *a* is a little quickſilver, which naturally falls down to the end *a* in the water, becauſe it is heavier than its bulk of water; and on the tube *b* is a ſmall cork which floats upon the top of the water at *e*, becauſe it is lighter; and is ſmall enough to have liberty to riſe or fall in the tube. While the board *b* with this machine upon it continues at reſt, the quickſilver lies at the bottom of the tube *a*, and the cork floats on the water near the top of the tube *b*. But, upon turning the winch, and putting the machine in motion, the contents of each tube will fly off towards the uppermoſt ends (which are fartheſt from the center of motion) the heavieſt with the greateſt force. Therefore, the quickſilver in the tube *a* will fly off quite to the end *f*, and occupy its bulk of ſpace there, excluding the water from that place, becauſe it is lighter than quickſilver; but the water in the tube *b* flying off to its higher end *e*, will exclude the cork from that place; and cauſe the cork to deſcend towards the lowermoſt end of the tube, where it will remain upon the loweſt end of the water near *b*; for the heavier body having the greater centrifugal force, will therefore poſſeſs the uppermoſt part of the tube; and the lighter body will keep between the heavier and the lowermoſt part.

The abſurdity of the *Carteſian* vortexes.

 This

This demonſtrates the abſurdity of the *Carteſian* doctrine of the planets moving round the ſun in vortexes: for, if the planet be more denſe or heavy than its bulk of the vortex, it will fly off therein, farther and farther from the ſun; if leſs denſe, it will come down to the loweſt part of the vortex, at the ſun: and the whole vortex itſelf muſt be ſurrounded with ſomething like a great wall, otherwiſe it would fly quite off, planets and all together.—But while gravity exiſts, there is no occaſion for ſuch vortexes; and when it ceaſes to exiſt, a ſtone thrown upwards will never return to the earth again.

If one body moves round another, both of them muſt move round their common center of gravity.

9. If a body be ſo placed upon the whirling-board of the machine (Fig. 1.) that the center of gravity of the body be directly over the center of the board, and the board be put into ever ſo rapid a motion by the winch *B*, the body will turn round with the board, but will not remove from the middle of it: for, as all parts of the body are in *equilibrio* round its center of gravity, and the center of gravity is at reſt in the center of motion, the centrifugal force of all parts of the body will be equal at equal diſtances from its center of motion; and therefore the body will remain in its place. But if the center of gravity be placed ever ſo little out of the center of motion, and the machine be turned ſwiftly round, the body will fly off towards that ſide of the board on which its center of gravity lies. Thus, if the wire *C* with its little ball *B* be taken away from the demi-globe *A*, and the flat ſide *ef* of this demi globe be laid upon the whirling-

Fig. 4.

whirling-board of the machine, ſo as their centers may coincide; if then the board be turned ever ſo quick by the winch, the demi-globe will remain where it was placed. But if the wire *C* be ſcrewed into the demi-globe at *d*, the whole becomes one body, whoſe center of gravity is now at or near *d*. Let the pin *c* be fixed in the center of the whirling-board, and the deep groove *b* cut in the flat ſide of the demi-globe be put upon the pin, ſo as the pin may be in the center of *A*, [See Fig. 5. where this groove is repreſented at *b*] and let the whirling-board be turned by the winch, which will carry the little ball *B* (Fig. 4.) with its wire *C*, and the demi-globe *A*, all round the center-pin *ci* and then, the centrifugal force of the little ball *B*, which weighs only one ounce, will be ſo great, as to draw off the demi-globe *A*, which weighs two pounds, until the end of the groove at *e* ſtrikes againſt the pin *c*, and ſo prevents *A* from going any farther: otherwiſe, the centrifugal force of *B* would have been great enough to have carried *A* quite off the whirling-board. Which ſhews, that if the ſun were placed in the very center of the orbits of the planets, it could not poſſibly remain there; for the centrifugal forces of the planets would carry them quite off, and the ſun with them; eſpecially when ſeveral of them happened to be in any one quarter of the heavens. For the ſun and planets are as much connected by the mutual attraction that ſubſiſts between them, as the bodies *A* and *B* are by the wire *C* which is fixed into them both. And even if there were but one ſingle planet

Fig. 5.

planet in the whole heavens to go round ever ſo large a ſun in the center of its orbit, its centrifugal force would ſoon carry off both itſelf and the ſun. For, the greateſt body placed in any part of free ſpace could be eaſily moved: becauſe if there were no other body to attract it, it could have no weight or gravity of itſelf; and conſequently, though it could have no tendency of itſelf to remove from that part of ſpace, yet it might be very eaſily moved by any other ſubſtance.—And perhaps it was this conſideration which made the celebrated ARCHIMEDES ſay, that if he had a proper place at ſome diſtance from the earth whereon to fix his machinery, he could move the whole earth.

10. As the centrifugal force of the light body *B* will not allow the heavy body *A* to remain in the center of motion, even though it be 24 times heavier than *B*; let us now take the ball *A* (Fig. 6.) which weighs only 6 ounces, and connect it by the wire *C* with the ball *B*, which weighs only one ounce; and let the fork *E* be fixed into the center of the whirling-board: then, hang the balls upon the fork by the wire *C* in ſuch a manner, that they may exactly balance each other; which will be when the center of gravity between them, in the wire at *d*, is ſupported by the fork. And this center of gravity is as much nearer to the center of the ball *A*, than to the center of the ball *B*, as *A* is heavier than *B*, allowing for the weight of the wire on each ſide of the fork. This done, let the machine be put into motion by the winch; and the balls *A* and *B* will go round their common center

Fig. 6.

center of gravity *d*, keeping their balance, because either will not allow the other to fly off with it. For, ſuppoſing the ball *B* to be only one ounce in weight, and the ball *A* to be ſix ounces; then, if the wire *C* were equally heavy on each ſide of the fork, the center of gravity *d* would be ſix times as far from the center of the ball *B* as from the center of the ball *A*, and conſequently *B* will revolve with a velocity ſix times as great as *A* does; which will give *B* ſix times as much centrifugal force as any ſingle ounce of *A* has: but then, as *B* is only one ounce, and *A* ſix ounces, the whole centrifugal force of *A* will exactly balance the whole centrifugal force of *B*: and therefore, each body will detain the other ſo as to make it keep in its circle. This ſhews that the ſun and planets muſt all move round the common center of gravity of the whole ſyſtem, in order to preſerve that juſt balance which takes place among them. For, the planets being as unactive and dead as the above balls, they could no more have put themſelves into motion than theſe balls can: nor have kept in their orbits without being balanced at firſt with the greateſt degree of exactneſs upon their common center of gravity, by the Almighty hand that made them and put them in motion.

Perhaps it may be here aſked, ſince theſe balls cannot go round unleſs their center of gravity be ſupported by the prop or fork *E*; what prop it is that ſupports the center of gravity of the ſolar ſyſtem, and conſequently bears the weight of all the bodies in it; and by what the prop itſelf is ſupported?

ſupported? The anſwer is eaſy and plain; for the center of gravity of our balls muſt be ſupported, becauſe they gravitate towards the earth, and would therefore fall to it: but as the ſun and planets gravitate only towards one another, they have nothing elſe to fall to; and therefore have no occaſion for any thing to ſupport their common center of gravity: and if they did not move round that center, and conſequently acquire a tendency to fly off from it by their motions, their mutual attractions would ſoon bring them together; and ſo the whole would become one maſs in the ſun: which would alſo be the caſe if their velocities round the ſun were not quick enough to create a centrifugal force equal to the ſun's attraction.

But after all this nice adjuſtment, it appears evident that the Deity cannot withdraw his regulating hand from his works, and leave them to be ſolely governed by the laws which he has impreſt upon them at firſt. For if he would leave them ſo, their order would in time come to an end; becauſe the planets muſt neceſſarily diſturb one another's motions by their mutual attractions, when ſeveral of them are in the ſame quarter of the heavens; as is often the caſe: and then, as they will attract the ſun more towards that quarter than when they are in a manner diſperſed equably around him, if he was not at that time made to deſcribe a portion of a larger circle round the common center of gravity, the balance would be deſtroyed; and as it could never reſtore itſelf again, the whole ſyſtem would begin to fall together, and would in time unite in

a maſs

a mafs at the fun.—Of this difturbance we have a very remarkable inftance in the comet which now appears; and which, in going laft up from the fun, went fo near to Jupiter, and was fo affected by his attraction, as to have the figure of its orbit much changed; and not only fo, but to have its period altered, and its courfe to be different in the heavens from what it was before.

11. Take away the fork and balls from the whirling-board, and place the trough *AB* thereon, fixing its center to the center of the whirling-board by the pin *H*. In this trough are two balls *D* and *E*, of unequal weights, connected by a wire *f*; and made to flide eafily upon the wire *C* ftretched from end to end of the trough, and made faft by nut-fcrews on the outfide of the ends. Let thefe balls be fo placed upon the wire *C*, that their common center of gravity *g* may be directly over the center of the whirling-board. Then, turn the machine by the winch, ever fo fwiftly, and the trough and balls will go round their center of gravity fo as neither of them will fly off; becaufe, on account of the equilibrium, each ball detains the other with an equal force acting againft it. But if the ball *E* be drawn a little more towards the end of the trough at *A*, it will remove the center of gravity towards that end from the center of motion; and then, upon turning the machine, the little ball *E* will fly off, and ftrike with a confiderable force againft the end *A*, and draw the great ball *B* into the middle of the trough. Or, if the great ball *D* be drawn towards the end *B* of the Fig. 7.

the

the trough, fo that the center of gravity may be a little towards that end from the center of motion, and the machine be turned by the winch, the great ball *D* will fly off, and ftrike violently againft the end *B* of the trough, and will bring the little ball *E* into the middle of it. If the trough be not made very ftrong, the ball *D* will break through it.

Of the tides.

12. The reafon why the tides rife at the fame abfolute time on oppofite fides of the earth, and confequently in oppofite directions, is made abundantly plain by a new experiment on the whirling-table. The caufe of their rifing on the fide next the moon every one underftands to be owing to the moon's attraction: but why they fhould rife on the oppofite fide at the fame time, where there is no moon to attract them, is perhaps not fo generally underftood. For it would feem that the moon fhould rather draw the waters (as it were) clofer to that fide, than raife them upon it, directly contrary

Fig. 8.

to her attractive force. Let the circle *abcd* reprefent the earth, with its fide *c* turned toward the moon, which will then attract the waters fo, as to raife them from *c* to *g*. But the queftion is, why fhould they rife as high at that very time on the

Fig. 9.

oppofite fide, from *a* to *e?* In order to explain this, let there be a plate *AB* fixed upon one end of the flat bar *DC*; with fuch a circle drawn upon it as *abcd* (in Fig. 8.) to reprefent the round figure of the earth and fea; and fuch an ellipfis as *defg* to reprefent the fwelling of the tide at *e* and *g*, occafioned by the influence of the moon. Over this

this plate *AB* let the three ivory balls *e*, *f*, *g*, be hung by the ſilk lines *h*, *i*, *k*, faſtened to the tops of the crooked wires *H*, *I*, *K*, in ſuch a manner that the ball at *e* may hang freely over the ſide of the circle *e*, which is fartheſt from the moon *M*, (at the other end of the bar;) the ball at *f* may hang freely over the center, and the ball at *g* hang over the ſide of the circle *g*, which is neareſt the moon. The ball *f* may repreſent the center of the earth, the ball *g* ſome water on the ſide next the moon, and the ball *e* ſome water on the oppoſite ſide. On the back of the moon *M* is fixt the ſhort bar *N* parallel to the horizon, and there are three holes in it above the little weights *p*, *q*, *r*. A ſilk thread *o* is tied to the line *k* cloſe above the ball *g*, and paſſing by one ſide of the moon *M*, goes through a hole in the bar *N*, and has the weight *p* hung to it. Such another thread *n* is tied to the line *i*, cloſe above the ball *f*, and paſſing through the center of the moon *M* and middle of the bar *N*, has the weight *q* hung to it, which is lighter than the weight *p*. A third thread *m* is tied to the line *h*, cloſe above the ball *e*, and paſſing by the other ſide of the moon *M*, through the bar *N*, has the weight *r* hung to it, which is lighter than the weight *q*.

The uſe of theſe three unequal weights is to repreſent the moon's unequal attraction at different diſtances from her. With whatever force ſhe attracts the center of the earth, ſhe attracts the ſide next her with a greater degree of force, and the ſide fartheſt from her with a leſs. So, if the weights

weights are left at liberty, they will draw all the three balls towards the moon with different degrees of force; and cauſe them to make the appearance
Fig. 10. ſhewn in Fig. 10; by which means they are evidently farther from each other than they would be if they hung at liberty by the lines *h*, *i*, *k*; becauſe the lines would then hang perpendicularly. This ſhews, that as the moon attracts the ſide of the earth which is neareſt her with a greater degree of force than ſhe does the center of the earth, ſhe will draw the water on that ſide more than ſhe draws the center, and ſo cauſe it to riſe on that ſide: and as ſhe draws the center more than ſhe draws the oppoſite ſide, the center will recede farther from the ſurface of the water on that oppoſite ſide, and ſo leave it as high there as ſhe raiſed it on the ſide next to her. For, as the center will be in the middle between the tops of the oppoſite elevations, they muſt of courſe be equally high on both ſides at the ſame time.

But upon this ſuppoſition the earth and moon would ſoon come together: and to be ſure they would, if they had not a motion round their common center of gravity, to create a degree of centrifugal force ſufficient to balance their mutual attraction. This motion they have; for as the moon goes round her orbit every month, at the diſtance of 240000 miles from the earth's center, and of 234000 miles from the center of gravity of the earth and moon, ſo does the earth go round the ſame center of gravity every month at the diſtance of 6000 miles from it; that is, from it

to

to the center of the earth. Now as the earth is (in round numbers) 8000 miles in diameter, it is plain that its ſide next the moon is only 2000 miles from the common center of gravity of the earth and moon; its center 6000 miles diſtant therefrom, and its fartheſt ſide from the moon 10000. Therefore the centrifugal forces of theſe parts are as 2000, 6000, and 10000, that is, the centrifugal force of any ſide of the earth when it is turned from the moon is five times as great as when it is turned towards the moon. And as the moon's attraction (expreſt by the number 6000) at the earth's center keeps the earth from flying out of this monthly circle, it muſt be greater than the centrifugal force of the waters next her, and conſequently ſufficient to raiſe them; but as her attraction on the oppoſite ſide is leſs than the centrifugal force of the water there, the exceſs of this force is ſufficient to raiſe the water juſt as high on the oppoſite ſide.—To prove this experimentally, let the bar *DC* with its furniture be fixed upon the whirling-board of the machine (Fig. 1.) by puſhing the pin *P* into the center of the board; which pin is in the center of gravity of the whole bar with its three balls *e*, *f*, *g*, and moon *M*. Now, if the whirling-board and bar be turned ſlowly round by the winch, until the ball *f* hangs over the center of the circle, as in Fig. 11. the ball *g* will be kept towards the moon by the heavieſt weight *p*, (Fig. 9.) and the ball *e*, on account of its greater centrifugal force, and the leſſer weight *r*, will fly off as far to the other ſide, as in Fig. 11. Fig. 9.

 And

And ſo, whilſt the machine is kept turning, the balls *e* and *g* will hang over the ends of the ellipſis *lfk*. So that the centrifugal force of the ball *e* will exceed the moon's attraction juſt as much as her attraction exceeds the centrifugal force of the ball *g*, whilſt her attraction juſt balances the centrifugal force of the ball *f* and makes it keep in its circle. And hence it is evident that the tides muſt riſe to equal heights at the ſame time on oppoſite ſides of the earth. This experiment, to the beſt of my knowledge, is entirely new.

The earth's motion demonſtrated. From the principles thus eſtabliſhed, it is evident that the earth moves round the ſun, and not the ſun round the earth; for the centrifugal law will never allow a great body to move round a ſmall one in any orbit whatever; eſpecially when we find that if a ſmall body moves round a great one, the great one muſt alſo move round the common center of gravity between them two. And it is well known that the quantity of matter in the ſun is 227000 times as great as the quantity of matter in the earth. Now, as the ſun's diſtance from the earth is at leaſt 81,000,000 of miles, if we divide that diſtance by 227000, we ſhall have only 357 for the number of miles that the center of gravity between the ſun and earth is diſtant from the ſun's center. And as the ſun's ſemidiameter is $\frac{1}{4}$ of a degree, which, at ſo great a diſtance as that of the ſun, muſt be no leſs than 381500 miles, if this be divided by 357, the quotient will be $1068\frac{2}{7}$, which ſhews that the common center of gravity is within the body of the ſun; and is only the $1068\frac{2}{7}$ part

part of his ſemidiameter from his center toward his ſurface.

All globular bodies which do not turn on their axes muſt be perfect ſpheres, becauſe all parts of their ſurfaces are equally attracted toward their centers. But all globes which do turn on their axes will be oblate ſpheroids; that is, their ſurfaces will be higher, or farther from the center, in the equatoreal than in the polar regions. For, as the equatoreal parts move quickeſt, they muſt have the greateſt centrifugal force; and will therefore recede fartheſt from the axis of motion. Thus, if two circular hoops *AB* and *CD*, made thin and flexible, and croſſing one another at right angles, be turned round their axis *EF* by means of the winch *m*, the wheel *n*, and pinion *o*; and the axis be looſe in the pole or interſection *e*, the middle parts *A*, *B*, *C*, *D* will ſwell out ſo as to ſtrike againſt the ſides of the frame at *F* and *G*, if the pole *e*, in ſinking to the pin *E*, be not ſtopt by it from ſinking farther: ſo that the whole will appear of an oval figure, the equatoreal diameter being conſiderably longer than the polar. That our earth is of this figure is demonſtrable from actual meaſurement of ſome degrees on its ſurface, which are found to be longer in the frigid zones than in the torrid: and the difference is found to be ſuch as proves the earth's equatoreal diameter to be 35 miles longer than its axis.—Since then, the earth is higher at the equator than at the poles, the ſea, which like all other fluids naturally runs downward (or towards the places which are neareſt the earth's center)

Fig. 12.

center) would run towards the polar regions, and leave the equatoreal parts dry, if the centrifugal force of the water, which carried it to thoſe parts, and ſo raiſed them, did not detain and keep it from running back again towards the poles of the earth.

L E C T. III.

Of the mechanical powers.

The foundation of all mechanics.

IF we conſider bodies in motion, and compare them together, we may do this either with reſpect to the quantities of matter they contain, or the velocities with which they are moved. The heavier any body is, the greater is the power required either to move it or to ſtop its motion: and again, the ſwifter it moves, the greater is its force. So that the whole *momentum* or quantity of force of a moving body is the reſult of its quantity of matter multiplied by the velocity with which it is moved. And when the products ariſing from the multiplication of the particular quantities of matter in any two bodies by their reſpective velocities are equal, the *momenta* or entire forces are ſo too. Thus, ſuppoſe a body, which we ſhall call *A*, to weigh 40 pounds, and to move at the rate of 2 miles in a minute; and another body, which we ſhall call *B*, to weigh only 4 pounds, and to move 20 miles in a minute; the entire forces with which theſe two bodies would ſtrike againſt any obſtacle would be equal to each other, and therefore it would require equal powers to ſtop them. For 40 multiplied

tiplied by 2 gives 80, the force of the body *A*; and 20 multiplied by 4 gives 80, the force of the body *B*.

Upon this eaſy principle depends the whole of mechanics: and it holds univerſally true, that when two bodies are ſuſpended by any machine, ſo as to act contrary to each other; if the machine be put into motion, and the perpendicular aſcent of one body multiplied into its weight, be equal to the perpendicular deſcent of the other body multiplied into its weight, theſe bodies, how unequal ſoever in their weights, will balance one another in all ſituations: for, as the whole aſcent of one is performed in the ſame time with the whole deſcent of the other, their reſpective velocities muſt be directly as the ſpaces they move through; and the exceſs of weight in one body is compenſated by the exceſs of velocity in the other.—Upon this principle it is eaſy to compute the power of any mechanical engine, whether ſimple or compound; for it is but only enquiring how much ſwifter the power moves than the weight (*i. e.* how much farther in the ſame time,) and juſt ſo much muſt the power be increaſed by the help of the engine.

How to compute the power of any mechanical engine.

In the theory of this ſcience, we ſuppoſe all planes perfectly even, all bodies perfectly ſmooth, levers to have no weight, cords to be extremely pliable, machines to have no friction; and in ſhort, all imperfections muſt be ſet aſide until the theory be eſtabliſhed, and then proper allowances are to be made.

The mechanic powers, what?

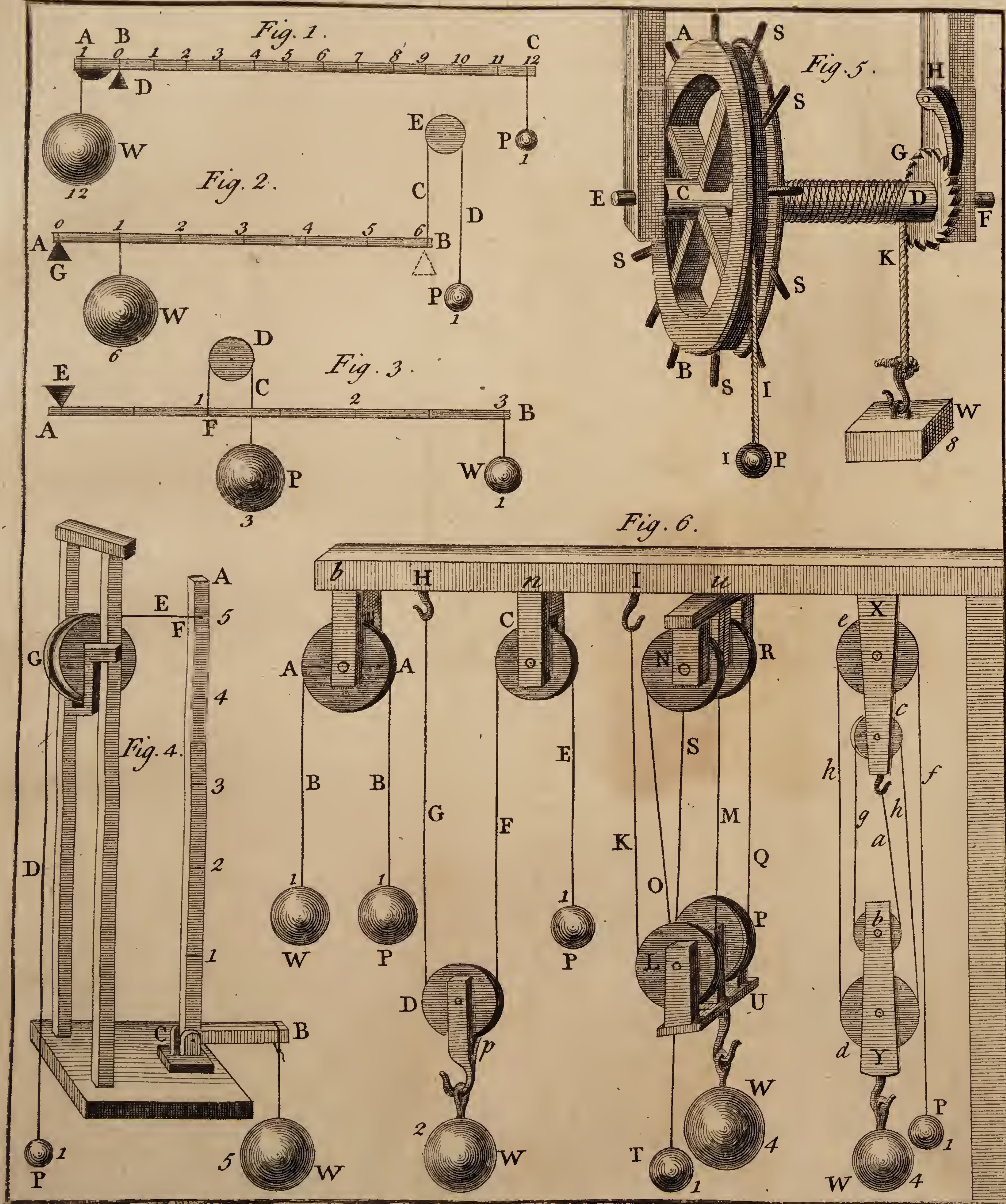
The ſimple *machines* uſually called *mechanical powers* are ſix in number, viz. the *lever*, the *wheel and axle*, the *pulley*, the *inclined plane*, the *wedge*, and the *ſcrew*.—They are called mechanical powers, becauſe they help us to raiſe weights, move heavy bodies, and overcome reſiſtances, which we could not effect without them.

The *lever*.

1. A *lever* is a bar of iron or wood, one part of which being ſupported by a prop, all the other parts turn upon that prop as their center of motion: and the velocity of every part or point is directly as its diſtance from the prop. Therefore, when the weight to be raiſed at one end is to the power applied at the other to raiſe it, as the diſtance of the power from the prop to the diſtance of the weight from the prop, the power and weight will exactly balance or counterpoiſe each other: and as a common lever has but very little friction on its prop, a very little additional power will be ſufficient to raiſe the weight.

There are four kinds of levers. 1. The common ſort, where the prop is placed between the weight and the power; but much nearer to the weight than to the power. 2. When the prop is at one end of the lever, the power at the other, and the weight between them. 3. When the prop is at one end, the weight at the other, and the power applied between them. 4. The bended lever, which differs only in form from the firſt ſort, but not in property. Thoſe of the firſt and ſecond kind are often uſed in mechanical engines; but there

PLATE V.



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there are few inſtances in which the third ſort is uſed.

A *common balance* is a lever of the firſt kind; but as both its ends are at equal diſtances from its center of motion, they move with equal velocities; and therefore, as it gives no mechanical advantage, it cannot properly be reckoned among the mechanical powers.

The *balance.*

A lever of the firſt kind is repreſented by the bar *ABC*, ſupported by the prop *D*. Its principal uſe is to looſen large ſtones in the ground, or raiſe great weights to ſmall heights, in order to have ropes put under them for raiſing them higher by other machines. The parts *AB* and *BC*, on different ſides of the prop *D*, are called the *arms* of the lever: the end *A* of the ſhorter arm *AB* being applied to the weight intended to be raiſed, or to the reſiſtance to be overcome; and the power applied to the end *C* of the longer arm *BC*.

Plate V. Fig. 1. The firſt kind of lever.

In making experiments with this machine, the ſhorter arm *AB* muſt be as much thicker than the longer arm *BC*, as will be ſufficient to balance it on the prop. This ſuppoſed, let *P* repreſent a power, whoſe intenſity is equal to 1 ounce, and *W* a weight whoſe intenſity is equal to 12 ounces. Then, if the power be 12 times as far from the prop as the weight is, they will exactly counterpoiſe; and a ſmall addition to the power *P* will cauſe it to deſcend, and raiſe the weight *W*; and the velocity with which the power deſcends will be to the velocity with which the weight riſes, as 12 to 1: that is, directly as their diſtances from the

prop; and consequently, as the spaces through which they move. Hence, it is plain that a man who by his natural strength, without the help of any machine, could support an hundred weight, will by the help of this lever be enabled to support twelve hundred. If the weight be less, or the power greater, the prop may be placed so much the farther from the weight; and then it can be raised to a proportionably greater height. For universally, if the intensity of the weight multiplied into its distance from the prop be equal to the intensity of the power multiplied into its distance from the prop, the power and weight will exactly balance each other; and a little addition to the power will raise the weight. Thus, in the present instance, the weight *W* is 12 ounces, and its distance from the prop is 1 inch; and 12 multiplied by 1 is 12; the power *P* is equal to 1 ounce, and its distance from the prop is 12 inches, which multiplied by 1 is 12 again; and therefore there is an equilibrium between them. So, if a power equal to 2 ounces be applied at the distance of 6 inches from the prop, it will just balance the weight *W*; for 6 multiplied by 2 is 12, as before. And a power equal to 3 ounces placed at 4 inches distance from the prop would do the same; for 3 times 4 is 12; and so on, in proportion.

The *steel-yard.* The *statera* or roman *steelyard* is a lever of this kind, contrived for finding the different weights of different sorts of bodies, or of a greater quantity of the same sort, by one single weight placed at different distances from the prop or center of motion

motion *D*. For, if a ſcale hangs at *A*, the extremity of the ſhorter arm *AB*, and is of ſuch a weight as will exactly counterpoiſe the longer arm *BC*; if this arm be divided into as many equal parts as it will contain, each equal to *AB*, the ſingle weight *P* (which we may ſuppoſe to be 1 pound) will ſerve for weighing any thing as heavy as itſelf, or as many times heavier as there are diviſions in the arm *BC*, or any quantity between its own weight and that quantity. As for example, if *P* be 1 pound, and placed at the firſt diviſion 1 in the arm *BC*, it will balance 1 pound in the ſcale at *A*: if it be removed to the ſecond diviſion at 2, it will balance 2 pounds in the ſcale: if to the third, 3 pounds; and ſo on to the end of the arm *BC*. If each of theſe integral diviſions be ſubdivided into as many equal parts as a pound contains ounces, and the weight *P* be placed at any of theſe ſubdiviſions, ſo as to counterpoiſe what is in the ſcale, the pounds and odd ounces therein are by that means aſcertained.

To this kind of lever may be reduced ſeveral ſorts of inſtruments, ſuch as ſciſſars, pinchers, ſnuffers; which are made of two levers acting contrary to one another: their prop or center of motion being the pin which keeps them together.

In common practice, the longer arm of this lever exceeds the weight of the ſhorter; which gains an advantage, becauſe it adds ſo much more to the power.

A lever of the ſecond kind has the weight between the prop and the power. In this, as well as the

The ſecond kind of lever.

the former, the advantage gained is as the diſtance of the power from the prop to the diſtance of the weight from the prop: for the reſpective velocities of the power and weight are in that proportion; and they will balance each other when the intenſity of the power multiplied by its diſtance from the prop is equal to the intenſity of the weight multiplied by its diſtance from the prop.

Fig. 2. Thus, if *AB* be a lever on which the weight *W* of 6 ounces hangs at the diſtance of 1 inch from the prop *G*, and a power *P* equal to the weight of 1 ounce hangs at the end *B*, 6 inches from the prop, by the cord *CD* going over the fixed pulley *E*, the power will juſt ſupport the weight: and a ſmall addition to the power will raiſe the weight, 1 inch for every 6 inches that the power deſcends.

This lever ſhews the reaſon why two men carrying a burden upon a ſtick between them, bear unequal ſhares of the burden in the inverſe proportion of their diſtances from it. For it is well known, that the nearer any of them is to the burden, the greater ſhare he bears of it: and if he goes directly under it, he bears the whole. So, if one man be at *G* and the other at *P*, having the pole or ſtick *AB* reſting on their ſhoulders; if the burden or weight *W* be placed five times as near the man at *G* as it is to the man at *P*, the former will bear five times as much weight as the latter. This is likewiſe applicable to the caſe of two horſes of unequal ſtrength, to be ſo yoaked as that each horſe may draw a part proportionable to his ſtrength; which is done by dividing the beam ſo, that

that the point of traction may be as much nearer to the ſtronger horſe than to the weaker as the ſtrength of the former is greater.

To this kind of lever may be reduced oars, rudders of ſhips, doors turning upon hinges, cutting knives which are fixed at the point of the blade, and the like.

If in this lever we ſuppoſe the power and weight to change places, ſo that the power may be between the weight and the prop, it will become a lever of the third kind: in which, that there may be a balance between the power and the weight, the intenſity of the power muſt exceed that of the weight juſt as much as the diſtance of the weight from the prop exceeds the diſtance of the power from it. Thus, let *E* be the prop of the lever *AB*, and *W* a weight of 1 pound placed 3 times as far from the prop as the power *P* acts at *F* by the cord *C* going over the fixed pulley *D*; in this caſe, the power muſt be equal to three pounds in order to ſupport the weight.

The third kind of lever.

Fig. 3.

To this ſort of lever are generally referred the bones of a man's arm: for when we lift a weight by the hand, the muſcle that exerts its force to raiſe that weight is fixed to the bone about one tenth part as far below the elbow as the hand is. And the elbow being the center round which the arm turns, the muſcle muſt therefore exert a force ten times as great as the weight raiſed.

As this kind of lever gives no advantage to the moving power, it is never uſed but in caſes of neceſſity; ſuch as that of a ladder, which being fixed

fixed at one end, is by the ſtrength of a man's arms reared againſt a wall. And in clock-work, where all the wheels may be reckoned levers of this kind, becauſe the power that moves every wheel except the firſt, acts upon it near the center of motion by means of a ſmall pinion, and the reſiſtance it has to overcome acts againſt the teeth round its circumference.

The fourth kind of lever. Fig. 4. The fourth kind of lever differs nothing from the firſt, but in being bended for the ſake of convenience. *ACB* is a lever of this ſort, bended at *C* which is its prop or center of motion. *P* is a power acting upon the longer arm *AC* at *F*, by means of the cord *DE* going over the pulley *G*; and *W* is a weight or reſiſtance acting upon the end *B* of the ſhorter arm *BC*. If the power be to the weight as *BC* to *CF*, they are in *equilibrio.* Thus, ſuppoſe *W* to be 5 pounds acting at the diſtance of one foot from the center of motion *C*, and *P* to be 1 pound acting at *F* five feet from the center *C*, the power and weight will juſt balance each other. A hammer drawing a nail is a lever of this ſort.

The wheel and axle. 2. The ſecond mechanical power is the *wheel and axle*, in which the power is applied to the circumference of the wheel, and the weight is raiſed by a rope which coils about the axle as the wheel is turned round. Fig. 5. Here it is plain that the velocity of the power muſt be to the velocity of the weight as the circumference of the wheel is to the circumference of the axle: and conſequently, the power and weight will balance each other when the intenſity

tenſity of the power is to the intenſity of the weight as the circumference of the axle is to the circumference of the wheel. Let *AB* be a wheel, *CD* its axle, and ſuppoſe the circumference of the wheel to be 8 times as great as the circumference of the axle; then, a power *P* equal to one pound hanging by the cord *I*, which goes round the wheel, will balance a weight *W* of 8 pounds hanging by the rope *X*, which goes round the axle. And as the friction on the pivots or gudgeons of the axle is but ſmall, a ſmall addition to the power will cauſe it to deſcend, and raiſe the weight: but the weight will riſe with only an eighth part of the velocity that the power deſcends, and conſequently, through no more than an eighth part of an equal ſpace, in the ſame time. If the wheel be pulled round by the handles *S*, *S*, the power will be increaſed in proportion to their length. And by this means, any weight may be raiſed as high as the operator pleaſes.

To this ſort of engine belong all cranes for raiſing great weights; and in this caſe, the wheel may have cogs all around it inſtead of handles, and a ſmall lantern or trundle may be made to work in the cogs, and be turned by a winch; which will again increaſe the power of the wheel as much as its number of cogs exceeds the number of ſtaves or rounds in the trundle, if the radius of the trundle and that of the winch be equal: but if the radius or length of the winch be double the radius or ſemidiameter of the trundle, it will again give a double power. So that, if the circumference of the wheel be 8 times

times the circumference of the axle, the power of the axle is eight times as great as the power of the wheel; and if the trundle makes 8 revolutions to one of the wheel, the power of the axle is 8 times 8 times, or 64 times, as great as that of the trundle: if the length of the winch be twice the semi-diameter of the trundle, this will again give a double power; and so make it twice 64 or 128 on the axle. Therefore, a man who by his natural strength could lift an hundred weight from the ground in his arms, will, by the help of such an engine as this, be able to raise 128 hundred weight, or 6 tons and almost an half. In this sort of machines it is requisite to have a ratchet-wheel *G* on one end of the axle, with a catch *H* to fall into its teeth; which will at any time support the weight, and keep it from descending, if the workman should through inadvertency or carlesness quit his hold whilst the weight is raising. And by this means, the danger is prevented which might otherwise happen by the running down of the weight when left at liberty.

The *pulley*. 3. The third mechanical power or engine consists either of one *moveable pulley* or a *system of pullies*, some in a block or case which is fixed, and others in a block which is moveable and rises with the weight. For though a single pulley that only turns on its axis, and rises not with the weight, may serve to change the direction of the power, yet it can give no mechanical advantage thereto: but is only as the beam of a balance whose arms are of equal length and weight. Thus, if the equal weights

Fig. 6.

weights *W* and *P* hang by the cord *B B* upon the pulley *A*, whofe block *b* is fixed to the beam *H I*, they will counterpoife each other, juft in the fame manner as if the cord were cut in the middle, and its two ends hung upon the hooks fixt in the pulley at *A* and *A*, equally diftant from its center.

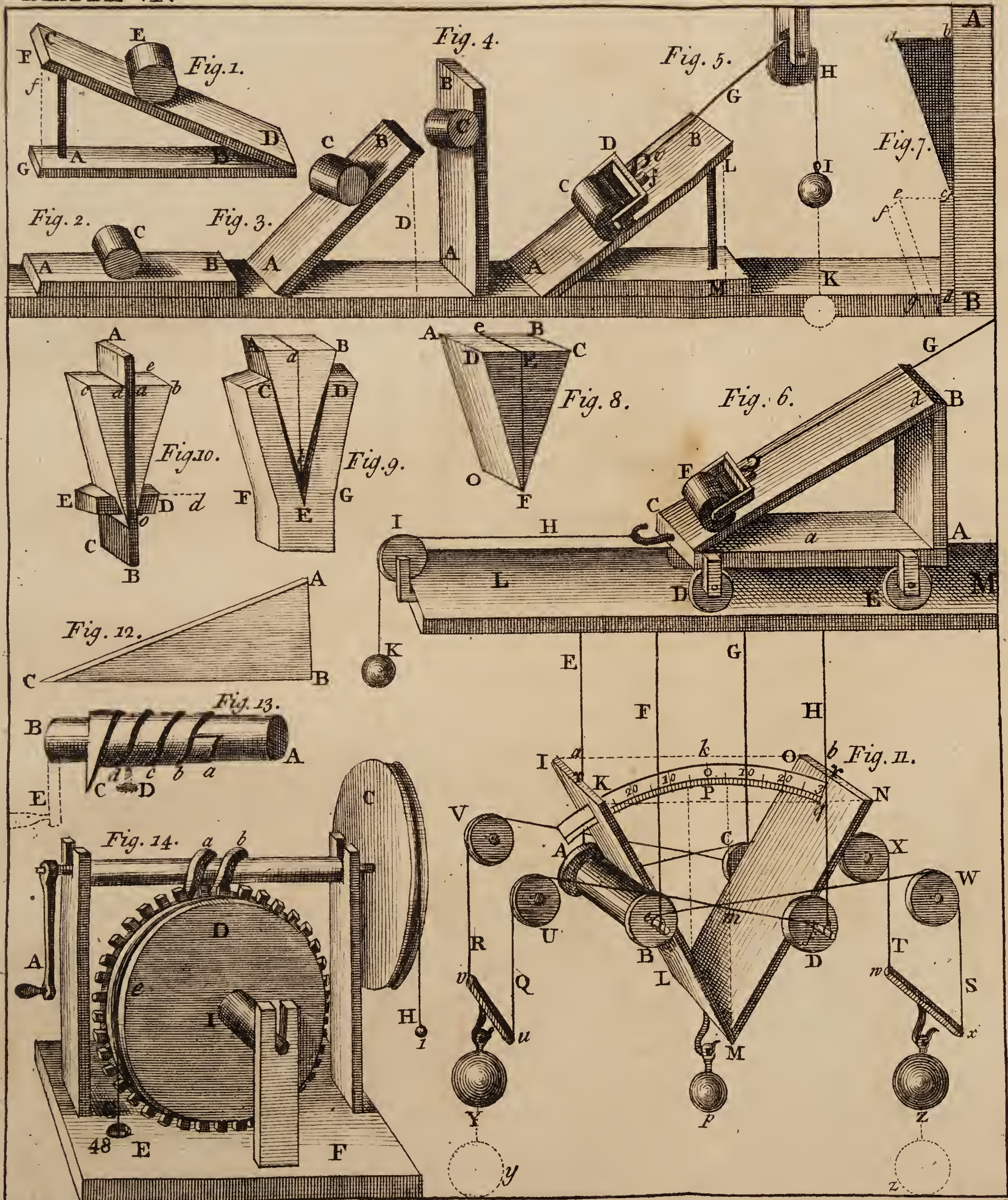
But if a weight *W* hang at the lower end of the moveable block *p* of the pulley *D*, and the cord *G F* go under the pulley, it is plain that the half *G* of the cord bears one half of the weight *W*, and the half *F* the other; for they bear the whole between them. Therefore, whatever holds the upper end of either rope fuftains one half of the weight: and if the cord at *F* be drawn up fo as to raife the pulley *D* to *C*, the cord will then be extended to its whole length, all but that part which goes under the pulley: and confequently, the power that draws the cord will have moved twice as far as the pulley *D* with its weight *W* rifes; on which account, a power whofe intenfity is equal to one half of the weight will be able to fupport it, becaufe if the power moves (by means of a fmall addition) its velocity will be double the velocity of the weight; as may be feen by putting the cord over the fixt pulley *C* (which only changes the direction of the power without giving any advantage to it) and hanging on the weight *P*, which is equal only to one half of the weight *W*; in which cafe there will be an equilibrium, and a little addition to *P* will caufe it to defcend, and raife *W* through a fpace equal to one half of that through which *P* defcends.—Hence, the advantage gained

gained will be always equal to twice the number of pullies in the moveable or undermoſt block. So, when the upper or fixt block *u* contains two pullies, which only turn on their axes, and the lower or moveable block *U* contains two pullies, which not only turn upon their axes but alſo riſe with the block and weight, the advantage gained by this is as 4 to the power. Thus, if one end of the rope *KMOQ* be fixed to a hook at *I*, and the rope paſs over the pullies *N* and *R*, and under the pullies *L* and *P*, and have a weight *T* of one pound hung to its other end at *T*, this weight will balance and ſupport a weight *W* of four pounds hanging by a hook at the moveable block *U*, allowing the ſaid block as a part of the weight. And if as much more power be added as is ſufficient to overcome the friction of the pullies, the power will deſcend with four times as much velocity as the weight riſes; and conſequently through four times as much ſpace.

The two pullies in the fixed block *X*, and the two in the moveable block *Y*, are in the ſame caſe with thoſe laſt mentioned; and give the ſame advantage to the power.

As a ſyſtem of pullies have no great weight, and lie in a ſmall compaſs, they are eaſily carried about; and can be applied, in a great many caſes, for raiſing weights, where other engines cannot. But they have a great deal of friction on three accounts: 1. becauſe the diameter of their axis bears a very conſiderable proportion to their own diameter; 2. becauſe in working they are apt to rub againſt one

PLATE VI.



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one another, or againſt the ſides of the block; 3. becauſe of the ſtiffneſs of the rope that goes round them.

4. The fourth mechanical power is the *inclined plane*; and the advantage gained by it is as great as its length exceeds its perpendicular height. Let *AB* be a plane parallel to the horizon, and *CD* a plane inclined to it; and ſuppoſe the whole length *CD* to be three times as great as the perpendicular height *GfF*: in this caſe, the cylinder *E* will be ſupported upon the plane *CD*, and kept from rolling down upon it, by a power equal to a third part of the weight of the cylinder. Therefore, a weight may be rolled up this inclined plane with a third part of the power which would be ſufficient to draw it up by the ſide of an upright wall. If the plane was four times as long as high, a fourth part of the power would be ſufficient; and ſo on, in proportion. Or, if a pillar is to be raiſed from a floor to the height *GF*, by means of the engine *ABDC*, (which will then act as a half wedge, where the reſiſtance gives way only on one ſide) the engine and pillar will be in *equilibrio* when the power applied at *GF* is to the weight of the pillar as *GF* to *CD*; and if the power be increaſed, ſo as to overcome the friction of the engine againſt the floor and pillar, the engine will be driven, and the pillar raiſed: and when the engine has moved its whole length upon the floor, the pillar will be raiſed to the whole height of the engine, from *G* to *F*.

The inclined plane.

Plate VI. Fig. 1.

The force wherewith a rolling body deſcends upon an inclined plane is to the force of its abſolute

gravity, by which it would defcend perpendicularly in a free fpace, as the height of the plane is to its length. For, fuppofe the plane *AB* to be parallel to the horizon, the cylinder *C* will keep at reft upon any part of the plane where it is laid. If the plane be fo elevated, that its perpendicular height *D* is equal to half its length *AB*, the cylinder will roll down upon the plane with a force equal to half its weight; for it would require a power equal to half its weight to keep it from rolling. If the plane *AB* be elevated, fo as to be perpendicular to the horizon, the cylinder *C* will defcend with its whole force of gravity, becaufe the plane contributes nothing to its fupport or hindrance; and therefore, it would require a power equal to its whole weight to keep it from defcending.

Fig. 2. Fig. 3. Fig. 4.

Fig. 5. Let the cylinder *C* be fitted to turn upon flender pivots in the frame *D*, in which there is a hook *e* with a line *G* tied to it: let this line go over the fixed pulley *H*, and have its other end tied to a hook in the weight *I*. If the weight of the body *I* be to the weight of the cylinder *C* added to that of its frame *D*, as the perpendicular height of the plane *LM* is to its length *AB*, the weight will juft fupport the cylinder upon the plane, and a fmall touch of a finger will either caufe it to afcend or defcend with equal eafe: then, if a little addition be made to the weight *I*, it will defcend, and draw the cylinder up the plane. In the time that the cylinder moves from *A* to *B*, it will rife through the whole height of the plane *ML*; and the weight will

will defcend from *H* to *K*, through a fpace equal to the whole length of the plane *AB*.

If the plane be made to move upon rollers, or friction-wheels, and the cylinder be fupported upon it; the fame power will draw the plane under the cylinder, which before drew the cylinder up the plane, provided the pivots of the axes of the friction-wheels be fmall, and the wheels themfelves be pretty large. For, let the machine *ABC* (equal in length and height to *ABM*, Fig. 5.) be provided with four wheels, whereof two appear at *D* and *E*, and the third under *C*, whilft the fourth is hid from fight by the horizontal board *a*. Let the cylinder *F* be laid upon the lower end of the inclined plane *CB*, and the line *G* be extended from the frame of the cylinder, about fix feet, parallel to the plane *CB*; and, in that direction, fixed to a hook in the wall; which will fupport the cylinder, and keep it from rolling off the plane. Let one end of the line *H* be tied to a hook at *C* in the machine, and the other end to a weight *K*, the fame as drew the cylinder up the plane before. If this line be put over the fixed pulley *I*, the weight *K* will draw the machine along the horizontal plane *L*, and under the cylinder *F*: and when the machine has been drawn its whole length, the cylinder will be raifed to *d*, equal to the perpendicular height *AB* above the horizontal part *a*. Fig. 6.

To the inclined plane may be reduced all hatchets, chifels, and other edge-tools which are chamfer'd only on one fide.

 5. The

The *wedge.* 5. The fifth mechanical power or engine is the *wedge*, which may be confidered as two equal inclined planes *DEF* and *CEF*, joined together at their bafes *e EF*: then, *DC* is the whole thicknefs of the wedge at its back *ABCD*, where the power is applied; *EF* is the depth or height of the wedge; *DF* the length of one of its fides, equal to *CF* the length of the other fide; and *OF* is its fharp edge, which is entered into the wood intended to be fplit by the force of a hammer or mallet, ftriking perpendicularly on its back. Thus, *ABb* is a wedge driven into the cleft *CDE* of the wood *FG*.

Fig. 8.

Fig. 9.

When the wood does not cleave at any diftance before the wedge, there will be an equilibrium between the power impelling the wedge downward, and the refiftance of the wood acting againft the two fides of the wedge, if the power be to the refiftance as half the thicknefs of the wedge at its back is to the length of either of its fides; that is, as *Aa* to *Ab*, or *Ba* to *Bb* (Fig. 9.) And if the power be increafed, fo as to overcome the friction of the wedge and the refiftance arifing from the cohefion or ftickage of the wood, the wedge will be drove in, and the wood fplit afunder.

But, when the wood cleaves at any diftance before the wedge (as it generally does) the power impelling the wedge will not be to the refiftance of the wood as half the thicknefs of the wedge is to the length of one of its fides; but as half its thicknefs is to the length of either fide of the cleft, eftimated from the top or acting part of the wedge.

 For,

For, if we ſuppoſe the wedge to be lengthened down from *b* to the bottom of the cleft at *E*, the ſame proportion will hold; namely, that the power will be to the reſiſtance as half the thickneſs of the wedge is to the length of either of its ſides: or, which amounts to the ſame thing, as the whole thickneſs of the wedge is to the length of both its ſides.

In order to prove what is here advanced concerning the wedge, let us ſuppoſe the wedge to be divided lengthwiſe into two equal parts; and then it will become two equally inclined planes; one of which, as *abc*, may be made uſe of as a half wedge for ſeparating the moulding *cd* from the wainſcot *AB*. It is evident, that when this half wedge has been driven its whole length *ac* between the wainſcot and moulding, its ſide *ac* will be at *ed*; and the moulding will be ſeparated to *fg* from the wainſcot. Now, from what has been already proved of the inclined plane, it appears, that to have an equilibrium between the power impelling the half wedge and the reſiſtance of the moulding, the former muſt be to the latter as *ab* to *ac*; that is, as the thickneſs of the back which receives the ſtroke is to the length of the ſide againſt which the moulding acts. Therefore, ſince the power upon the half wedge is to the reſiſtance againſt its ſide, as the half back *ab* is to the whole ſide *ac*, it is plain that the power upon the whole wedge (where the whole back is double the half back) muſt be to the reſiſtance againſt both its ſides, as the thickneſs of the whole back is to the length of both the Fig. 7.

 ſides;

ſides; ſuppoſing the wedge at the bottom of the cleft: or as the thickneſs of the whole back to the length of both ſides of the cleft, when the wood ſplits at any diſtance before the wedge. For, when the wedge is driven quite into the wood, and the wood ſplits at ever ſo ſmall a diſtance before its edge, the top of the wedge then becomes the acting part, becauſe the wood does not touch it any where elſe. And ſince the bottom of the cleft muſt be conſidered as that part where the whole ſtickage or reſiſtance is accumulated, it is plain, from the nature of the lever, that the farther the power acts from the reſiſtance, the greater is the advantage.

* Some writers, particularly *Graveſande*, *Worſter*, and *Emerſon* advance, that the power of the wedge is to the reſiſtance to be overcome, as the thickneſs of the back of the wedge is to the length only of one of its ſides; which ſeems very ſtrange: for, if we ſuppoſe *AB* to be a ſtrong inflexible bar of wood or iron fixt into the ground at *CB*, and *D* and *E* to be two blocks of marble lying on the ground on oppoſite ſides of the bar; it is evident that the block *D* may be ſeparated from the bar to the diſtance *ab* by driving the inclined plane or half wedge *abo* down between them; and the block *E* may be ſeparated to an equal diſtance on the other ſide in like manner by the half wedge *cdo*. But the power impelling each half wedge will be to the reſiſtance of the block againſt its ſide, as the thickneſs of that half wedge is to the length of its acting ſide. Therefore the power to drive both the half

Fig. 10.

half wedges is to both the refiftances, as both the half backs is to the length of both the acting fides, or as half the thicknefs of the whole back is to the length of either fide. And, if the bar be taken away, the blocks put clofe together, and the two half wedges joined to make one; it will require as much force to drive it down between the blocks, as is equal to the fum of the feparate powers acting upon the half wedges when the bar was between them.

To confirm this by an experiment, let two cylinders, as *AB* and *CD*, be drawn towards one another by lines running over fixed pullies, and a weight of 40 ounces hanging at the lines belonging to each cylinder: and let a wedge, of 40 ounces weight, having its back juft as thick as either of its fides is long, be put between the cylinders, which will then act againft each fide with a refiftance equal to 40 ounces, whilft its own weight endeavours to bring it down and feparate them. And here, the power of the wedge's gravity impelling it downward will be to the refiftance of both the cylinders againft the wedge, as the thicknefs of the wedge is to the length of both its fides; for there will then be an equilibrium between the weight of the wedge and the refiftance of the cylinders againft it, and it will remain at any height between them; requiring juft as much power to pufh it upward as to pull it downward.—If another wedge, of equal weight and depth with this, and only half as thick, be put between the cylinders, it will require twice as much weight to be hung at Fig. 11.

 the

the ends of the lines which draw them together, to keep the wedge from going down between them. That is, a wedge of 40 ounces, whofe back is only equal to half the length of one of its fides, will require 80 ounces to each cylinder, to keep it in an equilibrium between them; and twice 80 is 160, equal to four times 40. So that the power will be always to the refiftance, as the thicknefs of the back of the wedge is to the length (not of its one fide but) of both its fides.

Fig. 11. The beft way, though perhaps not the neateft that I know of, for making a wedge with its appurtenances for fuch experiments, is as follows. Let *IKLM* and *LMNO* be two flat pieces of wood, each about fifteen inches long and three or four in breadth, joined together by a hinge at *LM*; and let *P* be a graduated arch of brafs, on which the faid pieces of wood may be opened to any angle not more than 60 degrees, and then fixt at the given angle by means of the two fcrews *a* and *b*. Then, *IKNO* will reprefent the back of the wedge, *LM* its fharp edge which enters the wood, and the outfides of the pieces *IKLM* and *LMNO* the two fides of the wedge againft which the wood acts in cleaving. By means of the faid arch, the wedge may be opened fo, as to adjuft the thicknefs of its back in any proportion to the length of either of its fides, but not to exceed that length: and any weight as *p* may be hung to the wedge upon the hook *M*, which weight, together with the weight of the wedge itfelf, may be confidered as the impelling power; which is all the fame in experiment,

periment, whether it be laid upon the back of the wedge to puſh it down, or hung to its edge to pull it down.—Let *AB* and *CD* be two wooden cylinders, each about two inches thick where they touch the outſides of the wedge; and let their ends be made like two round flat plates, to keep the wedge from ſlipping off endwiſe between them. Let a ſmall cord with a loop on one end of it go over a pivot in the end of each cylinder, and the cords *S* and *T* belonging to the cylinder *AB* go over the fixt pullies *W* and *X*, and be faſtened at their other ends to the bar *wx*, on which any weight as *Z* may be hung at pleaſure. In like manner, let the cords *Q* and *R* belonging to the cylinder *BC* go over the fixt pullies *U* and *V* to the bar *uv*, on which a weight *Y* equal to *Z* may be hung. Theſe weights, by drawing the cylinders towards one another, may be conſidered as the reſiſtance of the wood acting equally againſt oppoſite ſides of the wedge: the cylinders themſelves being ſuſpended near and parallel to each other, by their pivots in loops on the lines *E*, *F*, *G*, *H*; which lines may be fixed to hooks in the cieling of the room. The longer theſe lines are, the better; and they ſhould never be leſs than four feet each. The farther alſo the pullies *W*, *V*, and *W*, *X* are from the cylinders, the truer will the experiments be: and they may turn upon pins fixed into the wall.

In this machine, the weights *Y* and *Z*, and the weight *p*, may be varied at pleaſure, ſo as to be adjuſted in proportion of the length of the wedge's ſides to the thickneſs of its back; and when they

are

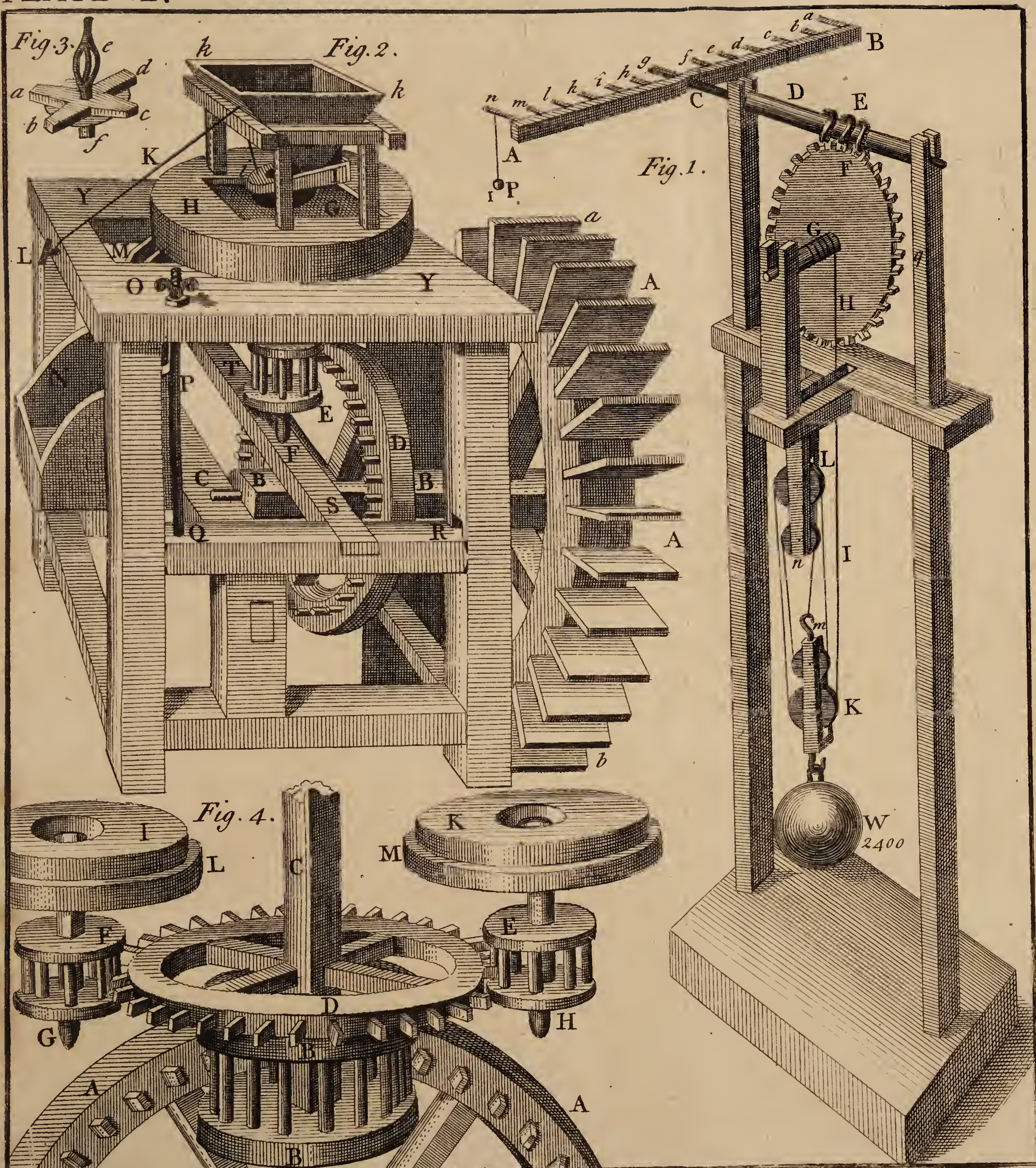
are ſo adjuſted, the wedge will be in *equilibrio* with the reſiſtance of the cylinders.

The wedge is a very great mechanical power, ſince not only wood but even rocks can be ſplit by it; which would be impoſſible to effect by the lever, wheel and axle, or pulley.

The *ſcrew.* 6. The ſixth, and laſt mechanical power is the *ſcrew*; which cannot properly be called a ſimple machine, becauſe it is never uſed without the application of the lever acting as a winch or handle in turning it: and then it becomes a compound engine, of a very great force either in preſſing the parts of bodies cloſe together, or in raiſing great weights. It may be conceived to be made Fig. 12, 13. by cutting a piece of paper *ABC* (Fig. 12.) into the form of an inclined plane or half wedge, and then coiling it round a cylinder *AB* (Fig. 13.) And here it is evident, that the winch *E* muſt turn the cylinder once round before the weight or reſiſtance *D* can be moved from one ſpiral winding to another, as from *d* to *c*: therefore, as much as the circumference of a circle deſcribed by the handle of the winch is greater than the interval or diſtance between the ſpirals, ſo much is the force of the ſcrew. That is, ſuppoſing the diſtance between the ſpirals to be half an inch, and the length of the winch twelve inches; the circle deſcribed by the handle where the power acts will be 76 inches nearly; or about 152 half inches, and conſequently 152 times the diſtance between the ſpirals: and therefore, a power at the handle, whoſe intenſity is equal to no more than a ſingle pound, will balance 152

pounds

PLATE VII.



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pounds acting againſt the ſcrew; and as much additional force, as is ſufficient to overcome the friction, will raiſe the 152 pounds; and the velocity of the power will be to the velocity of the weight, as 152 to 1. Hence it appears, that the longer the winch be made, and the nearer the ſpirals are to one another, ſo much the greater is the force of the ſcrew.

A machine for ſhewing the force or power of the ſcrew may be contrived in the following manner. Let the wheel *C* have a ſcrew *ab* on its axis, working in the teeth of the wheel *D*, which ſuppoſe to be 48 in number. It is plain that for every time the wheel *C* and ſcrew *ab* are turned round by the winch *A*, the wheel *D* will be moved one tooth by the ſcrew: and therefore, in 48 revolutions of the winch, the wheel *D* will be turned once round. Then, if the circumference of a circle deſcribed by the handle of the winch be equal to the circumference of a groove *e* round the wheel *D*, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove. Conſequently, if a line *G* goes round the groove, and has a weight of 48 pounds hung to it below the pedeſtal *EF*, a power equal to one pound at the handle will balance and ſupport the weight.—To prove this by experiment, let the circumferences of the grooves of the wheels *C* and *D* be equal to one another; and then, if a weight *H* of one pound be ſuſpended by a line going round the groove of the wheel *C*, it will balance a weight of 48 pounds hanging by the line *G*; and Fig. 14.

a ſmall

a ſmall addition to the weight *H* will cauſe it to deſcend, and ſo raiſe up the other weight.

If the line *G*, inſtead of going round the groove *e* of the wheel *D*, goes round its axle *I*; the power of the machine will be as much increaſed, as the circumference of the groove *e* exceeds the circumference of the axle: which, ſuppoſing it to be 6 times, then one pound at *H* will balance 6 times 48 or 288 pounds hung to the line on the axle: and hence, the power or advantage of this machine will be as 288 to 1. That is to ſay, a man who by his natural ſtrength could lift an hundred weight, will be able to raiſe 288 hundred, or $14\frac{8}{20}$ ton weight by this engine.

But the following engine is ſtill more powerful, on account of its having the addition of four pullies: and in it we may look upon all the mechanical powers as combined together, even if we take in the balance. For, as the axis *D* of the bar *AB* is in its middle at *C*, it is plain that if equal weights are ſuſpended upon any two pins equidiſtant from the axis *C*, they will counterpoiſe. It becomes a lever by hanging a ſmall weight *P* upon the pin *n*, and a weight ſo much heavier upon either of the pins *b*, *c*, *d*, *e*, or *f*, as is in proportion to the pin's being ſo much nearer the axis. The wheel-and-axle *FG* is evident; ſo is the ſcrew *E*, which takes in the inclined plane, and with it the half wedge. Part of a cord goes round the axle, the reſt under the lower pullies *K*, *m*, over the upper pullies *L*, *n*, and then it is tied to a hook at *m* in the

Plate VII. Fig. 1.

A combination of all the mechanical powers.

the lower or moveable block, on which hangs the weight *W*.

In this machine, if the wheel *F* has 30 teeth, it will be turned once round in 30 revolutions of the bar *AB*, which is fixt on the axis *D* of the ſcrew *E*: if the length of the bar is equal to twice the diameter of the wheel, the pins *a* and *n* at the ends of the bar will move 60 times as faſt as the teeth of the wheel: conſequently, one ounce at *P* will balance 60 ounces hung upon a tooth at *q* in the horizontal diameter of the wheel. Then, if the diameter of the wheel *F* is 10 times as great as the diameter of the axle *G*, the wheel will have 10 times the velocity of the axle; and therefore one ounce *P* at the end of the lever *AC* will balance 10 times 60 or 600 ounces hung to the rope *H* which goes round the axle. Laſtly, if four pullies be added, they will make the velocity of the lower block *K*, and weight *W*, four times leſs than the velocity of the axle: and this being the laſt power in the machine, which is 4 times as great as that gained by the axle, it makes the whole power of the machine 4 times 600, or 2400. So that a man who could lift one hundred weight in his arms, by his natural ſtrength, would be able to raiſe 2400 hundred weight by this engine.—But it is here as in all other mechanical caſes; for the time loſt is always as much as the power gained, becauſe the velocity with which the power moves will ever exceed the velocity with which the weight riſes, as much as the intenſity of the weight exceeds the intenſity of the power.

The

The friction of the screw itself is very considerable: and there are few combined engines, but what, upon account of the friction of the parts against one another, will require a third part of more power to work them when loaded, than what is sufficient to constitute a balance between the weight and the power.

LECT. IV.

Of mills, cranes, wheel-carriages, and the engine for driving piles.

AS these machines are so universally useful, it would be ridiculous to make any apology for describing them.

Plate VII, Fig. 2. A common mill.

In a common *breast mill*, where the fall of water may be about ten feet, *AA* is the great wheel, which is generally about 17 or 18 feet diameter, reckoned from the outermost edge of any float-board at *a* to that of its opposite float at *b*. To this wheel the water is conveyed through a channel, and so falling upon the wheel, turns it round.

On the axis *BB* of this wheel, and within the mill-house, is a wheel *D*, about 8 or 9 feet diameter, having 61 cogs which turn a trundle *E* containing 10 upright staves or rounds; and when these are the number of cogs and rounds, the trundle will make $6\frac{1}{10}$ revolutions for one revolution of the wheel.

The trundle is fixt upon a strong iron axis call'd the spindle, the lower end of which turns in a brass

brafs foot, fixt at *F*, in the horizontal beam *ST* called the bridge-tree; and the upper part of the fpindle turns in a wooden bufh fixt into the nether millftone which lies upon beams in the floor *YY*. The top part of the fpindle above the bufh is fquare, and goes into a fquare hole in a ftrong iron crofs *abcd*, (See Fig. 3.) called the rynd; under which, and clofe to the bufh, is a round piece of thick leather upon the fpindle, which it turns round at the fame time as it does the rynd.

The rynd is let into grooves in the under furface of the running millftone *G* (Fig. 2.) and fo turns it round in the fame time that the trundle *E* is turned round by the cog wheel *D*. This millftone has a large hole quite through its middle, called the eye of the ftone, through which the middle part of the rynd and upper end of the fpindle may be feen; whilft the four ends of the rynd lie hid below the ftone in their grooves.

The end *T* of the bridge-tree *TS* (which fupports the upper millftone *G* upon the fpindle) is fixed into a hole in the wall; and the end *S* is let into a beam *QR* called the brayer, whofe end *R* remains fixt, and its other end *Q* hangs by a ftrong iron rod *P* which goes through the floor *YY*, and has a fcrew-nut on its top at *O*; by the turning of which nut, the end *Q* of the brayer is raifed or depreffed at pleafure; and confequently the bridge-tree *TS* and upper millftone. By this means, the upper millftone may be fet as clofe to the under one, or raifed as high from it, as the miller pleafes. The nearer the millftones are to one another, the finer

finer they grind the corn, and the more remote from one another, the coarſer.

The upper millſtone *G* is incloſed in a round box *H*, which does not touch it any where; and is about an inch diſtant from its edge all around. On the top of this box ſtands a frame for holding the hopper *kk*, to which is hung the ſhoe *I* by two lines faſtened to the hind-part of it, fixed upon hooks in the hopper, and by one end of the crook-ſtring *K* faſtened to the fore-part of it at *i*; the other end being twiſted round the pin *L*. As the pin is turned one way, the ſtring draws up the ſhoe cloſer to the hopper, and ſo leſſens the aperture between them; and as the pin is turned the other way, it lets down the ſhoe, and enlarges the aperture.

If the ſhoe be drawn up quite to the hopper, no corn can fall from the hopper into the mill; if it be let a little down, ſome will fall: and the quantity will be more or leſs according as the ſhoe is more or leſs let down. For the hopper is open at bottom, and there is a hole in the bottom of the ſhoe, not directly under the bottom of the hopper, but forwarder towards the end *i*, over the middle of the eye of the millſtone.

Fig. 3. There is a ſquare hole in the top of the ſpindle, in which is put the feeder *e*: this feeder (as the ſpindle turns round) jogs the ſhoe three times in each revolution, and ſo cauſes the corn to run conſtantly down from the hopper, through the ſhoe, into the eye of the millſtone, where it falls upon the top of the rynd, and is, by the motion of the

rynd

rynd and the leather under it, thrown below the upper ſtone, and ground between it and the lower one. The violent motion of the ſtone creates a centrifugal force in the corn going round with it, by which means it gets farther and farther from the center, as in a ſpiral, in every revolution, until it be thrown quite out; and, being then ground, it falls through a ſpout *M* called the mill-eye, into the trough *N*.

When the mill is fed too faſt, the corn bears up the ſtone and is ground too coarſe; and beſides, it clogs the mill ſo as to make it go too ſlow. When the mill is too ſlowly fed, it goes too faſt, and the ſtones by their attrition ſtrike fire againſt one another. Both which inconveniencies are avoided by turning the pin *L* backwards or forwards, which draws up or lets down the ſhoe; and ſo regulates the feeding as the miller ſees convenient.

The heavier the running millſtone is, and the greater the quantity of water that falls upon the wheel, ſo much the faſter will the mill bear to be fed; and conſequently ſo much the more it will grind. And on the contrary, the lighter the ſtone, and the leſs the quantity of water, ſo much ſlower muſt the feeding be. But when the ſtone is conſiderably wore, and become light, the mill muſt be fed ſlowly at any rate; otherwiſe the ſtone will be too much borne up by the corn under it, which will make the meal coarſe.

The quantity of power required to turn a heavy millſtone is but very little more than what is ſufficient to turn a light one: for as it is ſup-

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ported upon the ſpindle by the bridge-tree *ST*, and the end of the ſpindle that turns in the braſs foot therein being but ſmall, the odds ariſing from the weight is but very inconſiderable in its action againſt the power or force of the water. And beſides, a heavy ſtone has the ſame advantage as a heavy fly; namely, that it regulates the motion much better than a light one.

In order to cut and grind the corn, both the upper and under millſtones have channels or furrows cut into them, proceeding obliquely from the center towards the circumference. And theſe furrows are each cut perpendicularly on one ſide and obliquely on the other into the ſtone; which gives each furrow a ſharp edge, and in the two ſtones they come, as it were, againſt one another like the edges of a pair of ſciſſars: and ſo cut the corn, to make it grind the eaſier when it falls upon the places between the furrows. Theſe are cut the ſame way in both ſtones when they lie upon their backs, which makes them run croſs ways to each other when the upper ſtone is inverted by turning its furrowed ſurface towards that of the lower. For, if the furrows of both ſtones lay the ſame way, a great deal of the corn would be drove onward in the lower furrows, and ſo come out from between the ſtones without ever being cut.

When the furrows become blunt and ſhallow by wearing, the running ſtone muſt be taken up, and both ſtones new dreſt with a chiſſel and hammer. And every time the ſtone is taken up, there muſt be ſome tallow put round the ſpindle upon the buſh, which

which will ſoon be melted by the heat that the ſpindle acquires from its turning and rubbing againſt the buſh, and ſo will get in betwixt them: otherwiſe the buſh would take fire in a very little time.

The buſh muſt embrace the ſpindle quite cloſe, to prevent any ſhake in the motion, which would make ſome parts of the ſtones grate and fire againſt each other; whilſt other parts of them would be too far aſunder, and by that means ſpoil the meal in grinding.

Whenever the ſpindle wears the buſh ſo as to begin to ſhake in it, the ſtone muſt be taken up, and a chiſſel drove into ſeveral parts of the buſh; and when it is taken out, wooden wedges muſt be drove into the holes; by which means the buſh will be made to embrace the ſpindle cloſe all around it again. In doing this, great care muſt be taken to drive equal wedges into the buſh on oppoſite ſides of the ſpindle; otherwiſe it will be thrown out of the perpendicular, and ſo hinder the upper ſtone from being ſet parallel to the under one, which is abſolutely neceſſary for making good work. When any accident of this kind happens, the perpendicular poſition of the ſpindle muſt be reſtored by adjuſting the bridge-tree *ST* by proper wedges put between it and the brayer *QR*.

It often happens, that the rynd is a little wrenched in laying down the upper ſtone upon it; or is made to ſink a little lower upon one ſide of the ſpindle than on the other; and this will cauſe one edge of the upper ſtone to drag all around upon the other,

whilſt the oppoſite edge will not touch. But this is eaſily ſet to rights, by raiſing the ſtone a little with a lever, and putting bits of paper, cards, or thin chips betwixt the rynd and the ſtone.

The diameter of the upper ſtone is generally about ſix feet, the lower ſtone about an inch more: and the upper ſtone when new contains about $22\frac{1}{2}$ cubic feet, which weighs ſomewhat more than 1900 pounds. A ſtone of this diameter ought never to go more than 60 times round in a minute; for if it turns faſter it will heat the meal.

The grinding ſurface of the under ſtone is a little convex from the edge to the center, and that of the upper ſtone a little more concave: ſo that they are fartheſt from one another in the middle, and come gradually nearer towards the edges. By this means, the corn at its firſt entrance between the ſtones is only bruiſed; but as it goes farther on towards the circumference or edge, it is cut ſmaller and ſmaller; and at laſt finely ground juſt before it comes out from between the ſtones.

The water-wheel muſt not be too large, for if it be, its motion will be too ſlow; nor too little, for then it will want power. And for a mill to be in perfection, the floats of the wheel ought to move with a third part of the velocity of the water, and the ſtone to turn round once in a ſecond of time.

Such a mill as this, with a fall of water about $7\frac{1}{2}$ feet, will require about 32 hogſheads every minute to turn the wheel with a third part of the velocity with which the water falls; and to overcome the

the refiftance arifing from the friction of the geers and attrition of the ftones in grinding the corn.

The greater fall the water has, the lefs quantity of it will ferve to turn the mill. The water is kept up in the mill-dam, and let out by a fluice called the penftock, when the mill is to go. When the penftock is drawn up by means of a lever, it opens a paffage through which the water flows to the wheel: and when the mill is to be ftopt, the penftock is let down, which ftops the water from falling upon the wheel.

A lefs quantity of water will turn an overfhot mill (where the wheel has buckets inftead of float boards) than a breaft mill where the fall of the water feldom exceeds half the height *Ab* of the wheel. So that, where there is but a fmall quantity of water, and a fall great enough for the wheel to lie under it, the bucket (or overfhot) wheel is always ufed. But where there is a large body of water, with a little fall, the breaft or float-board wheel muft take place. Where the water runs only upon a little declivity, it can act but flowly upon the under part of the wheel at *b*; in which cafe, the motion of the wheel will be very flow: and therefore, the floats ought to be very long, though not high, that a large body of water may act upon them; fo that what is wanting in velocity may be made up in power: and then the cog wheel may have a greater number of cogs in proportion to the rounds in the trundle, in order to give the millftone a fufficient degree of velocity.

They who have read what is ſaid in the firſt lecture, concerning the acceleration of bodies falling freely by the power of gravity acting conſtantly and uniformly upon them, may perhaps aſk, why ſhould the motion of the wheel be equable, and not accelerated, ſince the water acts conſtantly and uniformly upon it? The plain anſwer is, that the velocity of the wheel can never be ſo great as the velocity of the water that turns it; for, if it ſhould become ſo great, the power of the water would be quite loſt upon the wheel, and then there would be no proper force to overcome the friction of the geers and attrition of the ſtones. Therefore, the velocity with which the wheel begins to move, will increaſe no longer than till its *momentum* or force is balanced by the reſiſtance of the machine; and then the wheel will go on with an equable motion.

A *hand mill.* [If the cog wheel *D* be made about 18 inches diameter, with 30 cogs, the trundle as ſmall in proportion with 10 ſtaves, and the millſtones be each about two feet in diameter; and the whole work be put into a ſtrong frame of wood, as repreſented in the figure, the engine will be a hand-mill for grinding corn or malt in private families. And then, it may be turned by a winch inſtead of the wheel *A A*: the millſtone making three revolutions for every one of the winch. If a heavy fly be put upon the axle *B*, near the winch, it will aſſiſt greatly in regulating the motion.]

If the cogs of the wheel and rounds of the trundle could be put in as exactly as the teeth are cut in the wheels and pinions of a clock, then the

trundle

trundle might divide the wheel exactly: that is to say, the trundle might make a given number of revolutions for one of the wheel, without a fraction. But as any exact number is not neceſſary in mill-work, and the cogs and rounds cannot be ſet in ſo truly as to make all the intervals between them equal; a ſkilful mill-wright will always give the wheel what he calls *a hunting cog*; that is, one more than what will anſwer to an exact diviſion of the wheel by the trundle. And then, as every cog comes to the trundle, it will take the next ſtaff or round behind the one which it took in the former revolution: and by that means, will wear all the parts of the cogs and rounds which work upon one another equally, and to equal diſtances from one another in a little time; and ſo make a true uniform motion throughout the whole work. Thus, in the above water mill, the trundle has 10 ſtaves, and the wheel 61 cogs.

Sometimes, where there is a ſufficient quantity of water, the cog-wheel *AA* turns a large trundle Fig. 4. *BB*, on whoſe axis *C* is fixed the horizontal wheel *D*, with cogs all around its edge, turning two trundles *E* and *F* at the ſame time; whoſe axes or ſpindles *G* and *H* turn two millſtones *I* and *K*, upon the fixed ſtones *L* and *M*. And when there is not work for them both, either may be made to lie quiet, by taking out one of the ſtaves of its trundle, and turning the vacant place towards the cog wheel *D*. And there may be a wheel fixt on the upper end of the great upright axle *C* for turning a couple of boulting-mills; and other work for

 drawing

drawing up the ſacks, fanning and cleaning the corn, ſharpening of tools, &c.

A horſe mill. If, inſtead of the cog wheel *AA* and trundle *BB*, horizontal levers be fixed into the axle *C*, below the wheel *D*; then, horſes may be put to theſe levers for turning the mill: which is often done where water cannot be had for that purpoſe.

A wind mill. The working parts of a wind mill differ very little from thoſe of a water mill; only the former is turned by the action of the wind upon four ſails, every one of which ought (as is generally believed) to make an angle of $54\frac{2}{3}$ degrees with a plane perpendicular to the axis on which the arms are fixt for carrying them. It being demonſtrable, that when the ſails are ſet to ſuch an angle, and the axis turned towards the wind, it has the greateſt power upon them. But this angle anſwers only to the caſe of a vane or ſail juſt beginning to move*: for, when the vane has a certain degree of motion, it yields to the wind; and then that angle muſt be increaſed to give the wind its full effect.

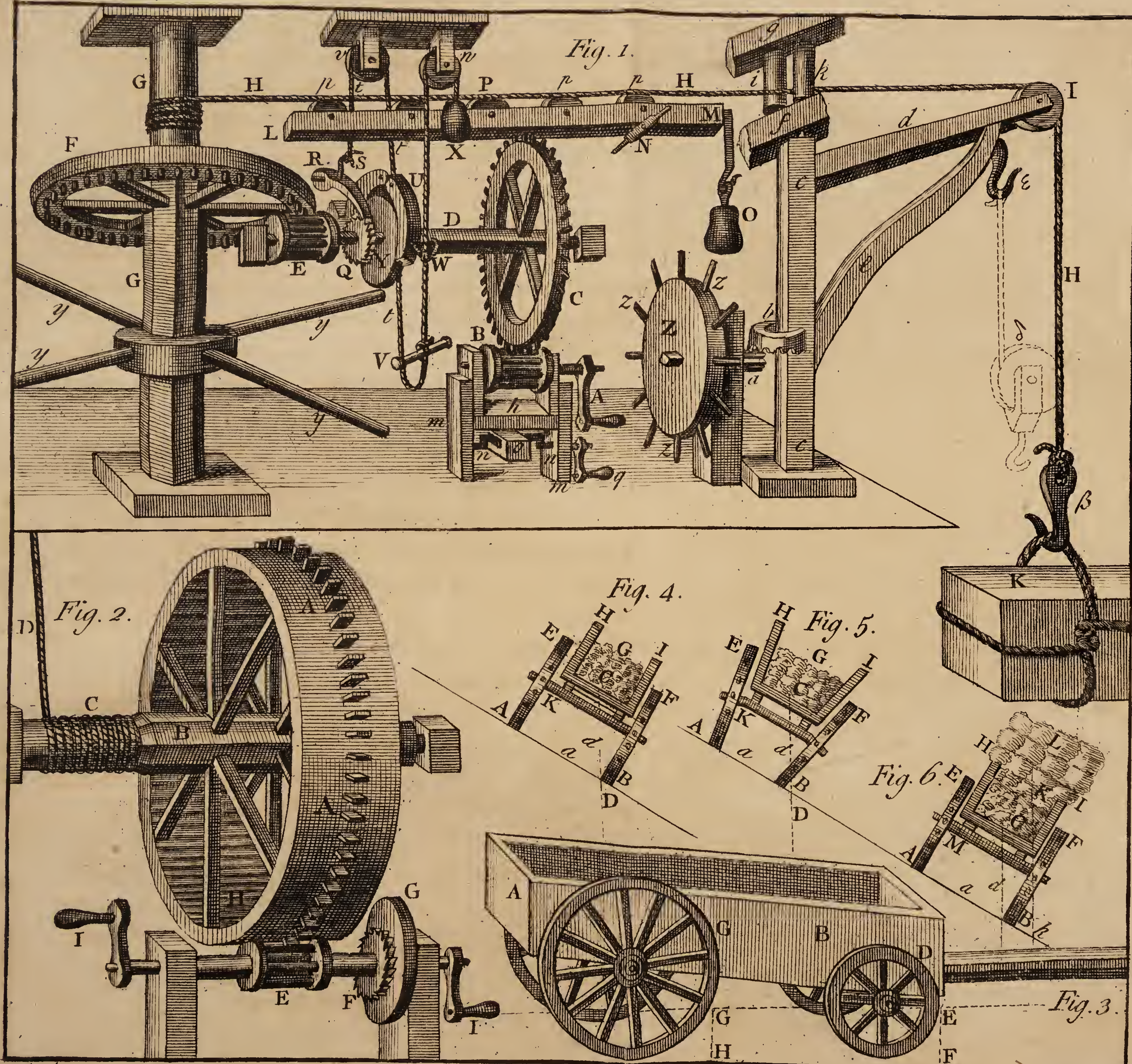
Again, the increaſe of this angle will be different, according to the different velocities from the axis to the extremity of the vane. At the axis it will be $54\frac{2}{3}$ degrees, and thence continually increaſe, giving the vane a twiſt, and ſo cauſing all the ribs of the vane to lie in different planes.

Laſtly, theſe ribs ought to decreaſe in length from the axis to the extremity, giving the vane a curvilineal form; ſo that no part of the force of

* See *MAC LAURIN*'s Fluxions, near the end.

any

PLATE VIII.



J. Ferguson delin.

J. Mynde sc.

any one rib be ſpent upon the reſt, but all move on independent of each other. All this is required to give the ſails of a wind mill their true form: and we ſee both the twiſt and the diminution of the ribs exemplified in the wings of birds.

It is almoſt incredible to think with what velocity the tips of the ſails move when acted upon by a moderate gale of wind. I have ſeveral times counted the number of revolutions made by the ſails in ten or fifteen minutes; and from the length of the arms from tip to tip, have computed, that if a hoop of that diameter was to run upon the ground with the ſame velocity that it would move if put upon the ſail-arms, it would go upwards of 30 miles in an hour.

As the ends of the ſails neareſt the axis cannot move with the ſame velocity that the tips or fartheſt ends do, although the wind acts equally ſtrong upon them; perhaps a better poſition than that of ſtretching them along the arms directly from the center of motion, might be to have them ſet perpendicularly acroſs the farther ends of the arms, and there adjuſted lengthwiſe to the proper angle. For, in that caſe, both ends of the ſails would move with the ſame velocity; and being farther from the center of motion, they would have ſo much the more power: and then, there would be no occaſion for having them ſo large as they are generally made; which would render them lighter, and conſequently, there would be ſo much the leſs friction on the thick neck of the axle where it turns in the wall.

A *crane*

A crane. Plate VIII. Fig. 1.

A *crane* is an engine by which great weights are raiſed to certain heights, or let down to certain depths. It conſiſts of wheels, axles, pullies, ropes, and a gib or gibbet. When the rope *H* is hooked to the weight *K*, a man turns the winch *A*, on the axis whereof is the trundle *B*, which turns the wheel *C*, on whoſe axis *D* is the trundle *E*, which turns the wheel *F* with its upright axis *G*, on which the great rope *HH* winds as the wheel turns; and going over a pulley *I* at the end of the arm *d* of the gib *ccde*, it draws up the heavy burthen *K*; which, being raiſed to a proper height, as from a ſhip to the quay, is then brought over the quay by pulling the wheel *Z* round by the handles *z*, *z*, which turns the gib by means of the half wheel *b* fixt on the gib-poſt *cc*, and the ſtrong pinion *a* fixt on the axis of the wheel *Z*. This wheel gives the man that turns it an abſolute command over the gib, ſo as to prevent it from taking any unlucky ſwing, ſuch as often happens when it is only guided by a rope tied to its arm *d*; and people are frequently hurt, ſometimes killed, by ſuch accidents.

The great rope goes between two upright rollers *i* and *k*, which turn upon gudgeons in the fixed beams *f* and *g*; and as the gib is turned towards either ſide, the rope bends upon the roller next that ſide. Were it not for theſe rollers, the gib would be quite unmanageable; for the moment it were turned ever ſo little towards any ſide, the weight *K* would begin to deſcend, becauſe the rope would be ſhortened between the pulley *I* and axis *G*; and ſo the gib would be pulled violently to that ſide, and either

either break itſelf to pieces, or every thing that came in its way. Theſe rollers muſt be placed ſo, that the ſides of them round which the rope bends, may keep the middle of the bended part directly over the center of the hole in which the upper gudgeon of the gib turns in the beam *f*. The truer theſe rollers are placed, the eaſier the gib is managed, and the leſs apt to ſwing either way by the force of the weight *K*.

A ratchet-wheel *Q* is fixt upon the axis *D*, near the trundle *E*; and into this wheel falls the catch or click *R*. This hinders the machine from running back by the weight of the burthen *K*, if the man who raiſes it ſhould happen to be careleſs, and ſo leave off working at the winch *A* ſooner than he ought to do.

When the burthen *K* is raiſed to its proper height from the ſhip, and brought over the quay by turning of the gib, it is let down gently upon the quay, or into a cart ſtanding thereon, in the following manner: A man takes hold of the rope *tt* (which goes over the pulley *v*, and is tied to a hook at *S* in the catch *R*) and ſo diſengages the catch from the ratchet wheel *Q*; and then, the man at the winch *A* turns it backward, and lets down the weight *K*. But if the weight pulls too hard againſt this man, another lays hold of the handle *V*, and by pulling it downward, draws the gripe *U* cloſe to the wheel *Y*, which, by rubbing hard againſt the gripe, hinders the too quick deſcent of the weight; and not only ſo, but even ſtops it at any time, if required. By this means, heavy

heavy goods may be either raiſed or let down at pleaſure, without any danger of hurting the men who work the engine.

When part of the goods are craned up, and the rope is to be let down for more, the catch *R* is firſt diſengaged from the ratchet wheel *Q*, by pulling the cord *t*; then the handle *q* is turned half round backward, which, by the crank *nn* in the piece *o*, pulls down the frame *h* between the guides *m* and *m* (in which it ſlides in a groove) and ſo diſengages the trundle *B* from the wheel *C*: and then, the heavy hook *β* at the end of the rope *H* deſcends by its own weight, and turns back the great wheel *F* with its trundle *E*, and the wheel *C*; and this laſt wheel acts like a fly againſt the wheel *F* and hook *β*; and ſo hinders it from going down too quick; whilſt the weight *X* keeps up the gripe *U* from rubbing againſt the wheel *Y*, by means of a cord going from the weight, over the pulley *w* to the hook *W* in the gripe: ſo that the gripe never touches the wheel, unleſs it be pulled down by the handle *V*.

When the crane is to be ſet at work again, for drawing up another burthen, the handle *q* is turned half round forwards; which, by the crank *nn*, raiſes up the frame *h*, and cauſes the trundle *B* to lay hold of the wheel *C*; and then, by turning the winch *A*, the burthen of goods *K* is drawn up as before.

The crank *nn* turns pretty ſtiff in the mortiſe near *o*, and ſtops againſt the farther end of it when it has got juſt a little beyond the perpendicular; ſo

ſo that it can never come back of itſelf: and therefore, the trundle *B* can never come away from the wheel *C*, until the handle *q* be turned half round.

The great rope runs upon rollers in the lever *LM*, which keep it from bending between the axle at *G* and the pulley *I*. This lever turns upon the axis *N* by means of the weight *O*, which is juſt ſufficient to keep its end *L* up to the rope; ſo that, as the great axle turns, and the rope coils round it, the lever riſes with the rope, and prevents the coilings from going over one another.

The power of this crane may be eſtimated thus, ſuppoſe the radius of the winch *A* to be twice the radius of the trundle *B*; this will make the power of the trundle upon the wheel *C* to be as 2: but if the trundle has 8 ſtaves, and the wheel 64 cogs, the trundle will increaſe the power of the wheel 8 times, which multiplied by 2, the power of the winch, is 16; and if the trundle *E* on the axis of the wheel *C* has 9 rounds, and the wheel *F* 72 cogs, the power is again 8 times increaſed; which makes the power of the wheel *F* as 8 times 16, or 128: and laſtly, if the diameter of the wheel *F* be 8 times the diameter of that part of its axle *G* on which the rope winds, this again increaſes the power 8 times, and ſo makes it 8 times 128, or 1024, which is the whole power of the machine; and ſo much more weight a man can raiſe by it than he could lift in his arms with an equal degree of ſtrength.

If this power be thought greater than what may be generally wanted, the wheels may be made with fewer

fewer cogs in proportion to the ſtaves in the trundles; and ſo the power may be of whatever degree is judged to be requiſite. But if the weight be ſo great as will require yet more power to raiſe it (ſuppoſe a double quantity) then the rope *H* may be put under a moveable pulley as δ, and the end of it tied to a hook in the gib at ε; which will give a double power to the machine, and ſo raiſe a double weight hook'd to the block of the moveable pulley.

When only ſmall burthens are to be raiſed, this may be quickly done by men puſhing the axle *G* round by the handſpikes *y,y,y,y*; having firſt diſengaged the trundle *B* from the wheel *C*: and then, this wheel will only act as a fly upon the wheel *F*; and the catch *R* will prevent its running back, if the men ſhould inadvertently leave off puſhing before the burthen be unhooked from β.

Laſtly, when very heavy burthens are to be raiſed, which might endanger the breaking of the cogs in the wheel *F*; their force againſt theſe cogs may be much abated by men puſhing at the handſpikes *y,y,y,y*, whilſt the man at *A* turns the winch.

I have only ſhewn the working parts of this crane, without the whole of the beams which ſupport them; knowing that theſe are eaſily ſuppoſed, and that if they had been drawn, they would have hid a great deal of the working parts from ſight, and alſo confuſed the figure.

Another *crane.*

Another very good *crane* is made in the following manner. *AA* is a great wheel turned by men walking

walking within it at *H*. On the part *C*, of its axle *BC*, the great rope *D* is wound as the wheel turns; and this rope draws up goods in the ſame way as the rope *HH* does in the abovementioned crane, the gib-work here being ſuppoſed to be of the ſame ſort. But theſe cranes are very dangerous to the men in the wheel; for, if any of the men ſhould chance to fall, the burthen will make the wheel run back and throw them all about within it; which often breaks their limbs, and ſometimes kills them. The late ingenious Mr. *Padmore* of Briſtol (whoſe contrivance the forementioned crane is, ſo far as I can remember its conſtruction after ſeeing it once about twelve years ago) obſerving this dangerous conſtruction, contrived a method for remedying it, by putting cogs all around the outſide of the wheel, and applying a trundle *E* to turn it; which increaſes the power as much as the number of cogs in the wheel is greater than the number of ſtaves in the trundle: and by putting a ratchet wheel *F* on the axis of the trundle, (as in the abovementioned crane) with a catch to fall into it, the great wheel is ſtopt from running back by the force of the weight, even if all the men in it ſhould leave off walking. And by one man working at the winch *I*, or two men at the oppoſite winches when needful, the men in the wheel are much aſſiſted, and much greater weights are raiſed, than could be by men only within the wheel. Mr. *Padmore* put alſo a gripe-wheel *G* upon the axis of the trundle, which being pinched in the ſame manner as deſcribed in the former crane, heavy burdens may Fig. 2.

be

be let down without the leaſt danger. And before this contrivance, the lowering of goods was always attended with the utmoſt danger to the men in the wheel; as every one muſt be ſenſible of, who has ſeen ſuch engines at work.

And it is ſurpriſing that the maſters of wharfs and cranes ſhould be ſo regardleſs of the limbs or even lives of their workmen, that, excepting Sir *James Creed*, there is ſcarce an inſtance of any who uſes this ſafe contrivance.

Wheel carriages. The ſtructure of *wheel carriages* is generally ſo well known, that it would be needleſs to deſcribe them. And therefore, we ſhall only point out ſome inconveniencies attending the common method of placing the wheels, and loading the waggons.

In coaches, and all other four wheel'd carriages, the fore-wheels are made of a leſs ſize than the hind ones, both on account of turning ſhort, and to avoid cutting the braces: otherwiſe, the carriage would go much eaſier if the fore-wheels were as high as the hind ones, and the higher the better, becauſe their motion would be ſo much the ſlower on their axles, and conſequently the friction proportionably taken off. But carriers and coachmen give another reaſon for making the fore-wheels much lower than the hind-wheels; namely, that when they are ſo, the hind-wheels help to puſh on the fore ones: which is too unphiloſophical and abſurd to deſerve a refutation, and yet for their ſatisfaction we ſhall ſoon ſhew by experiment that it has no exiſtence but in their own imaginations.

4 It

It is plain that the ſmall wheels muſt turn as much oftener round than the great ones, as their circumferences are leſs. And therefore, when the carriage is loaded equally heavy on both axles, the fore axle muſt endure as much more friction, and conſequently wear out as much ſooner, than the hind axle, as the fore-wheels are leſs than the hind ones. But the great misfortune is, that all the carriers to a man do obſtinately perſiſt, againſt the cleareſt reaſon and demonſtration, in putting the heavier part of the load upon the fore axle of the waggon; which not only makes the friction greateſt where it ought to be leaſt, but alſo preſſeth the fore-wheels deeper into the ground than the hind-wheels, notwithſtanding the fore-wheels, being leſs than the hind ones, are with ſo much the greater difficulty drawn out of a hole or over an obſtacle, even ſuppoſing the weights on their axles were equal. For the difficulty, with equal weights, will be as the depth of the hole or height of the obſtacle is to the ſemidiameter of the wheel. Thus, if we ſuppoſe the ſmall wheel *D* of the waggon *AB* to fall into a hole of the depth *EF*, which is equal to the ſemidiameter of the wheel, and the waggon to be drawn horizontally along; it is evident, that the point *E* of the ſmall wheel will be drawn directly againſt the top of the hole; and therefore, all the power of horſes and men will not be able to draw it out, unleſs the ground gives way before it. Whereas, if the hind-wheel *C* falls into ſuch a hole, it ſinks not near ſo deep in proportion to its ſemidiameter; and therefore, the point *G* of the Fig. 3.

large wheel will not be drawn directly, but obliquely, against the top of the hole; and so will be easily got out of it. Add to this, that since a small wheel will often sink to the bottom of a hole, in which a great wheel will go but a very little way, the small wheels ought in all reason to be loaded with less weight than the great ones: and then the heavier part of the load would be less jolted upward and downward, and the horses tired so much the less, as their draught raised the load to less heights.

It is true, that when the waggon road is much up-hill, there may be danger in loading the hind part much heavier than the fore part; for then the weight would overhang the hind axle, especially if the load be high, and endanger tilting up the forewheels from the ground. In this case, the safest way would be to load it equally heavy on both axles; and then, as much more of the weight would be thrown upon the hind axle than upon the fore one, as the ground rises from a level below the carriage. But as this seldom happens, and when it does, a small temporary weight laid upon the pole between the horses would overbalance the danger; and this weight might be thrown into the waggon when it comes to level ground, it is strange that an advantage so plain and obvious as would arise from loading the hind-wheels heaviest, should not be laid hold of, by complying with this method.

To confirm these reasonings by experiment, let a small model of a waggon be made, with its forewheels 2 $\frac{1}{2}$ inches in diameter, and its hind-

wheels

wheels $4\frac{1}{2}$; the whole model weighing about 20 ounces. Let this little carriage be loaded any how with weights, and have a ſmall cord tied to each of its ends, equally high from the ground it reſts upon; and let it be drawn along a horizontal board, firſt by a weight in a ſcale hung to the cord at the fore part; the cord going over a pulley at the end of the board to facilitate the draught, and the weight juſt ſufficient to draw it along. Then, turn the carriage, and hang the ſcale and weight to the hind cord, and it will be found to move along with the ſame velocity as at firſt: which ſhews, that the power required to draw the carriage is all the ſame, whether the great or ſmall wheels are foremoſt; and therefore the great wheels do not help in the leaſt to puſh on the ſmall wheels in the road.

Hang the ſcale to the fore cord, and place the fore-wheels (which are the ſmall ones) in two holes, cut three eighth parts of an inch deep into the board; then put a weight of 32 ounces into the carriage, over the fore axle, and an equal weight over the hind one: this done, put 44 ounces into the ſcale, which will be juſt ſufficient to draw out the fore-wheels: but if this weight be taken out of the ſcale, and one of 16 ounces put into its place, if the hind-wheels are placed in the holes, the 16 ounce weight will draw them out; which is little more than a third part of what was neceſſary to draw out the fore-wheels. This ſhews, that the larger the wheels are, the leſs power will draw the carriage, eſpecially on rough ground.

 Put

Put 64 ounces over the axle of the hind-wheels, and 32 over the axle of the fore ones, in the carriage; and place the fore-wheels in the holes: then, put 38 ounces into the ſcale, which will juſt draw out the fore-wheels; and when the hind ones come to the hole, they will find but very little reſiſtance, becauſe they ſink but a little way into it.

But ſhift the weights in the carriage, by putting the 32 ounces upon the hind axle, and the 64 ounces upon the fore one; and place the fore-wheels in the holes: then, if 76 ounces be put into the ſcale, it will be found no more than ſufficient to draw out theſe wheels; which is double the power required to draw them out, when the lighter part of the load was put upon them: which is a plain demonſtration of the abſurdity of putting the heavieſt part of the load in the fore part of the waggon.

Every one knows what an out-cry was made by the generality, if not the whole body, of the carriers, againſt the broad-wheel act; and how hard it was to perſuade them to comply with it, even though the government allowed them to draw with more horſes, and carry greater loads, than uſual. Their principal objection was, that as a broad wheel muſt touch the ground in a great many more points than a narrow wheel, the friction muſt of courſe be juſt ſo much the greater; and conſequently, there muſt be ſo many more horſes than uſual, to draw the waggon. I believe that the majority of people were of the ſame opinion, not conſidering, that if the whole weight of the waggon and

and load in it bears upon a great many points, each ſuſtains a proportionably leſs degree of weight and friction, than when it bears only upon a few points; ſo that what is wanting in one is made up in the other; and therefore will be juſt equal under equal degrees of weight, as may be ſhewn by the following plain and eaſy experiment.

Let one end of a piece of packthread be faſtened to a brick, and the other end to a common ſcale for holding weights: then, having laid the brick edgewiſe on a table, and let the ſcale hang under the edge of the table, put as much weight into the ſcale as will juſt draw the brick along the table. Then, taking back the brick to its former place, lay it flat on the table, and leave it to be acted upon by the ſame weight in the ſcale as before, which will draw it along with the ſame eaſe as when it lay upon its edge. In the former caſe, the brick may be conſidered as a narrow wheel on the ground; and in the latter, as a broad wheel. And ſince the brick is drawn along with equal eaſe, whether its broad ſide or narrow edge touches the table, it ſhews that a broad wheel might be drawn along the ground with the ſame eaſe as a narrow one, (ſuppoſing them equally heavy) even though they ſhould drag, and not roll, as they go along.

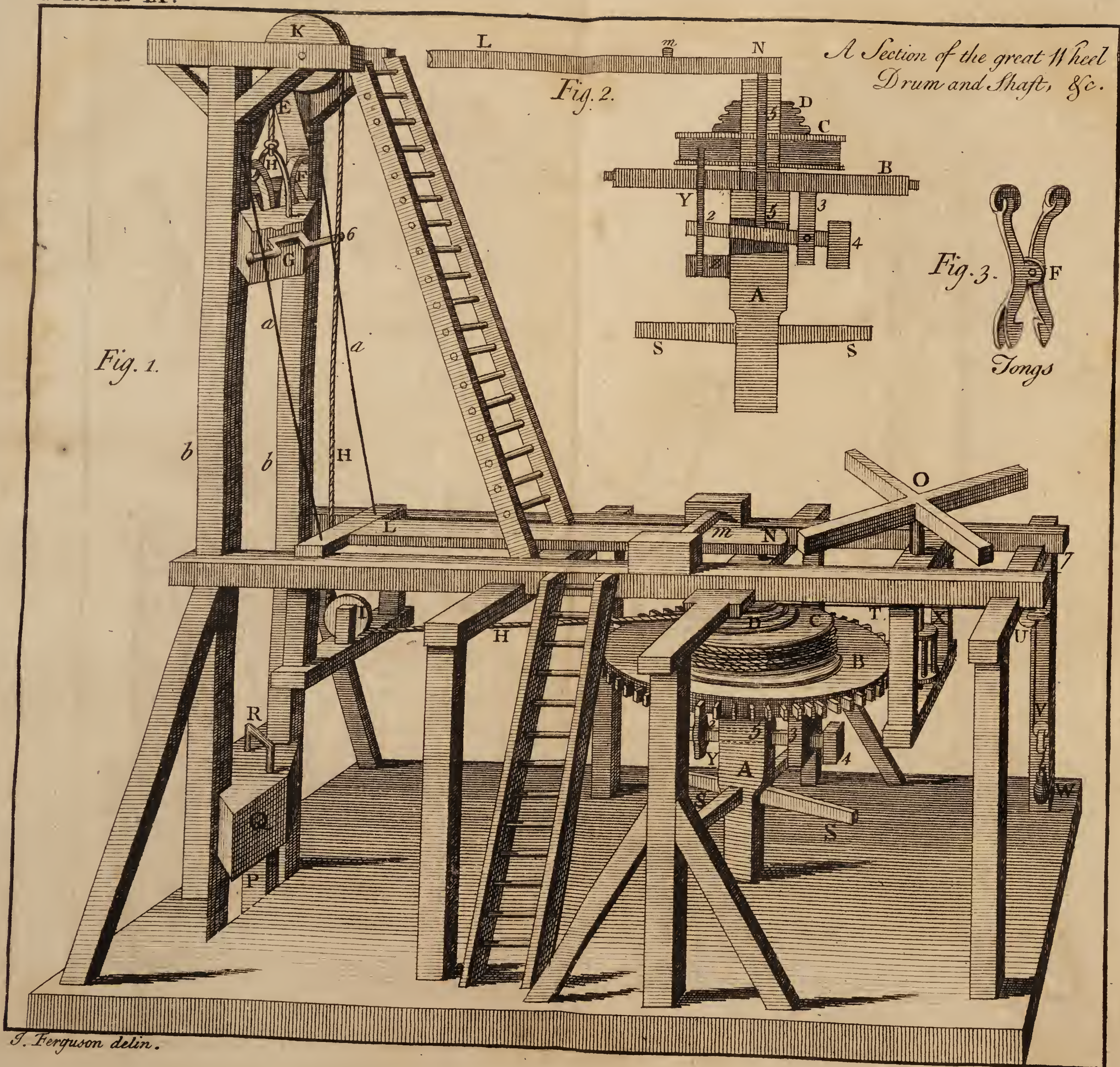
As narrow wheels are always ſinking into the ground, eſpecially when the heavieſt part of the load lies upon them, they muſt be conſidered as going conſtantly up-hill, even on level ground. And their edges muſt ſuſtain a great deal of friction by rubbing againſt the ſides of the ruts made by

them. But both theſe inconveniencies are avoided by broad wheels; which, inſtead of cutting and ploughing up the roads, roll them ſmooth and harden them; as experience teſtifies in places where they have been uſed, eſpecially either on wettiſh or ſandy ground: though after all it muſt be confeſt, that they will not do in ſtiff clayey croſs roads, becauſe they would ſoon gather up the weight of an ordinary load of clay.

If the wheels were always to go upon ſmooth and level ground, the beſt way would be to make the ſpokes perpendicular to the naves; that is, to ſtand at right angles to the axles, becauſe they would then bear the weight of the load perpendicularly, which is the ſtrongeſt way for wood. But becauſe the ground is generally uneven, one wheel often falls into a cavity or rut when the other does not; and then it bears much more of the weight than the other does: in which caſe, concave or diſhing wheels are beſt, becauſe when one falls into a rut, and the other keeps upon high ground, the ſpokes become perpendicular in the rut, and therefore have the greateſt ſtrength when the obliquity of the load throws moſt of its weight upon them; whilſt thoſe on high ground have leſs weight to bear, and therefore need not be at their full ſtrength. So that the uſual way of making the wheels concave is by much the beſt.

The axles of the wheels ought to be perfectly ſtraight, that the rims of the wheels may be parallel to each other; for then they will move eaſieſt, becauſe they will be at liberty to go on ſtraight forwards.

PLATE IX.



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forwards. But in the usual way of practice, the axles are bent downward at their ends; which brings the sides of the wheels next the ground nearer to one another than their opposite or higher sides are: and this not only makes the wheels to drag sidewise as they go along, and gives the load a much greater power of crushing them than when they are parallel to each other, but also endangers the overturning of the carriage when any wheel falls into a hole or rut; or when the carriage goes in a road which has one side lower than the other, as along the side of a hill. Thus, (in the hind view of a waggon or cart) let *AE* and *BF* be the great wheels parallel to each other, on their straight axle *K*, and *HCI* the carriage loaded with heavy goods from *C* to *G*. Then, as the carriage goes on in the oblique road *AaB*, the center of gravity of the whole machine and load will be at *C**; and the line of direction *CdD* falling within the wheel *BF*, the carriage will not overset. But if the wheels be inclined to each other at the ground, as *AE* and *BF*, and the machine be loaded as before, from *C* to *G*, the line of direction *CdD* falls without the wheel *BF*, and the whole machine tumbles over. When it is loaded with heavy goods (such as lead or iron) which lie low, it may travel safely upon an oblique road so long as the center of gravity is at *C*, and the line of direction *Cd* falls within the wheels; but if it be loaded high with lighter goods (such as woolpacks) from *C* to *L*, the center of gravity is raised from *C* to *K*, which throws the line of

Fig. 4.

Fig. 5.

Fig. 6.

* See page 14.

of direction *Kk* without the wheel *BF*, and then the load overſets the waggon.

If there be ſome advantage from ſmall fore-wheels, on account of the carriage turning more eaſily and ſhort than it can be made to do when they are large; there is at leaſt as great a diſadvantage attending them, which is, that as their axle is below the level of the horſes breaſts, the horſes not only have the loaded carriage to draw along, but alſo part of its weight to bear; which tires them ſooner, and makes them grow much ſtiffer in their hams, than they would be if they drew on a level with the fore axle. And for this reaſon, we find coach horſes ſoon become unfit for riding. So that on all accounts it is plain, that the fore-wheels of all carriages ought to be ſo high, as to have their axles even with the breaſts of the horſes; which would not only give the horſes a fair draught, but likewiſe cauſe the machine to be drawn by a leſs degree of power.

Plate IX. Fig. 1, 2. We ſhall conclude this lecture with a deſcription of Mr. *Vauloue*'s curious engine, which was made uſe of for driving the piles of Weſtminſter-bridge: and the reader may caſt his eyes upon the firſt and ſecond figures of the plate, in which the ſame letters of reference are annexed to the ſame parts, in order to explain thoſe in the ſecond, which are either partly or wholely hid in the firſt.

The *pile engine.* *A* is the great upright ſhaft or axle, on which are the great wheel *B* and drum *C*, turned by horſes joined to the bars *S*, *S*. The wheel *B* turns the trundle

trundle *X*, on the top of whoſe axis is the fly *O*, which ſerves to regulate the motion, and alſo to act againſt the horſes, and keep them from falling when the heavy ram *Q* is diſcharged to drive the pile *P* down into the mud in the bottom of the river. The drum *C* is looſe upon the ſhaft *A*, but is locked to the wheel *B* by the bolt *Y*. On this drum the great rope *HH* is wound; one end of the rope being fixed to the drum, and the other to the follower *G*, to which it is conveyed over the pullies *I* and *K*. In the follower *G* is contained the tongs *F* (See Fig. 3.) that takes hold of the ram *Q* by the ſtaple *R* for drawing it up. *D* is a ſpiral or fuſee fixt to the drum, on which is wound the ſmall rope *T* that goes over the pulley *U*, under the pulley *V*, and is faſtened to the top of the frame at 7. To the pulley-block *V* is hung the counterpoiſe *W*, which hinders the follower from accelerating as it goes down to take hold of the ram: for, as the follower tends to acquire velocity in its deſcent, the line *T* winds downwards upon the fuſee, on a larger and larger radius, by which means the counterpoiſe *W* acts ſtronger and ſtronger againſt it; and ſo allows it to come down with only a moderate and uniform velocity. The bolt *Y* locks the drum to the great wheel, being puſhed upward by the ſmall lever 2, which goes through a mortiſe in the ſhaft *A*, turns upon a pin in the bar 3 fixt into the great wheel *B*, and has a weight 4 which always tends to puſh up the bolt *Y* through the wheel into the drum. *L* is the great lever turning on the axis *m*, and reſting upon the forcing bar

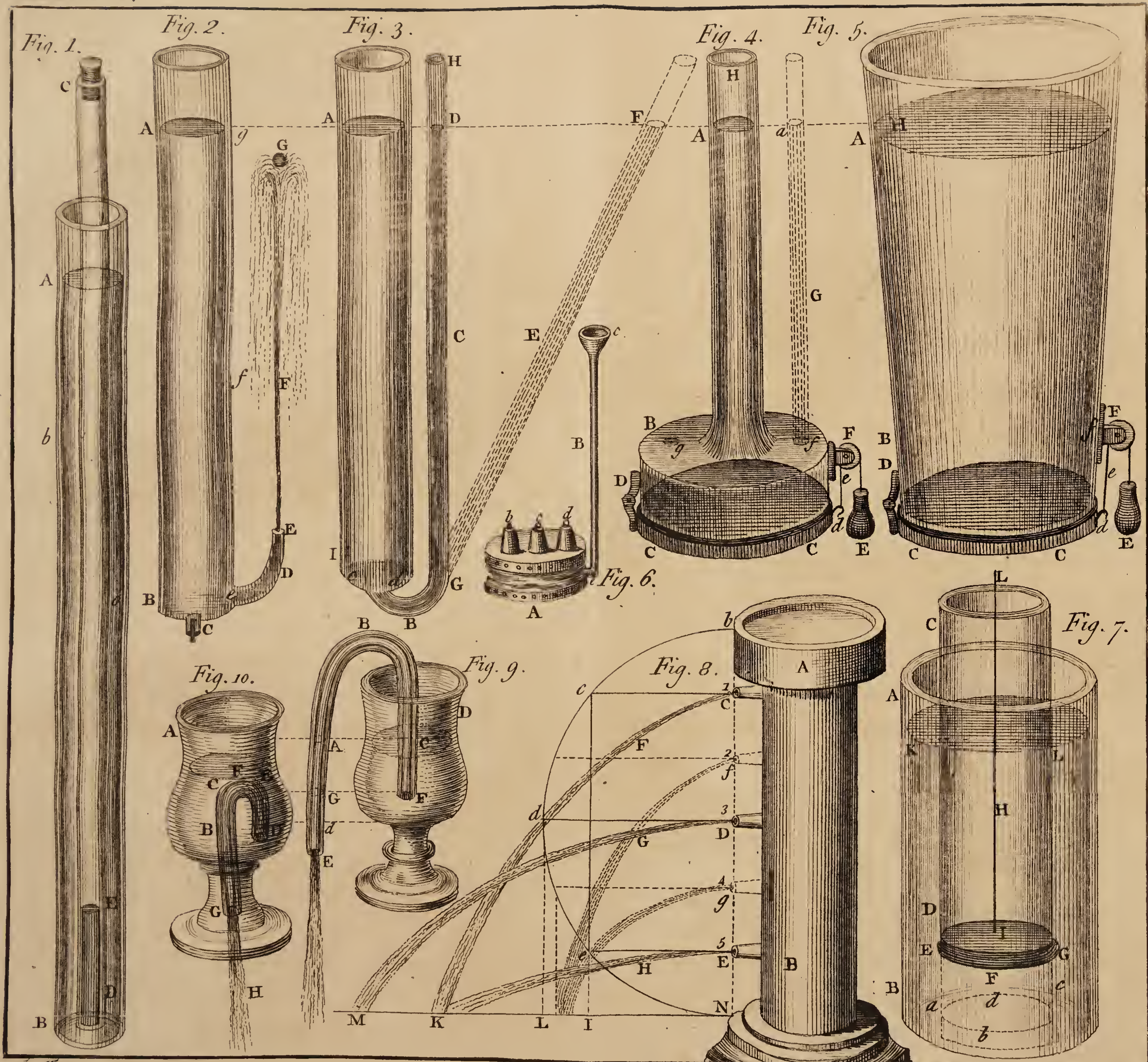
bar 5, 5, which goes down through a hollow in the ſhaft *A*, and bears upon the little lever 2.

By the horſes going round, the great rope *H* is wound about the drum *C*, and the ram *Q* is drawn up by the tongs *F* in the follower *G*, until the tongs comes between the inclined planes *E*; which, by ſhutting the tongs at the top, opens it at the foot, and diſcharges the ram, which falls down between the guides *b b* upon the pile *P*, and drives it by a few ſtrokes as far into the mud as it can go; after which, the top-part is ſawed off cloſe to the mud, by an engine for that purpoſe. Immediately after the ram is diſcharged, the piece 6 upon the follower *G* takes hold of the ropes *a*, *a*, which raiſe the end of the lever *L*, and cauſe its end *N* to deſcend and preſs down the forcing bar 5 upon the little lever 2, which, by pulling down the bolt *Y*, unlocks the drum *C* from the great wheel *B*; and then the follower, being at liberty, comes down by its own weight to the ram; and the lower ends of the tongs ſlip over the ſtaple *R*, and the weight of their heads cauſes them to fall outward, and ſhuts upon it. Then the weight 4 puſhes up the bolt *Y* into the drum, which locks it to the great wheel, and ſo the ram is drawn up as before.

As the follower comes down, it cauſes the drum to turn backward, and unwinds the rope from it, whilſt the horſes, great wheel, trundle, and fly go on with an uninterrupted motion: and as the drum is turning backward, the counterpoiſe *W* is drawn up, and its rope *T* wound upon the ſpiral fuſee *D*.

There

PLATE X.



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There are ſeveral holes in the under ſide of the drum, and the bolt *Y* always takes the firſt one that it finds when the drum ſtops by the falling of the follower upon the ram; until which ſtoppage, the bolt has not time to ſlip into any of the holes.

This engine was placed upon a barge on the water, and ſo was eaſily conveyed to any place deſired.—I never had the good fortune to ſee it, but drew this figure from a model which I made from a print of it; being not quite ſatisfied with the view which the print gives. I have been told that the ram was a ton weight, and that the guides *b b*, between which it was drawn up and let fall down, were 30 feet high. I ſuppoſe the great wheel may have had 100 cogs, and the trundle 10 ſtaves or rounds; ſo that the fly would make 10 revolutions for one of the great wheel.

L E C T. V.

Of hydroſtatics, and hydraulic machines, in general.

THE ſcience of *hydroſtatics* treats of the nature, gravity, preſſure, and motion of fluids in general; and of weighing ſolids in them.

A fluid is a body that yields to the leaſt preſſure, or difference of preſſures. Its particles muſt be exceedingly ſmall, becauſe they cannot be diſcerned by the beſt of microſcopes; they muſt be hard, becauſe no fluid, except air or ſteam, can be preſſed into a leſs ſpace than it naturally poſſeſſes; they muſt alſo be round and ſmooth, becauſe they are ſo eaſily Definition of a fluid.

eaſily

eaſily moved among one another; unleſs it can be ſhewn that their particles do not actually touch each other, which would indeed ſeem to be the caſe with ſuch as are pungent and ſharp to the taſte.

All bodies, both fluid and ſolid, preſs downwards by the force of gravity: but fluids have this wonderful property, that their preſſure upwards and ſidewiſe is equal to their preſſure downwards; and this is always in proportion to their perpendicular height, without any regard to their quantity: for, as each particle is quite free to move, it will move towards that part or ſide on which the preſſure is leaſt. And hence, no particle or quantity of a fluid can be at reſt till it is every way equally preſſed.

Plate X. Fig. 1. Fluids preſs as much upward as downward.

To ſhew by experiment that fluids preſs upward as well as downward, let *AB* be a long upright tube, filled with water near to its top; and *CD* a ſmall tube open at both ends, and immerſed into the water in the large one: if the immerſion be quick, you will ſee the water riſe in the ſmall tube to the ſame height that it ſtands in the great one, or until the ſurfaces of the water in both are on the ſame level: which ſhews that the water is preſt upward into the ſmall tube by the weight of what is in the great one; otherwiſe it could never riſe therein, contrary to its natural gravity; unleſs the diameter of the bore were ſo ſmall, that the attraction of the tube would raiſe the water; which will never happen if the tube be as wide as that in a common barometer. And, as the water riſes no higher in the ſmall tube than till its ſurface be

on

on a level with the surface of the water in the great one, this shews that the pressure is not in proportion to the quantity of water in the great tube, but in proportion to its perpendicular height therein: for there is much more water in the great tube, all around the small one, than what is raised to the same height in the small one, as it stands in the great.

Take out the small tube, and let the water run out of it; then it will be filled with air. Stop its upper end with the cork *C*, and it will be full of air all below the cork: this done, plunge it again to the bottom of the water in the great tube; and you will see the water rise up in it to the height *E*, which shews that the air is a body, otherwise it could not hinder the water from rising up to the same height as it did before, namely to *A*; and in so doing, it drove the air out at the top; but now the air is confined by the cork *C*: and it also shews that the air is a compressible body, for if it were not so, a drop of water could not enter into the tube.

The pressure of fluids being equal in all directions, it follows that the sides of a vessel are as much pressed by a fluid in it, all around in any given ring of points, as the fluid below that ring is pressed by the weight of all that stands above it. Hence, the pressure upon every point in the sides, immediately above the bottom, is equal to the pressure upon every point of the bottom. To shew this by experiment, let a hole be made at *e* in the side of the tube *AB* close by the bottom; and another Fig. 2.

another hole of the ſame ſize in the bottom at *C*; then pour water into the tube, keeping it full as long as you chooſe the holes ſhould run, and have two baſons ready to receive the water that runs through the two holes, until you think there is enough in each baſon; and you will find by meaſuring the quantities, that they are equal; which ſhews that the water run with equal ſpeed through both holes: and this it could not have done, unleſs it had been equally preſſed through them both. For, if a hole of the ſame ſize be made in the ſide of the tube, as about *f*, and if all three are permitted to run together, you will find that the quantity run through the hole at *f* is much leſs than what has run in the ſame time through either of the holes *C* or *e*.

In the ſame figure, let the tube be re-curved from the bottom at *C* into the ſhape *DE*, and the hole at *C* be ſtopt with a cork. Then, pour water into the tube to any height, as *Ag*, and it will ſpout up in a jet *EFG*, nearly as high as it is kept in the tube *AB*, by continuing to pour in as much there as runs through the hole *E*; which will be the caſe whilſt the ſurface *Ag* keeps at the ſame height. And if a little ball of cork *G* be laid upon the top of the jet, it will be ſupported thereby, and dance upon it.—The reaſon why the jet riſes not quite ſo high as the ſurface of the water *Ag*, is owing to the reſiſtance it meets with in the open air: for if a tube either great or ſmall was ſcrewed upon the pipe at *E*, the water would riſe in it until the ſurfaces of the water in both tubes were on the ſame level; as will be ſhewn by the next experiment.

Any

Any quantity of a fluid, how small soever, may be made to balance and support any quantity how great soever. This is deservedly termed *the hydrostatical paradox*, which we shall first shew by an experiment, and then account for it upon the principle above mentioned, namely, that *the pressure of fluids is directly as their perpendicular height, without any regard to their quantity.* The *hydrostatic paradox.*

Let a small glass tube DCG, open at both ends, and bended at B, be joined to the end of a great one AI at cd, where the great one is also open; so that these tubes may in their openings freely communicate with each other. Then pour water through a small necked funnel into the small tube at H; this water will run through the joining of the tubes at cd, and rise up into the great tube; and if you continue pouring until the surface of the water comes to any part, as A, in the great tube, and then leave off, you will see that the surface of the water in the small tube will be just as high, at D; so that the perpendicular altitude of the water will be the same in both tubes, however small the one be in proportion to the other. This shews, that the small column DCG balances and supports the great column Acd; which it could not do, if their pressures were not equal against one another in the recurved bottom at B.—If the small tube be made longer, and inclined in the situation GEF, the surface of the water in it will stand at F, on the same level with the surface A in the great tube; that is, the water will have the same perpendicular height in both tubes, although the Fig. 3.

column

column in the ſmall tube is longer than that in the great one; the former being oblique, and the latter perpendicular.

Since then, the preſſure of fluids is directly as their perpendicular heights, without any regard to their quantities, it appears, that whatever the figure or ſize of veſſels be, if they are of equal heights, and if the areas of their bottoms are equal, the preſſures of equal heights of water are equal upon the bottoms of theſe veſſels; even though the one ſhould hold a thouſand or ten thouſand times as much water as would fill the other. To confirm this part of the hydroſtatical paradox by an experiment, let two veſſels be prepared, of equal heights, but very unequal contents, ſuch as *AB* in Fig. 4. and *AB* in Fig. 5. Let each veſſel be open at both ends, and their bottoms *Dd*, *Dd* be of equal widths. Let a braſs bottom *CC* be exactly fitted to each veſſel, not to go into it, but for it to ſtand upon; and let a piece of wet leather be put between each veſſel and its braſs bottom, for the ſake of cloſeneſs. Join each bottom to its veſſel by a hinge *D*, ſo that it may open like the lid of a box; and let each bottom be kept up to its veſſel by equal weights *E* and *E*, hung to lines which go over the pullies *F* and *F* (whoſe blocks are fixed to the ſides of the veſſels at *f*) and the lines tied to hooks at *d* and *d*, fixed in the braſs bottoms oppoſite to the hinges *D* and *D*. Things being thus prepared and fitted, hold the veſſel *AB* (Fig. 5.) upright in your hands over a baſon on a table, and cauſe water to be poured into the veſſel

Fig. 4, 5.

ſlowly,

slowly, till the pressure of the water on its bottom bears it down from the vessel at the side *d*, and raises the weight *E*; and then the water will run out at *d*. Mark the height at which the surface *H* of the water stood in the vessel, when the bottom began to give way at *d*; and then, holding up the other vessel *AB* (Fig. 4.) in the same manner, cause water to be poured into it at *H*; and you will see that when the water rises to *A* in this vessel, just as high as it did in the former, its bottom will also give way at *d*, and it will lose all the water.

The natural reason of this surprising phenomenon is, that since all parts of a fluid at equal depths below the surface are equally pressed in all manner of directions, the water immediately below the fixed part *Bf* (Fig. 4.) will be pressed as much upward against its lower surface within the vessel, by the action of the column *Ag*, as it would be by a column of the same height, and of any diameter whatever; (as was evident by the experiment with the tube Fig. 3.) and therefore, since action and reaction are equal and contrary to one another, the water, immediately below the surface *Bf*, will be pressed as much downward by it, as if it was immediately touched and pressed by a column of the height *gA*, and of the diameter *Bf*: and therefore, the water in the cavity *BDdf* will be pressed as much downward upon its bottom *CC*, as the bottom of the other vessel (Fig. 5.) is pressed by all the water above it.

To illustrate this a little farther, let a hole be made at *f* in the fixed top *Bf*, and let a tube *G* Fig. 4.

be put into it; then, if water be poured into the tube *A*, it will (after filling the cavity *Bd*) riſe up into the tube *G*, until it comes to a level with that in the tube *A*; which is manifeſtly owing to the preſſure of the water in the tube *A*, upon that in the cavity of the veſſel below it. Conſequently, that part of the top *Bf*, in which the hole is now made, would, if corked up, be preſſed upward with a force equal to the weight of all the water which is ſupported in the tube *G*: and the ſame thing would hold at *g*, if a hole were made there. And ſo, if the whole cover or top *Bf* were full of holes, and had tubes put into them, the water in each tube would riſe to the ſame height as it is kept into the tube *A*, by pouring more into it, to make up the deficiency that it ſuſtains by ſupplying the others, until they were all full, if equally high: and then the water in the tube *A* would ſupport equal heights of water in all the reſt of the tubes. Or, if all the tubes except *A*, or any other one, were taken away, and a large tube equal in diameter to the whole top *Bf* were placed upon it, and cemented to it, and then if water were poured into the tube that was left in either of the holes, it would aſcend through all the reſt of the holes, until it filled the large tube to the ſame height that it ſtands in the ſmall one, after a ſufficient quantity had been poured into it: which ſhews, that the top *Bf* was preſſed upward by the water under it; and before any hole was made in it, with a force equal to that wherewith it is now preſſed downward by the weight of all the water above it in the great tube. And therefore,

therefore, the re-action of the fixed top Bf must be as great, in pressing the water downward upon the bottom CC, as the whole pressure of the water in the great tube would have been if the top had been taken away, and the water in that tube left to press directly upon the water in the cavity $BDdf$.

Perhaps the best machine in the world for demonstrating the upward pressure of fluids, is the hydrostatic bellows A; which consists of two thick oval boards, each about 16 inches broad and 18 inches long, covered with leather, to open and shut like a common bellows, but without valves; only a pipe B, three feet high, is fixed into the bellows at e. Let some water be poured into the pipe at c, which will run into the bellows, and separate the boards a little. Then, lay three weights b, c, d, each weighing 100 pounds, upon the upper board; and pour more water into the pipe B, which will run into the bellows, and raise up the board with all the weights upon it: and if the pipe be kept full, until the weights are raised as high as the leather which covers the bellows will allow them, the water will remain in the pipe, and support all the weights, even though it should weigh no more than a quarter of a pound, and they 300 pounds: nor will all their force be able to cause them to descend and force the water out at the top of the pipe.

Fig. 6.

The *hydrostatic bellows.*

The reason of this will be made evident, by considering what has been already said of the result of the pressure of fluids, from its being as their perpendicular height without any regard to their

 quantity.

quantity. For, if a hole be made in the upper board, and a tube be put into it, the water will rise in the tube to the same height that it has in the pipe; and would rise as high (by supplying the pipe) in as many tubes as the board could contain holes. Now, suppose only one hole to be made in any part of the board, of an equal diameter with the bore of the pipe *B*; and that the pipe holds just a quarter of a pound of water: if a person claps his finger upon the hole, and the pipe be filled with water, he will find his finger to be pressed upward with a force equal to a quarter of a pound. And as the same pressure is equal upon all equal parts of the board, each part whose area is equal to the area of the hole will be pressed upward with a force equal to that of a quarter of a pound: the sum of all which pressures against the under side of an oval board 16 inches broad, and 18 inches long, will amount to 300 pounds; and therefore, so much weight will be raised up, and supported, by a quarter of a pound of water in the pipe.

How a man may raise himself upward by his breath.

Hence, if a man stands upon the upper board, and blows into the bellows through the pipe *B*, he will raise himself upward upon the board: and the smaller the bore of the pipe is, the easier he will be able to raise himself. And then, by clapping his finger upon the top of the pipe, he can support himself as long as he pleases; provided the bellows be air-tight, so as not to lose what is blown into it.

This figure, I confess, ought to have been much larger than any other upon the plate; but it was not

not thought of, until all the rest were drawn: and it could not so properly come into any other plate.

Upon this principle of the upward pressure of fluids, a piece of lead may be made to swim in water, by immersing it to a proper depth, and keeping the water from getting above it. Let *CD* be a glass tube, open at both ends, and *EFG* a flat piece of lead, exactly fitted to the lower end of the tube, not to go within it, but for it to stand upon; with a wet leather between the lead and tube to make close work. Let this leaden bottom be half an inch thick, and held close to the tube by pulling the packthread *IHL* upward at *L* with one hand, whilst the tube is held in the other by the upper end *C*. In this situation, let the tube be immersed in water in the glass vessel *AB*, to the depth of three inches below the surface of the water at *K*; and then, the leaden bottom *EFG* will be plunged to the depth of somewhat more than eleven times its own thickness: holding the tube at that depth, you may let go the thread at *L*; and the lead will not fall from the tube, but will be kept to it by the upward pressure of the water below it, occasioned by the height of the water *K* above the level of the lead. The reason whereof is, that as lead is 11.33 times heavier than its bulk of water; and is therefore immersed to a depth somewhat more than 11.33 times its thickness, and no water getting into the tube between it and the lead, the column of water *EabcG* below the lead is pressed upward against it by the water *KDEGL* all

How lead may be made to swim in water. Fig. 7.

all around the tube; which being a little more than 11.33 times as high as the lead is thick, is ſufficient to balance and ſupport the lead at the depth *KE*. If a little water be poured into the tube upon the lead, it will increaſe the weight upon the column of water under the lead, and cauſe the lead to fall from the tube to the bottom of the glaſs veſſel; where it will lie in the ſituation *bd*. Or, if the tube be raiſed a little in the water, the lead will fall by its own weight, which will then be too great for the preſſure of the water around the tube upon the column of water below it.

How light wood may be made to lie at the bottom of water.

Let two pieces of wood be plained quite flat, ſo as no water may get in between them if they be put together: let one of the pieces as *bd* be cemented to the bottom of the veſſel *AB* (Fig. 7.) and the other piece be laid flat and cloſe upon it, and held down to it by a ſtick, whilſt water is poured into the veſſel; then remove the ſtick, and the upper piece of wood will not riſe from the lower one: for, as the upper one is preſſed down both by its own weight and the weight of all the water over it, whilſt the contrary preſſure of the water is kept off by the wood under it, it will lie as ſtill as a ſtone would do in its place. But if it be raiſed ever ſo little at any edge, ſome water will then get under it; which being acted upon by the water above, will immediately preſs it upward; and as it is lighter than its bulk of water, it will riſe, and ſwim upon the ſurface of the water.

All fluids weigh juſt as much in their own element as they do in open air. To prove this by experiment,

experiment, let as much ſhot be put into a phial, as, when corked, will make it ſink in water: and being thus charged, let it be weighed, firſt in air, and then in water, and the weights in both caſes wrote down. Then, as the phial hangs ſuſpended in water, and counterpoiſed, pull out the cork, that water may run into it, and it will deſcend, and pull down that end of the beam. This done, put as much weight into the oppoſite ſcale as will reſtore the equipoiſe; which weight will be found to anſwer exactly to the additional weight of the phial when it is again weighed in air with the water in it.

The velocity of ſpouting water.

The velocity with which water ſpouts out at a hole in the ſide or bottom of a veſſel, is as the * ſquare root of the depth or diſtance of the hole below the ſurface of the water. For, in order to make double the quantity of a fluid run through one hole as through another of the ſame ſize, it will require four times the preſſure of the other, and therefore four times its depth below the ſurface of the water: and for the ſame reaſon, three times the quantity running in an equal time through the ſame ſort of hole, muſt run with three times the velocity, which will require nine times the preſſure; and conſequently muſt be nine times as deep below the ſurface of the fluid: and ſo on.— To prove this by an experiment, let two pipes, as

* The ſquare root of any number is that which being multiplied by itſelf produces the ſaid number. Thus, 2 is the ſquare root of 4, and 3 is the ſquare root of 9: for 2 multiplied by 2 produces 4, and 3 multiplied by 3 produces 9, &c.

Fig. 8. *C* and *g*, of equal ſiz'd bores, be fixed into the ſide of the veſſel *AB*; the pipe *g* being four times as deep below the ſurface of the water at *b* in the veſſel as the pipe *C* is: and whilſt theſe pipes run, let water be conſtantly poured into the veſſel, to keep the ſurface ſtill at the ſame height. Then, if a cup that holds a pint be placed at *K* to receive the water that ſpouts from the pipe *C*, and at the ſame moment a cup that holds a quart be placed near *L* to receive the water that ſpouts from the pipe *g*, both cups will be filled at the ſame time by their reſpective pipes.

The horizontal diſtance to which water will ſpout from pipes.

The horizontal diſtance, to which a fluid will ſpout from a horizontal pipe, in any part of the ſide of an upright veſſel below the ſurface of the fluid, is equal to twice the length of a perpendicular to the ſide of the veſſel, drawn from the mouth of the pipe to a ſemicircle deſcribed upon the altitude of the fluid: and therefore, the fluid will ſpout to the greateſt diſtance poſſible from a pipe, whoſe mouth is at the center of the ſemicircle; becauſe a perpendicular to its diameter (ſuppoſed parallel to the ſide of the veſſel) drawn from that point, is the greateſt that can poſſibly be drawn from any part of the diameter to the circumference of the ſemicircle. Fig. 8. Thus, if the veſſel *AB* be full of water, the horizontal pipe *D* be in the middle of its ſide, and the ſemicircle *Nedcb* be deſcribed upon *D* as a center, with the radius or ſemidiameter *DgN*, or *Dfb*, the perpendicular *Dd* to the diameter *NDb* is the greateſt that can be drawn from any part of the diameter to the circumference *Nedcb*.

Nedcb. And if the veſſel be kept full, the jet *G* will ſpout from the pipe *D*, to the horizontal diſtance *NM*, which is double the length of the perpendicular *Dd*. If two other pipes, as *C* and *E*, be fixed into the ſide of the veſſel at equal diſtances above and below the pipe *D*, the perpendiculars *Cc* and *Ee*, from theſe pipes to the ſemicircle, will be equal; and the jets *F* and *H* ſpouting from them will each go to the horizontal diſtance *NK*; which is double the length of either of the equal perpendiculars *Cc* or *Dd*.

If either of theſe pipes be elevated 45 degrees above the level or horizontal poſition, it will ſpout the water ſtill farther; and at that elevation will ſpout it to the greateſt horizontal diſtance poſſible; taking this diſtance on a level with the pipe. At any greater or leſs elevation, it will ſpout to a leſs diſtance: and at equal elevations above or below 45 degrees, it will ſpout to equal diſtances.

Fluids by their preſſure may be conveyed over hills and valleys in bended pipes, to any height not greater than the level of the ſprings from whence they flow. For it has been already ſhewn that they will riſe to the ſame level in bended pipes *. But when they are deſigned to be raiſed higher than the ſprings, forcing engines muſt be uſed; which ſhall be deſcribed when we come to treat of pumps.

How water may be conveyed over hills and valleys.

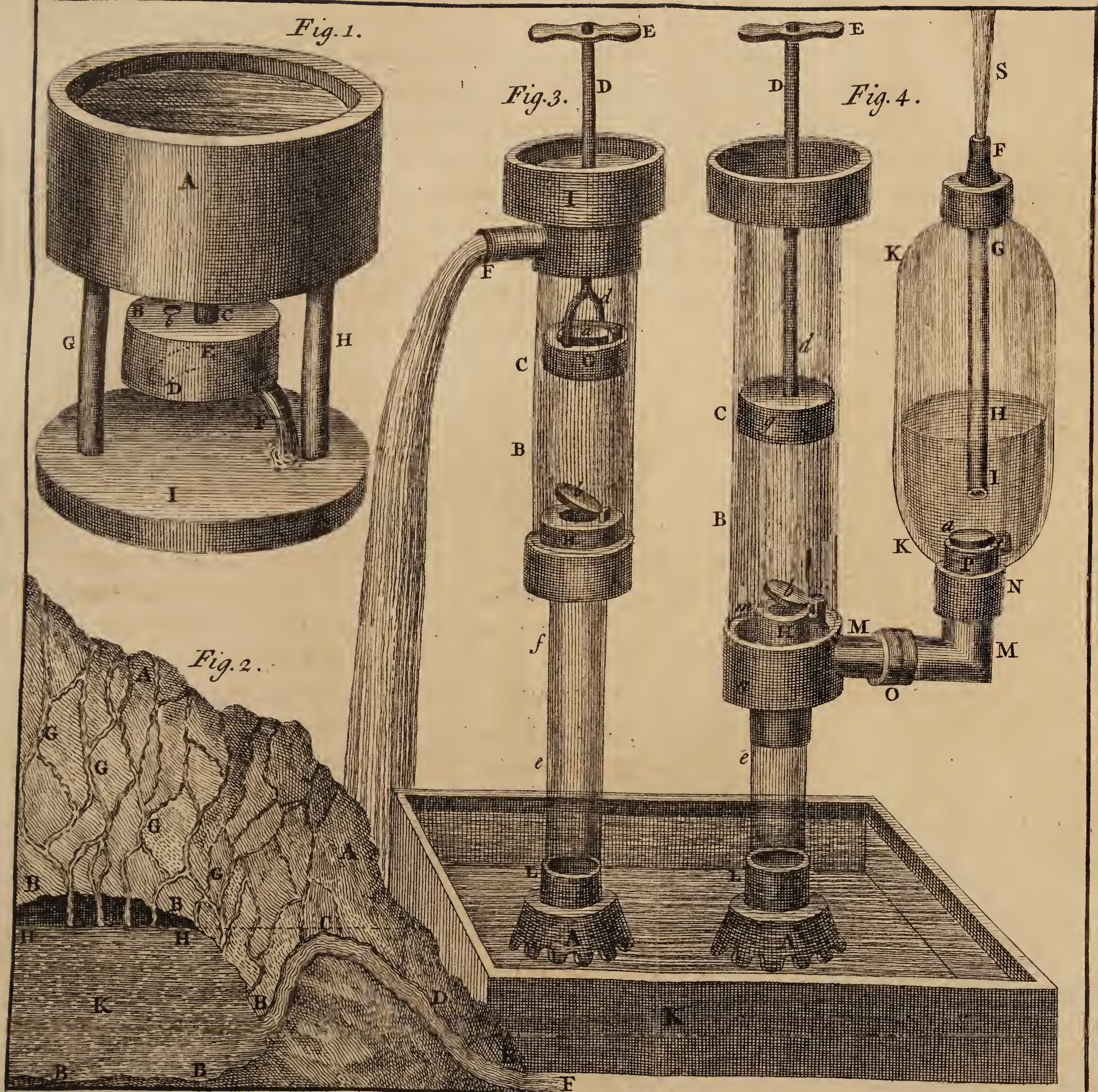
* Page 111.

A *ſyphon*, generally uſed for decanting liquors, is a bended pipe whoſe legs are of unequal lengths; and the ſhorteſt leg muſt always be put into the liquor intended to be decanted, that the perpendicular altitude of the column of liquor in the other leg

leg may be longer than the column in the immerſed leg, eſpecially above the ſurface of the water. For, if both columns were equally high in that reſpect, the atmoſphere, which preſſes as much upward as downward, and therefore acts as much upward againſt the column in the leg that hangs without the veſſel, as it acts downward upon the ſurface of the liquor in the veſſel, would hinder the running of the liquor through the ſyphon, even though it were brought over the bended top by ſuction. So that there is nothing left to determine the motion of the liquor, but the ſuperior weight of the column in the longer leg, on account of its having the greater perpendicular height.

Fig. 9. Let *D* be a cup filled with water to *C*, and *ABC* a ſyphon, whoſe ſhorter leg *BCF* is immerſed in the water from *C* to *F*. If the end of the other leg were no lower than the line *AC*, which is level with the ſurface of the water, the ſyphon would not work, even though the air ſhould be drawn out of it at the mouth *A*. For although the ſuction would draw ſome water at firſt, yet the water would ſtop at the moment the ſuction ceaſed; becauſe the air would act as much upward againſt the water at *A*, as it acted for it by preſſing downward on the ſurface at *C*. But if the leg *AB* comes down to *G*, and the air be drawn out at *G* by ſuction, the water will immediately follow, and continue to run, until the ſurface of the water in the cup comes down to *F*; becauſe, until then, the perpendicular height of the column *BAG* will be greater than that of the column *CB*; and conſequently, its weight will

PLATE XI.



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will be greater, until the ſurface comes down to *F*; and then the ſyphon will ſtop, though the leg *CF* ſhould reach to the bottom of the cup at *b*. For which reaſon, the leg that hangs without the cup is always made long enough to reach below the level of its bottom; as from *d* to *E*: and then, when the ſyphon is emptied of air by ſuction at *E*, the water immediately follows, and by its continuity brings away the whole from the cup; juſt as pulling one end of a thread will make the whole clue follow.

If the perpendicular height of a ſyphon, from the ſurface of the water to its bend at *B*, be more than 33 feet, it will draw no water, even though the other leg were much longer, and the ſyphon quite emptied of air; becauſe the weight of a column of water 33 feet high is equal to the weight of as thick a column of air, reaching from the ſurface of the earth to the top of the atmoſphere; ſo that there will be an equilibrium, and conſequently, though there would be weight enough of air upon the ſurface *C* to make the water aſcend in the leg *CB* almoſt to the height *B*, if the ſyphon were emptied of air, yet the weight would not be ſufficient to force the water over the bend; and therefore, it could never be brought into the leg *BAC*.

Let a hole be made in the bottom of the cup *A*, and the longer leg of the bended ſyphon *BCED* be cemented into the hole, ſo that the end *D* of the ſhorter leg *DE* may almoſt touch the bottom of the cup within. Then, if water be poured into this

Fig. 10. *Tantalus's* cup.

this cup, it will riſe in the ſhorter leg by its upward preſſure, extruding the air all the way before it through the longer leg: and when the cup is filled above the bend of the ſyphon at *F*, the preſſure of the water in the cup will force it over the bend of the ſyphon; and it will deſcend in the longer leg *CBG*, and run through the bottom, until the cup be emptied.

This is generally called *Tantalus*'s cup, and the legs of the ſyphon in it are almoſt cloſe together; and a little hollow ſtatue, or figure of a man, is ſometimes put over the ſyphon to conceal it; the bend *E* being within the neck of the figure as high as the chin. So that poor thirſty *Tantalus* ſtands up to the chin in water, imagining it will riſe a little higher, and he may drink; but inſtead of that, when the water comes up to his chin, it immediately begins to deſcend, and ſo, as he cannot ſtoop to follow it, he is left as much pained with thirſt as ever.

The fountain at command. Plate XI. Fig. 1.

The device called *the fountain at command* acts upon the ſame principle with the ſyphon in the cup. Let two veſſels *A* and *B* be joined together by the pipe *C* which opens into them both. Let *A* be open at top, *B* cloſe both at top and bottom, (ſave only a ſmall hole at *b* to let the air get out of the veſſel *B*;) and *A* be of ſuch a ſize, as to hold about ſix times as much water as *B*. Let a ſyphon *DEF* be ſoldered to the veſſel *B*, ſo that the part *DEe* may be within the veſſel, and *F* without it; the end *D* almoſt touching the bottom of the veſſel, and the end *F* below the level of *D*: the veſſel *B* hanging

hanging at *A* by the pipe *C* (ſoldered into both) and the whole ſupported by the pillars *G* and *H* upon the ſtand *I*. The bore of the pipe muſt be conſiderably leſs than the bore of the ſyphon.

The machine being thus conſtructed, let the veſſel *A* be filled with water, which will run through the pipe *C*, and fill the veſſel *B*. When *B* is filled above the top of the ſyphon at *E*, the water will run through the ſyphon, and be diſcharged at *F*. But ſince the bore of the ſyphon is larger than the bore of the pipe, the ſyphon will run faſter than the pipe, and will ſoon empty the veſſel *B*; upon which the water will ceaſe from running through the ſyphon at *F*, until the pipe *C* re-fills the veſſel *B*; and then it will begin to run as before. And thus the ſyphon will continue to run and ſtop alternately, until all the water in the veſſel *A* has run through the pipe *C*.—So that after a few trials, one may eaſily gueſs about what time the ſyphon will ſtop, and when it will begin to run: and then, to amuſe others, he may call out *ſtop*, or *run*, accordingly.

Intermitting ſprings. Fig. 2.

Upon this principle, we may eaſily account for *intermitting* or *reciprocating ſprings*. Let *AA* be part of a hill, within which there is a cavity *BB*; and from this cavity a vein running in the direction *BCDE*. The rain that falls upon the ſide of the hill will ſink and ſtrain through the ſmall pores and crannies *G*, *G*, *G*, *G*; and fill the cavity with water *K*. When the water riſes to the level *HHC*, the vein *BCDE* will be filled, and begin to run at *F*, like a ſyphon; which running will continue until

until the cavity be emptied, and then it will ſtop until the cavity be filled again.

The common pump. The *common ſucking pump*, with which we draw water out of wells, is an engine both pneumatic and hydraulic. It conſiſts of a pipe open at both ends, in which is a moveable piſton, bucket, or ſucker, as big as the bore of the pipe in that part wherein it works; and is leathered round, ſo as to fit the bore exactly; and may be moved up and down, without ſuffering any air to come between it and the pipe or pump barrel.

We ſhall explain the conſtruction both of this and the forcing pump by pictures of glaſs models, in which, both the action of the piſtons and motion of the valves are ſeen.

Fig. 3. Hold the model *ABC* upright in the veſſel of water *K*, the water being deep enough to riſe at leaſt as high as from *A* to *L*. The valve *a* on the moveable bucket *G*, and the valve *b* on the fixed box *H*, (which quite fills the bore of the pipe or barrel at *H*) will each lie cloſe, by its own weight, upon the hole in the bucket and box, until the engine begins to work. The valves are made of braſs, and covered underneath with leather for cloſing the holes the more exactly: and the bucket *G* is raiſed and depreſſed alternately by the handle *E* and rod *Dd*, the bucket being ſuppoſed at *B* before the working begins.

Take hold of the handle *E*, and thereby draw up the bucket from *B* to *C*, which will make room for the air in the pump all the way below the bucket

to

to dilate itſelf, by which its ſpring is weakened, and then its force is not equivalent to the weight or preſſure of the outward air upon the water in the veſſel *K:* and therefore, at the firſt ſtroke, the outward air will preſs up the water through the notched foot *A*, into the lower pipe, about as far as *e:* this will condenſe the rarefied air in the pipe between *e* and *C* to the ſame ſtate it was in before; and then, as its ſpring within the pipe is equal to the force or preſſure of the outward air, the water will riſe no higher by the firſt ſtroke; and the valve *b*, which was raiſed a little by the dilation of the air in the pipe, will fall, and ſtop the hole in the box *H*; and the ſurface of the water will ſtand at *e*. Then, depreſs the piſton or bucket from *C* to *B*, and as the air in the part *B* cannot get back again through the valve *b*, it will (as the bucket deſcends) raiſe the valve *a*, and ſo make its way through the upper part of the barrel *d* into the open air. But upon raiſing the bucket *G* a ſecond time, the air between it and the water in the lower pipe at *e* will be again left at liberty to fill a larger ſpace; and ſo its ſpring being again weakened, the preſſure of the outward air on the water in the veſſel *K* will force more water up into the lower pipe from *e* to *f*; and when the bucket is at its greateſt height *C*, the lower valve *b* will fall, and ſtop the hole in the box *H* as before. At the next ſtroke of the bucket or piſton, the water will riſe through the box *H* towards *B*, and then the valve *b*, which was raiſed by it, will fall when the bucket *G* is at its greateſt height. Upon depreſſing the bucket again, the water

water cannot be puſhed back through the valve *b*, which keeps cloſe upon the hole whilſt the piſton deſcends. And upon raiſing the piſton again, the outward preſſure of the air will force the water up through *H*, where it will raiſe the valve, and follow the bucket to *C*. Upon the next depreſſion of the bucket *G*, it will go down into the water in the barrel *B*; and as the water cannot be driven back through the now cloſe valve *b*, it will raiſe the valve *a* as the bucket deſcends, and will be lifted up by the bucket when it is next raiſed. And now, the whole ſpace below the bucket being full, the water above it cannot ſink when it is next depreſſed; but upon its depreſſion, the valve *a* will riſe to let the bucket go down; and when it is quite down, the valve *a* will fall by its weight, and ſtop the hole in the bucket. When the bucket is next raiſed, all the water above it will be lifted up, and begin to run off by the pipe *F*. And thus, by raiſing and depreſſing the bucket alternately, there is ſtill more water raiſed by it; which getting over the pipe *F*, into the wide top *I*, will ſupply the pipe, and make it run with a continued ſtream.

So, at every time the bucket is raiſed, the valve *b* riſes and the valve *a* falls; and at every time the bucket is depreſſed, the valve *b* falls and *a* riſes.

As it is the preſſure of the air or atmoſphere which cauſes the water to riſe, and follow the piſton or bucket *G* as it is drawn up; and ſince a column of water 33 feet high is of equal weight with as thick a column of the atmoſphere, from the earth to the very top of the air; therefore, the perpendicular

dicular height of the piſton or bucket from the ſurface of the water in the well, muſt always be leſs than 33 feet; otherwiſe the water will never get above the bucket. But, when the height is leſs, the preſſure of the atmoſphere will be greater than the weight of the water in the pump, and will therefore raiſe it above the bucket: and when the water has once got above the bucket, it may be lifted thereby to any height, if the rod *D* be made long enough, and a ſufficient degree of ſtrength be employed, to raiſe it with the weight of the water above the bucket; without ever lengthening the ſtroke.

The force required to work a pump, will be as the height to which the water is raiſed, and as the ſquare of the diameter of the pump-bore, in that part where the piſton works. So that, if two pumps be of equal heights, and one of them twice as wide in the bore as the other, the wideſt will raiſe four times as much water as the narroweſt; and will therefore require four times as much ſtrength to work it.

The wideneſs or narrowneſs of the pump, if any other part beſides that in which the piſton works, does not make the pump either more or leſs difficult to work; except what difference may ariſe from the friction of the water in the bore, which is always greater in a narrow bore than in a wide one, becauſe of the greater velocity of the water: upon which account, the wider the better.

The pump rod is never raiſed directly by ſuch a handle as *E* at the top, but by means of a lever,

whoſe longer arm (at the end of which the power is applied) generally exceeds the length of the ſhorter arm five or ſix times; and, by that means, gives five or ſix times as much advantage to the power. Upon theſe principles, it will be eaſy to find the dimenſions of a pump that ſhall work with a given force, and draw water from any given depth. But, as theſe calculations have been generally neglected by pump-makers (either for want of ſkill or induſtry) the following table was calculated by the late ingenious Mr. *Booth* for their benefit. In this calculation, he ſuppoſed the handle of the pump to be a lever increaſing the power five times; and had often found that a man can work a pump four inches diameter, and 30 feet high above the bucket, and diſcharge $27\frac{1}{2}$ gallons of water (Engliſh wine meaſure) in a minute. Now, if it be required to find the diameter of a pump, that ſhall raiſe water with the ſame eaſe from any other height above the bucket; look for that height in the firſt column, and over-againſt it in the ſecond you have the diameter or width of the pump; and in the third, you find the quantity of water which a man of ordinary ſtrength can diſcharge in a minute.

Height

Height of the pump above the bucket.	Diameter of the bore where the bucket works.		Water difcharged in a minute, Englifh wine meafure.	
Feet.	Inches.	100 parts	Gallons.	Pints.
10	6	.93	81	6
15	5	.65	54	4
20	4	.90	40	8
25	4	.38	32	6
30	4	.00	27	2
35	3	.70	23	3
40	3	.47	20	4
45	3	.26	18	1
50	3	.10	16	3
55	2	.95	14	7
60	2	.83	13	5
65	2	.71	12	4
70	2	.62	11	5
75	2	.53	10	7
80	2	.44	10	2

The *forcing pump* raifes water through the box *H* in the fame manner as the fucking pump does, when the plunger or pifton *g* is lifted up by the rod *Dd*. But this plunger has no hole through it, to let the water in the barrel *BC* get above it, when it is depreffed to *B*, and the valve *b* (which rofe by the afcent of the water through the box *H* when the plunger *g* was drawn up) falls down and ftops the hole in *H*, the moment that the plunger is raifed to its greateft height. Therefore, as the water between the plunger *g* and box *H* can neither get through the plunger upon its defcent, nor back again into the lower part of the pump *Le*, but has Fig. 4. The *forcing pump.*

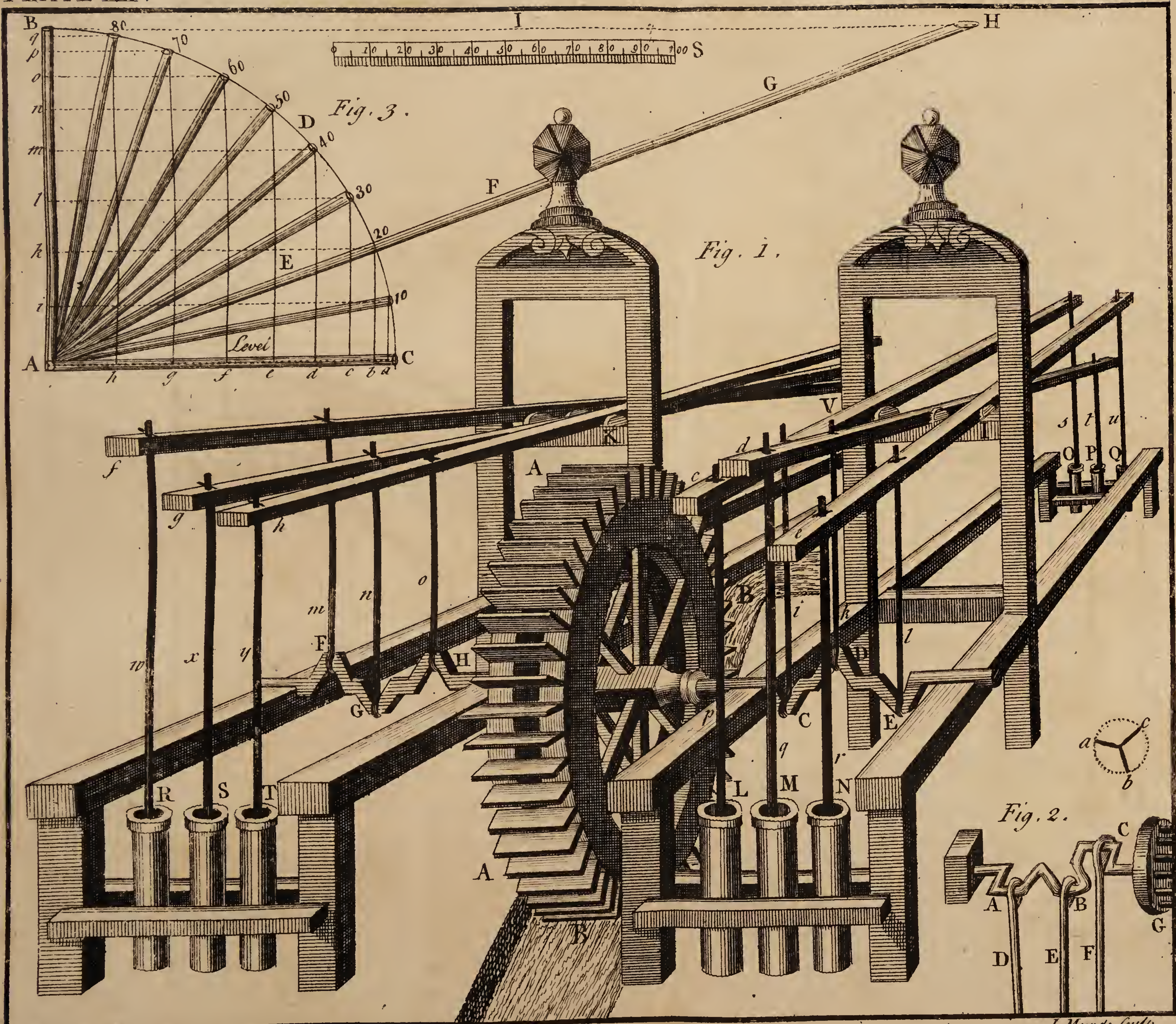
a free paſſage by the cavity *mn* into the pipe *MM*, which opens into the air-veſſel *KK* at *P*; the water is forced through the pipe *MM* by the deſcent of the plunger, and driven into the air veſſel; and in running up through the pipe at *P*, it opens the valve *a*; which ſhuts at the moment the plunger begins to be raiſed, becauſe the action of the water againſt the under ſide of the valve then ceaſes.

The water, being thus forced into the air-veſſel *KK*, by repeated ſtrokes of the plunger, gets above the lower end of the pipe *GHI*, and then begins to condenſe the air in the veſſel *KK*. For, as the pipe *GH* is fixed air-tight into the veſſel below *F*, and the air has no way to get out of the veſſel but through the mouth of the pipe at *I*, and cannot get out when the mouth *I* is covered with water; and is more and more condenſed as the water riſes upon the pipe, the air then begins to act forcibly by its ſpring againſt the ſurface of the water *H*: and this action drives the water up through the pipe *IHGF*, from whence it ſpouts in a jet *S* to a great height; and is ſupplied by alternately raiſing and depreſſing of the plunger *g*, which conſtantly forces the water that it raiſes, through the valve *H*, along the pipe *MM*, into the air veſſel *KK*.

The higher that the ſurface of the water *H* is raiſed in the air-veſſel, the leſs ſpace will the air, which before filled that veſſel, be condenſed into; and therefore the force of its ſpring will be ſo much the ſtronger upon the water, and will drive it with the greater force through the pipe at *F*: and as the ſpring of the air continues whilſt the plunger *g* is

riſing,

PLATE XII.



J. Ferguson delin.

J. Mynde sculp.

riſing, the ſtream or jet *S* will be uniform as long as the action of the plunger continues: and when the valve *b* opens, to let the water follow the plunger upward, the valve *a* ſhuts, to hinder the water, which is forced into the air veſſel, from running back by the pipe *MM* into the barrel of the pump.

If there were no air-veſſel to this engine, the pipe *GHI* would be joined to the pipe *MMN* at *P*; and then, the jet *S* would ſtop every time the plunger is raiſed, and run only when the plunger is depreſſed.

Mr. *Newſham*'s water-engine, for extinguiſhing fire, conſiſts of two forcing pumps, which alternately drive water into a cloſe veſſel of air; and by forcing the water into that veſſel, the air in it is thereby condenſed, and compreſſes the water ſo ſtrongly, that it ruſhes out with great impetuoſity and force through a pipe that comes down into it; and makes a continued ſtream by the condenſation of the air upon its ſurface in the veſſel.

By means of forcing pumps, water may be raiſed to any height above the level of a river or ſpring; and machines may be contrived to work theſe pumps either by a running ſtream, a fall of water, or by horſes. An inſtance in each ſort will be ſufficient to ſhew the method.

Firſt, by a running ſtream, or a fall of water. Plate XII.
Let *AA* be a wheel, turned by the fall of water Fig. 1.
BB; and have any number of cranks (ſuppoſe ſix) as *C*, *D*, *E*, *F*, *G*, *H*, on its axis, according to the ſtrength of the fall of water, and the height to which the water is intended to be raiſed by the

 engine.

A pump engine to go by water.

engine. As the wheel turns round, theſe cranks move the levers *c*, *d*, *e*, *f*, *g*, *h*, up and down, by the iron rods *i*, *k*, *l*, *m*, *n*, *o*; which alternately raiſe and depreſs the piſtons by the other iron rods *p*, *q*, *r*, *s*, *t*, *u*, *w*, *x*, *y*, in twelve pumps; nine whereof, as *L*, *M*, *N*, *O*, *P*, *Q*, *R*, *S*, *T*, appear in the plate; the other three being hid behind the work at *V*. And as pipes may go from all theſe pumps, to convey the water (drawn up by them to a ſmall height) into a cloſe ciſtern, from which the main pipe proceeds, the water will be forced into this ciſtern by the deſcent of the piſtons. And as each pipe, going from its reſpective pump into the ciſtern, has a valve at its end in the ciſtern, theſe valves will hinder the return of the water by the pipes; and therefore, when the ciſtern is once full, each piſton upon its deſcent will force the water (conveyed into the ciſtern by a former ſtroke) up the main pipe, to the height the engine was intended to raiſe it: which height depends upon the quantity raiſed, and the power that turns the wheel. When the power upon the wheel is leſſened by any defect of the quantity of water turning it, a proportionable number of the pumps may be laid aſide, by diſengaging their rods from the vibrating levers.

This figure is a repreſentation of the engine erected at *Blenheim* for the Duke of *Marlborough* by the late ingenious Mr. *Alderſea*. The water wheel is $7\frac{1}{2}$ feet in diameter, according to Mr. *Switzer*'s account in his Hydraulics.

When

When ſuch a machine is placed in a ſtream that runs upon a ſmall declivity, the motion of the levers and action of the pumps will be but ſlow; ſince the wheel muſt go once round for each ſtroke of the pumps. But, when there is a large body of ſlow running water, a cog or ſpur wheel may be placed upon each ſide of the water wheel *AA*, upon its axis, to turn a trundle upon each ſide; the cranks being upon the axis of the trundle. And by proportioning the cog wheels to the trundles, the motion of the pumps may be made quicker, according to the quantity and ſtrength of the water upon the firſt wheel; which may be as great as the workman pleaſes, according to the length and breadth of the float-boards or wings of the wheel. In this manner, the engine for raiſing water at *London-Bridge* is conſtructed; in which, the water wheel is 20 feet diameter, and the floats 14 feet long.

Where a ſtream, or ~~fall of water~~ cannot be had, and gentlemen want to have water raiſed, and brought to their houſes from a rivulet or ſpring; this may be effected by a horſe-engine, working three forcing pumps which ſtand in a reſervoir filled by the ſpring or rivulet: the piſtons being moved up and down in the pumps by means of a triple crank *ABC*, which, as it is turned round by the trundle *G*, raiſes and depreſſes the rods *D*, *E*, *F*. The trundle may be turned by ſuch a wheel as *F* in Fig. 1. of Plate VIII. having levers *y*, *y*, *y*, *y*, on its upright axle, to which horſes may be joined for working the engine. And if the wheel has three

A pump engine to go by horſes.

Fig. 2.

times as many cogs as the trundle has ſtaves or rounds, the trundle and cranks will make three revolutions for every one of the wheel: and as each crank will fetch a ſtroke in the time it goes round, the three cranks will make nine ſtrokes for every turn of the great wheel.

The cranks ſhould be made of caſt iron, becauſe *that* will not bend; and they ſhould each make an angle of 120 with both of the others, as at *a*, *b*, *c*; which is (as it were) a view of their *radii*, in looking endwiſe at the axis: and then, there will be always one or other of them going downward, which will puſh the water forward with a continued ſtream into the main pipe. For, when *b* is almoſt at its loweſt ſituation, and is therefore juſt beginning to loſe its action upon the piſton which it moves, *c* is beginning to move downward, which will by its piſton continue the propelling force upon the water: and when *c* is come down to the poſition of *b*, *a* will be in the poſition of *c*.

The trundle may be either on the upper or under ſide of the rim of the great wheel, at the diſcretion of the workman.

The more perpendicularly the piſton-rods move up and down in the pumps, the freer and better will their ſtrokes be: but a little deviation from the perpendicular will not be material. Therefore, when the pump-rods *D*, *E*, and *F* go down into a deep well, they may be moved directly by the cranks, as is done in a very good horſe-engine of this ſort at Sir *James Creed*'s, which forces up water

ter about 40 feet from a well under ground, to a refervoir on the top of his houfe at *Greenwich*. But when the cranks are only at a fmall height above the pumps, the piftons muft be moved by vibrating levers, as in the above engine at *Blenheim*: and the longer the levers are, the nearer will the ftrokes be to a perpendicular.

Let us fuppofe, that in fuch an engine as Sir *James Creed*'s, the great wheel is 12 feet diameter, the trundle 4 feet, and the radius or length of each crank to be 9 inches, working a pifton in its pump. Let there be three pumps in all, and the bore of each pump be four inches diameter. Then, if the great wheel has three times as many cogs as the trundle has ftaves, the trundle and cranks will go three times round for each revolution of the horfes and wheel, and the three cranks will make nine ftrokes of the pumps in that time, each ftroke being 18 inches (or double the length of the crank) in a 4 inch bore. Let the diameter of the horfe walk be 18 feet, and the perpendicular height to which the water is raifed above the furface of the well be 64 feet.

A calculation of the quantity of water that may be raifed by a horfe-engine.

If the horfes go at the rate of two miles an hour (which is very moderate walking) they will turn the great wheel 187 times round in an hour.

In each turn of the wheel the piftons make 9 ftrokes in the pumps, which amount to 1683 in an hour.

Each ftroke raifes a column of water 18 inches long, and 4 inches thick, in the pump-barrels; which column, upon the defcent of the pifton, is forced

forced into the main pipe, whofe perpendicular altitude above the furface of the well is 64 feet.

Now, fince a column of water 18 inches long and 4 inches thick, contains 226.18 cubic inches, this number multiplied by 1683 (the ftrokes in an hour) gives 380661 for the number of cubic inches of water raifed in an hour.

A gallon, in wine meafure, contains 231 cubic inches, by which divide 380661, and it quotes 1468 in round numbers, for the number of gallons raifed in an hour; which, divided by 63, gives $26\frac{1}{2}$ hogfheads.—If the horfes go fafter, the quantity raifed will be fo much greater.

In this calculation it is fuppofed that no water is wafted by the engine. But as no forcing engine can be fuppofed to lofe lefs than a fifth part of the calculated quantity of water, between the piftons and barrels, and by the opening and fhutting of the valves, the horfes ought to walk almoft $2\frac{1}{2}$ miles *per* hour, to fetch up this lofs.

A column of water 4 inches thick, and 64 feet high, weighs $349\frac{9}{16}$ pounds averdupoife, or $424\frac{5}{12}$ pounds troy; and this weight, together with the friction of the engine, is the refiftance that muft be overcome by the ftrength of the horfes.

The horfe-tackle fhould be fo contrived, that the horfes may rather pufh on than drag the levers after them. For if they draw, in going round the walk, the outfide traces will rub againft their fides and hams; which will hinder them from drawing at right angles to the levers, and fo make them pull at a difadvantage. But if they pufh the lever before

before their breasts, instead of dragging it behind them, they can always walk at right angles to it.

It is no ways material what the diameter of the main or conduct pipe be: for the whole resistance of the water therein, against the horses, will be according to the height to which it is raised, and the diameter of that part of the pump in which the piston works; as we have already observed. So that by the same pump, an equal quantity of water may be raised in (and consequently made to run from) a pipe of a foot diameter, with the same ease as in a pipe of five or six inches: or rather with more ease, because its velocity in a large pipe will be less than in a small one; and therefore its friction against the sides of the pipe will be less also.

And, the force required to raise water depends not upon the length of the pipe, but upon the perpendicular height to which it is raised therein above the level of the spring. So that the same force, Fig. 3. which would raise water to the height *AB* in the upright pipe *AiklmnopqB*, will raise it to the same height or level *BIH* in the oblique pipe *AEFGH*. For the pressure of the water at the end *A* of the latter, is no more than its pressure against the end *A* of the former.

The weight or pressure of water at the lower end of a pipe, is always as the sine of the angle to which the pipe is elevated above the level or horizon. For, although the water in the upright pipe *AB* would require a force applied immediately to the lower end *A* equal to the weight of all the water in it to support the water, and a little more

more to drive it up, and out of the pipe; yet, if that pipe be inclined from its upright poſition to an angle of 80 degrees (as in *A* 80) the force required to ſupport or to raiſe the ſame cylinder of water will then be as much leſs, as the ſine 80 *b* is leſs than the radius *AB*; or as the ſine of 80 degrees is leſs than the ſine of 90. And ſo, decreaſing as the ſine of the angle of elevation leſſens, until it arrives at its level *AC* or place of reſt, where the force of the water is nothing at either end of the pipe. For, although the abſolute weight of the water is the ſame in all poſitions, yet its preſſure at the lower end decreaſes, as the ſine of the angle of elevation decreaſes; as will appear plainly by a farther conſideration of the figure.

Let two pipes, *AB* and *BC*, of equal lengths and bores, communicate with one another at *A*; and let the pipe *AB* be divided into 100 equal parts, as the ſcale *S* is; whoſe length is equal to the length of the pipe.—Upon this length, as a radius, deſcribe the quadrant *BCD*, and divide it into 90 equal parts or degrees.

Let the pipe *AC* be filled with water, and elevated to 10 degrees upon the quadrant; then, part of the water that is in it will riſe in the pipe *AB*, and if it be kept full of water, it will raiſe the water in the pipe *AB* from *A* to *i*; that is, to a level *i* 10 with the mouth of the pipe at 10: and the upright line *a* 10, equal to *Ai*, will be the ſine of 10 degrees elevation; which, being meaſured upon the ſcale *S*, will be about 17.3 of ſuch parts as the pipe contains 100 in length: and therefore, the force

or

or preſſure of the water at *A*, in the pipe *A* 10, will be to the force or preſſure at *A* in the pipe *AB*, as 17.3 to 100.

Let the ſame pipe be elevated to 20 degrees in the quadrant, and if it be kept full of water, part of that water will run into the pipe *AB*, and riſe therein to the height *Ak*, which is equal to the length of the upright line *b* 20, or to the ſine of 20 degrees elevation; which, being meaſured upon the ſcale *S*, will be 34.2 of ſuch parts as the pipe contains 100 in length. And therefore, the preſſure of the water at *A*, in the full pipe *A* 20, will be to its preſſure, if that pipe were raiſed to the perpendicular ſituation *AB*, as 34.2 to 100.

Elevate the pipe to the poſition *A* 30 on the quadrant, and if it be ſupplied with water, the water will riſe from it, into the pipe *AB*, to the height *Al*, or to the ſame level with the mouth of the pipe at 30. The ſine of this elevation, or of the angle of 30 degrees, is *c* 30; which is juſt equal to half the length of the pipe, or to 50 of ſuch parts of the ſcale, as the length of the pipe contains 100. Therefore, the preſſure of the water at *A*, in a pipe elevated 30 degrees above the horizontal level, will be equal to one half of what it would be, if the ſame pipe ſtood upright in the ſituation *AB*.

And thus, by elevating the pipe to 40, 50, 60, 70, and 80 degrees on the quadrant, the ſines of theſe elevations will be *d* 40, *e* 50, *f* 60, *g* 70, and *h* 80; which will be equal to the heights *Am*, *An*, *Ao*, *Ap*, and *Aq*: and theſe heights meaſured upon the ſcale *S* will be 64.3, 76.6, 86.6, 93.9, and

and 98.5; which expreſs the preſſure at *A* in all theſe elevations, conſidering the preſſure in the upright pipe *AB* as 100.

Becauſe it may be of uſe to have the lengths of all the ſines of a quadrant from 0 degrees to 90, we ſhall here annex a table ſhewing the length of the ſine of every degree in ſuch parts as the whole pipe (equal to the radius of the quadrant) contains 1000. Then the ſines will be integral or whole parts in length. But if you ſuppoſe the length of the pipe to be divided only into 100 equal parts, the laſt figure of each part or ſine muſt be cut off as a decimal; and then thoſe which remain at the left hand of this ſeparation will be integral or whole parts.

Sine of	Parts	Sine of	Parts	Sine of	Parts	Sine of	Parts	Sine of	Parts
D. 1	17	D.19	325	D.37	602	D.55	819	D.73	956
2	39	20	342	38	616	56	829	74	961
3	52	21	358	39	629	57	839	75	966
4	70	22	375	40	643	58	848	76	970
5	87	23	391	41	656	59	857	77	974
6	104	24	407	42	669	60	866	78	978
7	122	25	423	43	682	61	875	79	982
8	139	26	438	44	695	62	883	80	985
9	156	27	454	45	707	63	891	81	988
10	173	28	469	46	719	64	898	82	990
11	191	29	485	47	731	65	906	83	992
12	208	30	500	48	743	66	913	84	994
13	225	31	515	49	755	67	920	85	996
14	242	32	530	50	766	68	927	86	997
15	259	33	545	51	777	69	933	87	998
16	275	34	559	52	788	70	939	88	999
17	292	35	573	53	799	71	945	89	999
18	309	36	588	54	809	72	951	90	1000

Thus, if the radius of the quadrant (ſuppoſed to be equal to the length of the pipe *AC*) be divided

vided into 1000 equal parts, and the elevation be 45 degrees, the ſine of that elevation will be equal to 707 of theſe parts: but if the radius be divided only into 100 equal parts, the ſame ſine will be only 70.7 or $70\frac{7}{10}$ of theſe parts. For, as 1000 is to 707, ſo is 100 to 70.7.

As it is of the greateſt importance to all engine-makers, to know what quantity and weight of water will be contained in an upright round pipe of a given diameter and height; ſo as by knowing what weight is to be raiſed, they may proportion their engines to that force; we ſhall ſubjoin tables ſhewing the number of cubic inches of water contained in an upright pipe of a round bore, of any diameter from one inch to ſix and an half; and of any height from one foot to two hundred: together with the weight of the ſaid number of cubic inches, both in troy and averdupoiſe ounces. The number of cubic inches divided by 231 will reduce the water to gallons in wine meaſure, and divided by 282 will reduce it to the meaſure of ale gallons. Alſo, the troy ounces divided by 12 will reduce the weight to troy pounds, and the averdupoiſe ounces divided by 16 will reduce the weight to averdupoiſe pounds.

And here I muſt repeat it again, that the weight or preſſure of the water, acting againſt the power that works the engine, muſt always be eſtimated according to the perpendicular height to which it is to be raiſed, without any regard to the length of the conduct pipe when it has an oblique poſition; and as if the diameter of that pipe were juſt equal

to

to the diameter of that part of the pump in which the piſton works. Thus, by the following tables, the preſſure of the water, againſt an engine whoſe pump is of a 4½ inch bore, and the perpendicular height of the water in the conduct pipe is 80 feet, will be equal to 8057.5 troy ounces, and to 8848.2 averdupoiſe ounces; which makes 671.4 troy pounds, and 553 averdupoiſe.

One inch diameter.				1½ inch diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoiſe ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoiſe ounces.
1	9.42	4.97	5.46	1	21.20	11.19	12.29
2	18.85	9.95	10.92	2	42.41	22.38	24.58
3	28.27	14.92	16.38	3	63.62	33.57	36.87
4	37.70	19.81	21.85	4	84.82	44.76	49.16
5	47.12	24.87	27.31	5	106.03	55.95	6[illegible].45
6	56.55	29.84	32.77	6	127.23	67.14	73.73
7	65.97	34.82	38.23	7	147.44	78.34	86.02
8	75.40	39.79	43.69	8	169.65	89.53	98.31
9	84.82	44.76	49.16	9	190.85	100.72	110.60
10	94.25	49.7	54.62	10	212.06	111.91	122.89
20	188.49	99.48	109.24	20	424.11	223.82	245.78
30	282.74	149.21	163.86	30	6[illegible]6.17	335.73	368.67
40	376.99	198.95	218.47	40	848.23	447.64	491.57
50	471.24	248.69	273.09	50	1060.29	559.55	614 46
60	565.49	298.43	327.71	60	1272.35	671.46	737.35
70	659.73	348.17	382.33	70	1484.40	783.37	860.24
80	753.98	397.90	436.95	80	1696.46	895.28	983.14
90	843.23	447.64	491.57	90	1908.52	1007.19	1106.03
100	942.48	49[illegible].38	546.19	100	2120.58	1119.09	1228.92
200	1884.96	994.76	1092.38	200	4241.15	2238.18	2457.84

Example. *Required the number of cubic inches, and the weight of the water, in an upright pipe* 278 *feet high, and* 1½ *inch diameter?*

Here,

Here, the nearest single decimal figure is only taken into the account; and the whole being reduced by division, amounts to $25\frac{1}{2}$ wine gallons in measure; to $259\frac{1}{4}$ pounds troy, and to $213\frac{1}{2}$ pounds averdupoise.

	Feet	Cubic inches	Troy oz.	Averd. oz.
	200	4241.1	2238.2	2457.8
	70	1484.4	783.3	860.2
	8	169.6	89.5	98.3
Ans.	278	5895.1	3111.0	3416.3

2 Inches diameter.				$2\frac{1}{2}$ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.
1	37.70	19.89	21.85	1	58.90	31.08	34.14
2	75.40	39.79	43.69	2	117.81	62.17	68.27
3	113.09	59.68	65.54	3	176.71	93.26	102.41
4	150.79	79.58	87.39	4	235.62	124.34	136.55
5	188.49	99.47	109.24	5	294.52	155.43	170.68
6	226.19	119.37	131.08	6	353.43	186.52	204.82
7	263.89	139.26	152.93	7	412.33	217.60	238.96
8	301.59	159.16	174.78	8	471.24	248.69	273.09
9	339.29	179.05	196.63	9	530.14	279.77	307.23
10	376.99	198.95	218.47	10	589.05	310.86	341.37
20	753.98	397.90	436.95	20	1178.10	621.72	682.73
30	1130.97	596.85	665.42	30	1767.15	932.58	1024.10
40	1507.96	795.80	873.90	40	2356.20	1243.44	1365.47
50	1884.96	994.75	1092.37	50	2545.25	1554.30	1706.83
60	2261.95	1193.70	1310.84	60	3534.29	1865.16	2048.20
70	2638.94	1392.65	1529.32	70	4123.34	2176.02	2389.57
80	3015.93	1591.60	1747.80	80	4712.39	2486.88	2730.94
90	3392.92	1790.55	1966.27	90	5301.44	2797.74	3072.30
100	3769.91	1989.50	2184.75	100	5890.49	3108.60	2413.67
200	7539.82	3979.00	4369.50	200	11780.98	6217.20	4827.34

These tables were first calculated to six decimal places for the sake of exactness; but in transcribing them, there are no more than two decimal figures taken into the account, and sometimes but one; because there is no necessity for computing to hun-

3 Inches diameter.				3½ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.
1	84.8	44.76	49.16	1	115.4	60.9	66.9
2	169.6	89.53	98.31	2	230.9	121.8	133.8
3	254.5	134.29	147.47	3	346.4	182.8	200.7
4	239.3	179.05	196.63	4	461.8	243.7	267.6
5	424.1	223.82	245.78	5	577.3	304.6	334.5
6	508.9	268.58	294.94	6	692.7	365.6	401.4
7	593.7	313.35	344.10	7	808.2	426.5	468.3
8	698.6	358.11	393.25	8	923.6	487.4	535.3
9	763.4	402.87	442.41	9	1039.1	548.3	602.2
10	848.2	447.64	491.57	10	1154.5	609.2	669.1
20	1696.5	895.28	983.14	20	2309.1	1218.6	1338.2
30	2544.7	1342.92	1474.70	30	3463.6	1827.8	2007.2
40	3392.9	1790.55	1966.27	40	4618.1	2437.1	2676.3
50	4241.1	2238.19	2457.84	50	5772.7	3046.4	3345.4
60	5089.4	2685.83	2949.41	60	6927.2	3655.7	4014.5
70	5937.6	3133.47	3440.98	70	8081.7	4265.0	4683.5
80	6785.8	3581.11	3932.55	80	9236.3	4874.3	5352.6
90	7634.1	4028.75	4424.12	90	10390.8	5483.6	6021.7
100	8482.3	4476.39	4915.68	100	11545.4	6092.8	6690.8
200	16964.6	8952.78	9821.36	200	23090.7	12185.7	13381.5

dreth parts of an inch or of an ounce in practice. And as they never appeared in print before, it may not be amiss to give the reader an account of the principles upon which they were constructed.

The solidity of cylinders are found by multiplying the areas of their bases by their altitudes. And ARCHIMEDES gives the following proportion

4 Inches diameter.				4½ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoiſe ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoiſe ounces.
1	150.8	79.6	87.4	1	100.8	100.7	110.6
2	301.6	159.2	174.8	2	381.7	201.4	221.2
3	452.4	238.7	262.2	3	572.6	302.2	331.8
4	603.2	318.3	349.6	4	763.4	402.9	442.4
5	753.9	497.9	436.9	5	954.3	503.6	553.0
6	904.8	477.5	524.3	6	1145.1	604.3	663.6
7	1055.6	557.1	611.7	7	1337.9	705.0	774.2
8	1206.4	636.6	699.1	8	1526.8	805.7	884.8
9	1357.2	716.2	786.5	9	1717.7	906.5	995.4
10	1508.0	795.8	873.9	10	1908.5	1007.2	1106.0
20	3115.9	1591.6	1747.8	20	3817.0	2014.4	2212.1
30	4523.9	2387.4	2621.7	30	5725.6	3021.6	3818.1
40	6031.9	3183.2	3495.6	40	7634.1	4028.7	4424.1
50	7539.8	3997.0	4369.5	50	9542.6	5035.9	5530.1
60	9047.8	4774.8	5243.3	60	11451.1	6043.1	6636.2
70	10555.8	5570.6	6117.3	70	13359.6	7050.3	7742.2
80	12063.7	6366.4	6991.2	80	15268.2	8057.5	8848.2
90	13571.7	7162.2	7865.1	90	17176.7	9064.7	9954.3
100	15079.7	7958.0	8739.0	100	19085.2	10071.9	11060.3
200	30159.3	15916.0	17478.0	200	38170.4	20143.8	22120.6

tion for finding the area of a circle, and the ſolidity of a cylinder raiſed upon that circle.

As 1 is to 0.785339, ſo is the ſquare of the diameter to the area of the circle. And, as 1 is to 0.785339, ſo is the ſquare of the diameter multiplyed by the height to the ſolidity of the cylinder. By this analogy the ſolid inches and parts of an

5 Inches diameter.				5½ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoife ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoife ounces.
1	235.6	124.3	136.5	1	285.1	150.4	164.3
2	471.2	248.7	273.1	2	570.2	300.9	328.5
3	706.8	373.0	409.6	3	855.2	451.4	492.8
4	492.5	497.4	546.2	4	1140.4	601.8	657.1
5	1178.1	621.8	682.7	5	1425.5	752.3	821.3
6	1413.7	746.1	819.3	6	1710.6	902.7	985.6
7	1649.3	870.4	955.8	7	1995.7	1053.2	1149.9
8	1884.9	994.7	1092.4	8	2280.8	1203.6	1314.1
9	2120.6	1119.1	1228.9	9	2565.9	1354.1	1478.4
10	2356.2	1243.4	1365.5	10	2851.0	1504.6	1642.7
20	4712.4	2486.9	2730.9	20	5702.0	3009.1	3285.4
30	7068.6	3730.3	4096.4	30	8553.0	4513.7	4928.1
40	2424.8	4973.8	5461.9	40	11404.0	6018.2	6570.8
50	11781.0	6217.2	6827.3	50	14255.0	7522.9	8213.5
60	14137.2	7460.6	8192.8	60	17106.0	9027.4	9856.2
70	16493.4	8704.1	9558.3	70	19957.0	10531.9	11498.9
80	18849.6	9947.5	10923.7	80	22808.0	12036.5	13141.6
90	21205.8	11190.9	12289.2	90	25659.0	13541.1	14784.3
100	23561.9	12434.4	13654.7	100	28510.0	15045.6	16426.9
200	47123.9	24868.8	27309.3	200	57020.0	30091.2	32853.8

inch in the tables, are calculated to a cylinder 200 feet high, of any diameter from 1 inch to $6\frac{1}{2}$ inches; and may be continued at pleafure.

The weight of running water.

And as to the weight of a cubic foot of running water, it has been often found upon trial, by Dr. *Wyberd* and others, to be 76 pounds troy, which is equal to 62.5 pounds averdupoife. Therefore, fince

6 Inches diameter.				6½ Inches diameter.			
Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.	Feet high.	Solidity in cubic inches.	Weight in troy ounces.	In averdupoise ounces.
1	339.3	179.0	196.6	1	398.2	210.1	230.7
2	678.6	358.1	393.3	2	797.4	420.3	461.4
3	1017.9	537.2	589.9	3	1195.6	630.4	692.1
4	1357.2	716.2	786.5	4	1593.8	840.6	922.8
5	1696.5	895.3	983.1	5	1991.9	1050.8	1153.6
6	2035.7	1074.3	1179.8	6	2390.1	1260.9	1384.3
7	2375.0	1253.4	1376.4	7	2788.3	1471.1	1615.0
8	2714.3	1432.4	1573.0	8	3186.5	1681.2	1845.7
9	3053.6	1611.5	1769.6	9	3584.7	1891.3	2076.4
10	3392.9	1790.6	1966.3	10	3982.9	2101.5	2307.1
20	6785.8	3581.1	3932.5	20	7965.8	4202.9	4614.3
30	10178.8	5371.7	5898.8	30	11948.8	6304.4	6921.4
40	13571.7	7162.2	7865.1	40	15931.7	8405.9	9228.6
50	16964.6	8952.8	9831.4	50	19914.6	10507.4	11535.7
60	20357.5	10743.3	11797.6	60	23897.6	12608.9	13842.9
70	23750.5	12533.9	13763.9	70	27880.5	14710.4	16150.0
80	27143.4	14324.4	15730.2	80	31863.4	16811.8	18457.2
90	30536.3	16115.0	17696.5	90	35846.3	18913.3	20764.3
100	33929.2	17905.6	19662.7	100	39829.3	21014.8	23071.5
200	67858.4	35811.2	39325.4	200	79658.6	42029.6	46143.0

ſince there are 1728 cubic inches in a cubic foot, a troy ounce of water contains 1.8949 cubic inch; and an averdupoiſe ounce of water 1.72556 cubic inch. Conſequently, if the number of cubic inches contained in any given cylinder, be divided by 1.8949, it will give the weight in troy ounces; and divided by 1.72556, will give the weight in

 averdupoiſe

averdupoise ounces. By this method, the weights shewn in the tables were calculated; and are near enough for any common practice.

The *fire-engine*.

The *fire-engine* comes next in order to be explained: but as it would be difficult, even by the best plates, to give a particular description of its several parts, so as to make the whole intelligible, I shall only explain the principles upon which it is constructed.

1. Whatever weight of water is to be raised, the pump-rod must be loaded with weights sufficient for that purpose, if it be done by a forcing-pump, as is generally the case: and the power of the engine must be sufficient for the weight of the rod, in order to bring it up.

2. It is known, that the atmosphere presses upon the surface of the earth with a force equal to 15 pounds upon every square inch.

3. When water is heated to a certain degree, the particles thereof repel one another, and constitute an elastic fluid, which is generally called *steam* or *vapour*.

4. Hot steam is very elastic; and when it is cooled by any means, particularly by its being mixed with cold water, its elasticity is destroyed immediately, and it is reduced to water again.

5. If a vessel be filled with hot steam, and then closed so, as to keep out the external air, and all other fluids; when that steam is by any means condensed, cooled, or reduced to water, *that* water will fall to the bottom of the vessel; and the cavity of the vessel will be almost a perfect vacuum.

6. Whenever

6. Whenever a vacuum is made in any veſſel, the air by its weight will endeavour to ruſh into the veſſel, or to drive in any other body that will give way to its preſſure; as may be eaſily ſeen by a common ſyringe. For, if you ſtop the bottom of a ſyringe, and then draw up the piſton, if it be ſo tight as to drive out all the air before it, and leave a vacuum within the ſyringe, the piſton being let go, will be drove down with a great force.

7. The force with which the piſton is drove down, when there is a vacuum under it, will be as the ſquare of the diameter of the bore in the ſyringe. That is to ſay, it will be driven down with four times as much force in a ſyringe of a two inch bore, as in a ſyringe of one inch: for the areas of circles are always as the ſquares of their diameters.

8. The preſſure of the atmoſphere being equal to 15 pounds upon every ſquare inch, it will be equal to about 12 pounds upon every circular inch. So that if the bore of the ſyringe be round, and one inch in diameter, the piſton will be preſt down into it by a force nearly equal to 12 pounds: but if the bore be 2 inches diameter, the piſton will be preſt down with 4 times that force.

And hence it is eaſy to find with what force the atmoſphere preſſes upon any given number either of ſquare or circular inches.

Theſe being the principles upon which this engine is conſtructed, we ſhall next deſcribe the chief working parts of it: which are, 1. A boiler. 2. A cylinder and piſton. 3. A beam or lever.

The *boiler* is a large veſſel, generally made of iron or copper; and commonly ſo big, as to contain about 2000 gallons.

The *cylinder* is made about 40 inches diameter, bored ſo ſmooth, and its piſton fitting ſo cloſe, that little or no water can get between the piſton and ſides of the cylinder.

Things being thus prepared, the cylinder is placed upright, and the ſhank of the piſton is fixed to one end of the *beam*, which turns on a center like a common balance.

The boiler is placed under the cylinder, with a communication between them, which can be opened and ſhut occaſionally.

The boiler is filled about half full of water, and a ſtrong fire is made under it: then, if the communication between the boiler and the cylinder be opened, the cylinder will be filled with hot ſteam; which would drive the piſton quite out at the top of it. But there is a contrivance by which the piſton, when it is near the top of the cylinder, ſhuts the communication at the top of the boiler within.

This is no ſooner ſhut, than another is opened, by which a little cold water is thrown upwards in a jet into the cylinder, which mixing with the hot ſteam, condenſes it immediately; by which means a vacuum is made in the cylinder, and the piſton is preſſed down by the weight of the atmoſphere; and ſo lifts up the loaded pump-rod at the other end of the beam.

If

If the cylinder be 42 inches in diameter, the piſton will be preſſed down with a force greater than 20000 pounds, and will conſequently lift up that weight at the oppoſite end of the beam: and as the pump-rod with its plunger is fixed to that end, if the bore where the plunger works were 10 inches diameter, the water would be forced up through a pipe of 180 yards perpendicular height.

But, as the parts of this engine have a good deal of friction, and muſt work with a conſiderable velocity, and there is no ſuch thing as making a perfect vacuum in the cylinder, it is found that no more than 8 pounds of preſſure muſt be allowed for, on every circular inch of the piſton in the cylinder, that it may make about 16 ſtrokes in a minute, about 6 feet each.

Where the boiler is very large, the piſton will make between 20 and 25 ſtrokes in a minute, and each ſtroke 7 or 8 feet; which, in a pump of 9 inches bore, will raiſe upwards of 300 hogſheads of water in an hour.

It is found by experience that a cylinder, 40 inches diameter, will work a pump 10 inches diameter, and 100 yards long: and hence we can find the diameter and length of a pump, that can be worked by any other cylinder.

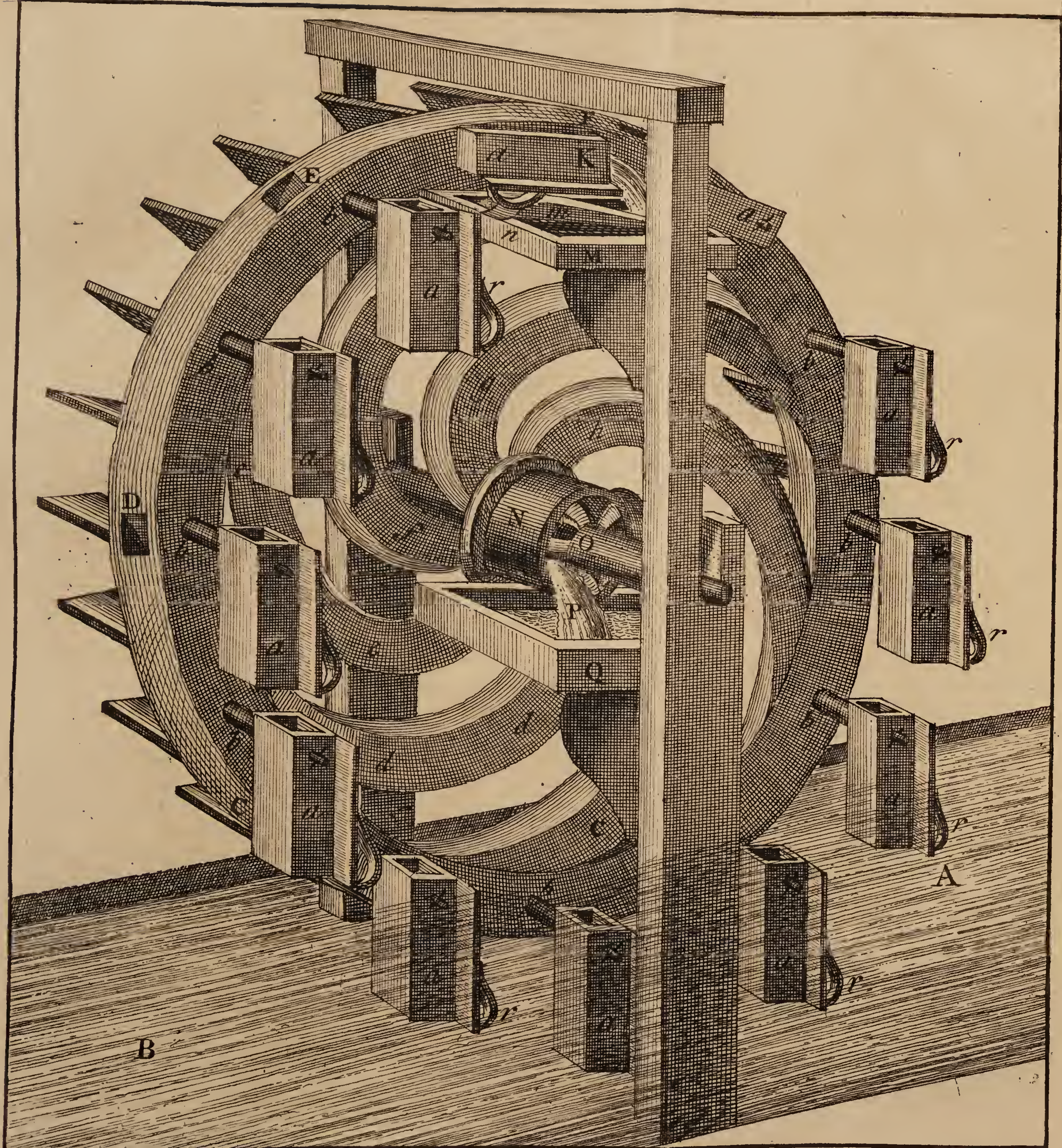
For the conveniency of thoſe who would make uſe of this engine for raiſing water, we ſhall ſubjoin part of a table calculated by Mr. *Beighton*, ſhewing how any given quantity of water may be raiſed in an hour, from 48 to 440 hogſheads; at any given depth, from 15 to 100 yards; the machine

chine working at the rate of 16 ſtrokes *per* minute, and each ſtroke being 6 feet long.

One example of the uſe of this table, will make the whole plain. Suppoſe it were required to draw 150 hogſheads *per* hour, at 90 yards depth; in the ſecond column from the right hand, I find the neareſt number, viz. 149 hogſheads 40 gallons, againſt which, on the right hand, I find the diameter of the bore of the pump muſt be 7 inches; and in the ſame collateral line, under the given depth 90, I find 27 inches, the diameter of the cylinder fit for that purpoſe.—And ſo for any other.

A Table

PLATE XIII.



J. Ferguson delin.

J. Mynde sc.

A Table shewing the power of the engine for raising water by fire.

This table is calculated to the meafure of ale gallons, at 282 cubic inches *per* gallon.

The depth to be drawn in yards.													In one hour.		Diam. of pump.
15	20	25	30	35	40	45	50	60	70	80	90	100	Hogfh.	Gal.	Inches.
18½	21¾	24	26½	28½	30½	32½	34¼	37¼	40	—	—	—	440		12
17	19¾	22	34¾	26¼	28	29¾	31¼	34¼	37	39½	—	—	369	33	11
15½	18	20	22	23¾	25¼	27	28¾	31¼	33¾	36	38¾	40	304	48	10
14	16¼	18	20	21½	23	24¼	25	28	30½	33	35	36½	247	7	9
13½	15¼	17¼	19	20¼	21¾	23	24	26¼	28½	31	32½	35¼	221	15	8½
12½	14½	16½	18½	19	20½	21½	23	25	27	29	30½	32½	195	22	8
12	14	15½	17½	18¾	19¾	21	22	24¼	26	28	29½	31½	182	13	7¾
11	13¾	15	16½	18	19	20	21¼	23¼	25	27	28½	30¼	172	30	7½
10¾	13	14	15½	16¾	18¾	19	20½	22	24	25½	27	28½	149	40	7
10	12	13	14	15½	16¼	18	19	20	22	23	24½	26¼	128	54	6½
9½	11	12	13	14	15½	16	17	19	20½	22	23	24½	110	1	6
	10	11	12	13	14	15	15¾	17	19	20	21	22¼	94	30	5½
		10	11	11¾	13	13¾	14	15½	16¾	18½	19¼	20¼	66	61	5
			10	10	11¼	12	13½	14	15	16	17	18¼	60	60	4½
				9	10	11	11½	12	13½	14	15	16	48	51	4

Diameter of the cylinder in inches.

Water

Plate XIII. The *Persian* wheel.

Water may be raiſed by means of a ſtream *AB* turning a wheel *CDE*, according to the order of the letters, with buckets *a, a, a, a,* &c. hung upon it by ſtrong pins *b, b, b, b,* &c. fixed in the ſide of the rim: but the wheel muſt be made as high as the water is intended to be raiſed above the level of that part of the ſtream in which the wheel is placed. As the wheel turns, the buckets on the right hand go down into the water, and are thereby filled, and go up full on the left hand, until they come to the top at *K*; where they ſtrike againſt the end *n* of the fixed trough *M*, and are thereby overſet, and empty the water into the trough; from which it may be conveyed in pipes to the place which it is deſigned for: and as each bucket gets over the trough, it falls into a perpendicular poſition again, and goes down empty, until it comes to the water at *A*, where it is filled as before. On each bucket is a piece *r*, which going over the top or crown of the bar *m* (fixed to the trough *M*) raiſes the bottom of the bucket above the level of its mouth, and ſo cauſes it to empty all its water into the trough.

Sometimes this wheel is made to raiſe water no higher than its axis; and then, inſtead of buckets hung upon it, its ſpokes *c, d, e, f, g, h,* are made of a bent form, and hollow within; theſe hollows opening into the holes *C, D, E, F,* in the outſide of the wheel, and alſo into thoſe at *O* in the box *N* upon the axis. So that, as the holes *C D*, &c. dip into the water, it runs into them; and as the wheel turns, the water riſes in the hollow ſpokes *c, d*, &c.

and

and runs out in a stream *P* from the holes at *O*, and falls into the trough *Q*, from whence it is conveyed by pipes. And this is a very easy way of raising water, because the engine requires neither men nor horses to turn it.

The art of weighing different bodies in water, and thereby finding their specific gravities, or weights, bulk for bulk, was invented by ARCHIMEDES; of which, we have the following account. Of the specific gravities of bodies.

Hiero, king of *Syracuse*, having employed a goldsmith to make a crown, and given him a mass of pure gold for that purpose, suspected that the workman had kept back part of the gold for his own use, and made up the weight by allaying the crown with copper. But the king not knowing how to find out the truth of that matter, referred it to *Archimedes*; who having studied a long time in vain, found it out at last by chance. For, going into a bathing tub of water, and observing that he thereby raised the water higher in the tub than it was before, he concluded instantly that he had raised it just as high as any thing else could have done, that was exactly of his bulk: and considering that any other body of equal weight, and of less bulk than himself, could not have raised the water so high as he did; he immediately told the king, that he had found a method by which he could discover whether there were any cheat in the crown. For, since gold is the heaviest of all known metals, it must be of less bulk, according to its weight, than any other metal. And therefore, he desired that a mass of pure gold, equally heavy with the crown when

when weighed in air, ſhould be weighed againſt it in water; and if the crown was not allayed, it would counterpoiſe the maſs of gold when they were both immerſed in water, as well as it did when they were weighed in air. But upon making the trial, he found that the maſs of gold weighed much heavier in water than the crown did. And not only ſo, but that, when the maſs and crown were immerſed ſeparately in one veſſel of water, the crown raiſed the water much higher than the maſs did; which ſhewed it to be allayed with ſome lighter metal that increaſed its bulk. And ſo, by making trials with different metals, all equally heavy with the crown when weighed in air, he found out the quantity of allay in the crown.

The ſpecific gravities of bodies are as their weights, bulk for bulk; thus, a body is ſaid to have two or three times the ſpecific gravity of another, when it contains two or three times as much matter in the ſame ſpace.

A body immerſed in a fluid will ſink to the bottom, if it be heavier than its bulk of the fluid. If it be ſuſpended therein, it will loſe as much of what it weighed in air, as its bulk of the fluid weighs. Hence, all bodies of equal bulk, which would ſink in fluids, loſe equal weights when ſuſpended therein. And unequal bodies loſe in proportion to their bulks.

The *hydroſtatic balance.*

The *hydroſtatic balance* differs very little from a common balance that is nicely made: only it has a hook at the bottom of each ſcale, on which ſmall weights may be hung by horſe-hairs, or by ſilk threads.

threads. So that a body, suspended by the hair or thread, may be immersed in water without wetting the scale from which it hangs.

How to find the specific gravity of any body.

If the body thus suspended under the scale, at one end of the balance, be first counterpoised in air by weights in the opposite scale, and then immersed in water, the equilibrium will be immediately destroyed. Then, if as much weight be put into the scale from which the body hangs, as will restore the equilibrium (without altering the weights in the opposite scale) that weight which restores the equilibrium, will be equal to the weight of a quantity of water as big as the immersed body. And if the weight of the body in air be divided by what it loses in water, the quotient will shew how much that body is heavier than its bulk of water. Thus, if a guinea suspended in air, be counterbalanced by 129 grains in the opposite scale of the balance; and then, upon its being immersed in water, it becomes so much lighter, as to require $7\frac{1}{4}$ grains put into the scale over it, to restore the equilibrium, it shews that a quantity of water, of equal bulk with the guinea, weighs $7\frac{1}{4}$ grains, or 7.25; by which divide 129, (the aerial weight of the guinea) and the quotient will be 17.793; which shews that the guinea is 17.793 times heavier than its bulk of water. And thus, any piece of gold may be tried, by weighing it first in air and then in water; and if upon dividing the weight in air by the loss in water, the quotient comes out to be 17.793, the gold is good; if the quotient be 18, or between 18 and 19, the gold

gold is very fine; but if it be less than $17\frac{1}{2}$, the gold is too much allayed, by being mixed with some other metal.

If silver be tried in this manner, and found to be 11 times heavier than water, it is very fine; if it be $10\frac{1}{2}$ times heavier, it is standard; but if it be of any less weight compared with water, it is mixed with some lighter metal, such as tin.

By this method, the specific gravities of all bodies that will sink in water, may be found. But as to those which are lighter than water, as most sorts of wood are; the following method may be taken, to shew how much lighter they are than their respective bulks of water.

Let an upright stud be fixed into a thick flat piece of brass, and in this stud let a small lever, whose arms are equally long, turn upon a fine pin as an axis. Let the thread which hangs from the scale of the balance be tied to one end of the lever, and a thread from the body to be weighed, tied to the other end. This done, put the brass and lever into a vessel; then pour water into the vessel, and the body will rise and float upon it, and draw down the end of the balance from which it hangs: then, put as much weight in the opposite scale as will raise that end of the balance, so as to pull the body down into the water by means of the lever; and this weight in the scale will shew how much the body is lighter than its bulk of water.

There are some things which cannot be weighed in this manner, such as quicksilver, fragments of diamonds, &c. because they cannot be suspended

in

in threads; and must therefore be put into a glass bucket, hanging by a thread from the hook of one scale, and counterpoised by weights put into the opposite scale. Thus, suppose you want to know the specific gravity of quicksilver, with respect to that of water; let the empty bucket be first counterpoised in air, and then the quicksilver put into it and weighed. Write down the weight of the bucket, and also of the quicksilver; which done, empty the bucket, and let it be immersed in water as it hangs by the thread, and counterpoised therein by weights in the opposite scale: then, pour the quicksilver into the bucket in the water, which will cause it to preponderate; and, put as much weight into the scale as will restore the balance to an equipoise; and this weight will be the weight of a quantity of water equal in bulk to the quicksilver. Lastly, divide the weight of the quicksilver in air by the weight of its bulk of water, and the quotient will shew how much the quicksilver is heavier than its bulk of water.

If a piece of brass, glass, lead, or silver, be immersed and suspended in different sorts of fluids, its different losses of weight therein will shew how much it is heavier than its bulk of the fluid; *that* fluid being the lightest, in which the immersed body loses least of its aerial weight. A solid bubble of glass is generally used for finding the specific gravities of fluids.

Hence we have an easy method of finding the specific gravities both of solids and fluids, with regard to their respective bulks of common pump

water, which is generally made a standard for comparing all the others by.

In constructing tables of specific gravities with accuracy, the gravity of water must be represented by unity or 1.000, where three cyphers are added, to give room for expressing the ratios of other gravities, as in the following table.

A Table of the specific gravities of several solid and fluid bodies.

A cubic inch of	Troy weight.			Averdup.		Comparative weight.
	oz.	pw.	gr.	oz.	drams.	
Very fine gold — —	10	7	3,83	1	5,80	19,637
Standard gold — —	9	19	6,44	10	14,90	18,888
Guinea gold — —	9	7	17,18	10	4,76	17,793
Moidore gold — —	9	0	19,84	9	14,71	17,140
Quickfilver — —	7	7	11,61	8	1,45	14,019
Lead — — —	5	19	17,55	6	9,08	11,325
Fine silver — —	5	16	23,23	6	6,66	11,087
Standard silver — —	5	11	3,36	6	1,54	10,535
Copper — — —	4	13	7,04	5	1,89	8,843
Plate brass — — —	4	4	9,60	4	10,09	8,000
Steel — — —	4	2	20,12	4	8,70	7,852
Iron — — —	4	0	15,20	4	6,77	7,645
Block tin — —	3	17	5,68	4	3,79	7,321
Speltar — — —	3	14	12,86	4	1,42	7,065
Lead ore — —	3	11	17,76	3	14,96	6,800
Glass of antimony —	2	15	16,89	3	0,89	5,280
German antimony —	2	2	4,80	2	5,04	4,000
Copper ore — —	2	1	11,83	2	4,43	3,775
Diamond — — —	1	15	20,88	1	15,48	3,400
Clear glass — — —	1	13	5,58	1	13,16	3,150
Lapis lazuli — — —	1	12	5,27	1	12,27	3,054
Welch asbestos — —	1	10	17,57	1	10,97	2,913

The

The Table concluded.

A cubic inch of	Troy weight. oz.	pw.	gr.	Averdup. oz.	drams.	Comparative weight.
White marble — —	1	8	13,41	1	9,06	2,707
Black ditto — —	1	8	12,65	1	9,02	2,704
Rock cryſtal — —	1	8	1,00	1	8,61	2,658
Green glaſs — —	1	7	15,38	1	8,26	2,620
Cornelian ſtone —	1	7	1,21	1	7,73	2,568
Flint — — —	1	6	19,63	1	7,53	2,542
Hard paving ſtone —	1	5	22,87	1	6,77	2,460
Live ſulphur — —	1	1	2,40	1	2,52	2,000
Nitre — — —	1	0	1,08	1	1,59	1,900
Alabaſter — —	0	19	18,74	1	1,35	1,875
Dry ivory — —	0	19	6,09	1	0,89	1,825
Brimſtone — —	0	18	23,76	1	0,66	1,800
Allum — — —	0	17	21,92	0	15,72	1,714
Ebony — — —	0	11	18,82	0	10,34	1,117
Human blood — —	0	11	2,89	0	9,76	1,054
Amber — — —	0	10	20,79	0	9,54	1,030
Cow's milk — —	0	10	20,79	0	9,54	1,030
Sea water — —	0	10	20,79	0	9,54	1,030
Pump water — —	0	10	13,30	0	9,26	1,000
Spring water — —	0	10	12,94	0	9,25	0,999
Diſtilled water — —	0	10	11,42	0	9,20	0,993
Red wine — —	0	10	11,42	0	9,20	0,993
Oil of amber — —	0	10	7,63	0	9,06	0,978
Proof ſpirits — —	0	9	19,73	0	8,62	0,931
Dry oak — — —	0	9	18,00	0	8,56	0,925
Olive oil — —	0	9	15,17	0	8,45	0,913
Pure ſpirits — —	0	9	3,27	0	8,02	0,866
Spirit of turpentine —	0	9	2,76	0	7,99	0.864
Oil of turpentine —	0	8	8,53	0	7,33	0,772
Dry crabtree — —	0	8	1,69	0	7,08	0,765
Saſſafras wood — —	0	5	2,04	0	4,46	0,482
Cork — — —	0	2	12,77	0	2,21	0,240

Take away the comma from the numbers in the right hand column, or (which is the ſame) multiply

 them

them by 1000, and they will ſhew how many ounces averdupoiſe are contained in a cubic foot of each body.

In troy weight, 24 grains make a pennyweight, 20 pennyweight make an ounce, and 12 ounces a pound. In averdupoiſe weight, 16 drams make an ounce, and 16 ounces a pound. The troy pound contains 5760 grains, and the averdupoiſe pound 7000; and hence, the averdupoiſe dram weighs 27.34375 grains, and the averdupoiſe ounce 437.5.

Becauſe it is often of uſe to know how much any given quantity of goods in troy weight do make in averdupoiſe weight, and the reverſe; we ſhall here annex two tables for converting theſe weights into one another. Thoſe from page 144 to page 149 are near enough for common hydraulic purpoſes; but the two following are better where accuracy is required in comparing the weights with one another: and by trial I find, that 175 troy ounces are preciſely equal to 192 averdupoiſe ounces, and 175 troy pounds are equal to 144 averdupoiſe. And although there are ſeveral leſſer integral numbers, which come very near to agree together, yet I have found none leſs than the above to agree exactly. Indeed 41 troy ounces are ſo nearly equal to 45 averdupoiſe ounces, that the latter contains only $7\frac{1}{2}$ grains more than the former: and 45 troy pounds weigh only $7\frac{3}{10}$ drams more than 37 averdupoiſe.

A Table

A Table for reducing troy weight into averdupoise weight.

Troy weight.	Averdupoise. lb.	oz	drams.	Troy weight.	Aver. Drams.
Pounds—4000	3291	6	13.68	Penny wt. 19	16.67
300	2528	9	2.26	18	15.79
2000	1645	11	6.84	17	14.92
1000	822	13	11.42	16	14.04
900	740	9	2.28	15	13.16
800	658	4	9.14	14	12.29
700	576	0	0.00	13	11.41
600	493	11	6.85	12	10.53
500	411	6	13.71	11	9.65
400	329	2	4.57	10	8.78
300	246	13	11.42	9	7.90
200	164	9	2.28	8	7.02
100	82	4	9.15	7	6.14
90	74	0	13.62	6	5.27
80	65	13	4.11	5	4.39
70	57	9	9.60	4	3.51
60	49	5	15.08	3	2.63
50	41	2	4.57	2	1.75
40	32	14	10.05	1	0.88
30	24	10	15.54	Grains— 23	.84
20	16	7	5.03	22	.80
10	8	3	10.52	21	.77
9	7	6	7.86	20	.73
8	6	9	5.21	19	.69
7	5	12	2.56	18	.66
6	4	14	15.90	17	.62
5	4	1	13.25	16	.58
4	3	4	10.60	15	.55
3	2	7	7.95	14	.51
2	1	10	5.30	13	.47
1	0	13	2.65	12	.44
Ounces— 11		12	1.09	11	.40
10		10	15.54	10	.36
9		9	13.99	9	.33
8		8	12.43	8	.29
7		7	10.88	7	.26
6		6	9.32	6	.22
5		5	7.77	5	.18
4		4	6.22	4	.15
3		3	4.66	3	.11
2		2	3.11	2	.07
1		1	1.55	1	.04

A Table for reducing averdupoise weight into troy weight.

Averdupoise weight.	Troy weight. lb.	oz.	pw.	gr.	Averd. weight.	Troy weight. lb.	oz.	pw.	gr.
Pounds 6000	7291	8	0	0	Ounces 15	1	1	13	10.50
5000	6076	4	13	8	14	1	0	15	5
4000	4861	1	6	16	13		11	16	23.50
3000	3645	10	0	0	12		10	18	18
2000	2430	6	13	8	11		10	0	12.50
1000	1215	3	6	16	10		9	2	7
900	1093	9	0	0	9		8	4	1.50
800	972	2	13	8	8		7	5	20
700	850	8	6	16	7		6	7	14.50
600	729	2	0	0	6		5	9	9
500	607	7	13	8	5		4	11	3.50
400	486	1	6	16	4		3	12	22
300	364	7	0	0	3		2	14	16.50
200	243	0	13	8	2		1	16	11
100	121	6	6	16	1		0	18	5.50
90	109	4	10	0	Drams 15			17	2.10
80	97	2	13	8	14			15	22.76
70	85	0	16	16	13			14	19.42
60	72	11	0	0	12			13	15.08
50	60	9	3	8	11			12	12.74
40	48	7	6	16	10			11	9.40
30	36	5	10	0	9			10	6.06
20	24	3	13	8	8			9	2.72
10	12	1	16	16	7			7	23.38
9	10	11	5	0	6			6	20.04
8	9	8	13	8	5			5	16.70
7	8	6	1	16	4			4	13.36
6	7	3	10	0	3			3	10.02
5	6	0	18	8	2			2	6.68
4	4	10	6	16	1			1	3.34
3	3	7	15	0	$\frac{3}{4}$			0	20.51
2	2	5	3	8	$\frac{1}{2}$				13.67
1	1	2	11	16	$\frac{1}{4}$				6.83

The

The two following examples will be sufficient to explain these two tables, and shew their agreement.

Ex. I. *In 6835 pounds 6 ounces 9 pennyweights 6 grains troy, Qu. How much averdupoise weight?* (See page 165.)

Averdupoise.

		lb.	oz.	drams.
Pounds troy—	4000	3291	6	13.68
	2000	1645	11	6.84
	800	658	4	9.14
	20	16	7	5.02
	10	8	3	10.51
	5	4	1	13.25
oz.	6		6	9.32
pw.	9			7.90
gr.	6			.22
Answer.		5624	10	11.88

Ex. II. *In 5624 pounds 10 ounces 12 drams averdupoise, Qu. How much troy weight?* (See page 166)

Troy.

		lb.	oz.	pw.	gr.
Pounds averd.	5000	6076	4	13	8
	600	729	2	0	0
	20	24	3	13	8
	4	4	10	6	16
oz.	10		9	2	7
dr.	12			13	15.08
Answer.		6835	6	9	6.08

How to find out the quantity of adulteration in metals.

The uſe of the table of ſpecific gravities will beſt appear by an example. Suppoſe a body to be compounded of gold and ſilver, and it is required to find the quantity of each metal in the compound.

Firſt find the ſpecific gravity of the compound, by weighing it in air and in water, and dividing its aerial weight by what it loſes thereof in water, the quotient will ſhew its ſpecific gravity, or how many times it is heavier than its bulk of water. Then, ſubtract the ſpecific gravity of ſilver (found in the table) from that of the compound, and the ſpecific gravity of the compound from that of gold; the firſt remainder ſhews the bulk of gold, and the latter the bulk of ſilver, in the whole compound: and if theſe remainders be multiplied by the reſpective ſpecific gravities, the products will ſhew the proportion of weights of each metal in the body. Example.

Suppoſe the ſpecific gravity of the compounded body be 13; that of ſtandard ſilver (by the table) is 10.5, and that of gold 19.63: therefore 10.5 from 13, remains 2.5, the proportional bulk of the gold; and 13 from 19.63, remains 6.63 the proportional bulk of ſilver in the compound. Then, the firſt remainder 2.5, multiplied by 19.63 the ſpecific gravity of gold, produces 49.075 for the proportional weight of gold; and the laſt remainder 6.63 multiplied by 10.5, the ſpecific gravity of ſilver, produces 69.615 for the proportional weight of ſilver in the whole body. So that, for every

every 49.07 ounces or pounds of gold, there are 69.6 pounds or ounces of silver in the body.

Hence it is easy to know whether any suspected metal be genuine, or allayed, or counterfeit; by finding how much it is heavier than its bulk of water, and comparing the same with the table: if they agree, the metal is good; if they differ, it is allayed or counterfeited.

How to try spirituous liquors.

A cubical inch of good brandy, rum, or other proof spirits, weighs 235.7 grains; therefore, if a true inch cube of any metal weighs 235.7 grains less in spirits than in air, it shews the spirits are proof. If it loses less of its aerial weight in spirits, they are above proof; if it loses more, they are under. For, the better the spirits are, they are the lighter, and the worse, the heavier. All bodies expand with heat and contract with cold, but some more and some less than others. And therefore the specific gravities of bodies are not precisely the same in summer as in winter. It has been found, that a cubic inch of good brandy is 10 grains heavier in winter than in summer; as much spirit of nitre, 20 grains; vinegar 6 grains, and spring water 3. Hence it is most profitable to buy spirits in winter, and sell them in summer, since they are always bought and sold by measure. It has been found, that 32 gallons of spirits in winter will make 33 in summer.

The *thermometer.*

The expansion of all fluids is proportionable to the degree of heat; that is, with a double or triple heat a fluid will expand two or three times as much.

Upon

Upon these principles depends the construction of the thermometer, in which the globe or bulb, and part of the tube, are filled with a fluid, which, when joined to the barometer, is spirits of wine tinged, that it may be the more easily seen in the tube. But when thermometers are made by themselves, quicksilver is generally used.

In the thermometer, a scale is always fitted to the tube, to shew the expansion of the quicksilver, and consequently the degree of heat. And, as *Farenheit*'s scale is most in esteem at present, I shall explain the construction and graduation of thermometers according to that scale.

First, let the globe or bulb, and part of the tube, be filled with a fluid; then immerse the bulb in water just freezing, or snow just thawing; and even with that part in the scale where the fluid then stands in the tube, place the number 32, to denote the freezing point: then put the bulb under your arm-pit, when your body is of a moderate degree of heat, so that it may acquire the same degree of heat with your skin; and when the fluid has risen as far as it can by that heat, there place the number 97: then divide the space between these numbers into 65 equal parts, and continue those divisions both above 97 and below 32, and number them accordingly.

This may be done in any part of the world; for it is found that the freezing point is always the same in all places, and the heat of the human body differs but very little: so that the thermometers made in this manner will perfectly agree with one another;

another; and the heat of several bodies will be shewn by them, and expressed by the number upon the scale, thus.

Air, in severe cold weather, in our climate, from 15 to 25. Air in winter, from 26 to 42. Air in spring and autumn, from 43 to 53. Air at midsummer, from 65 to 68. Extreme heat of the summer sun, from 86 to 100. Butter just melting, 95. Alcohol boils with 174 or 175. Brandy with 190. Water 212. Oil of turpentine 550. Tin melts with 408, and lead with 540. Milk freezes about 30, vinegar 28, and blood 27.

A body specifically lighter than a fluid will swim upon its surface, in such a manner, that a quantity of the fluid equal in bulk with the immersed part of the body, will be as heavy as the whole body. Hence, the lighter a fluid is, the deeper a body will sink in it; upon which depends the construction of the *hydrometer* or water-poise.

How the weight of a ship in water may be estimated.

From this we can easily find the weight of a ship, or any other body that swims in water. For, if we multiply the number of cubic feet which are under the surface, by 62.5, the number of pounds in one foot of fresh water; or by 63, the number of pounds in a foot of salt water; the product will be the weight of the ship and all that is in it. For, since it is the weight of the ship that displaces the water, it must continue to sink until it has removed as much water as is equal to it in weight: and therefore the part immersed must be equal in bulk

bulk to ſuch a portion of the water as is equal to the weight of the whole ſhip.

To prove this by experiment, let a ball of ſome light wood, ſuch as fir or peartree, be put into water contained in a glaſs veſſel; and let the veſſel be put into a ſcale at one end of a balance, and counterpoiſed by weights in the oppoſite ſcale: then, marking the height of the water in the veſſel, take out the ball; and fill up the veſſel with water to the ſame height that it ſtood at when the ball was in it; and the ſame weight will counterpoiſe it as before.

From the veſſel's being filled up to the ſame height at which the water ſtood when the ball was in it, it is evident that the quantity poured in is equal in magnitude to the immerſed part of the ball; and from the ſame weight counterpoiſing, it is plain that the water poured in is equal in weight to the whole ball.

LECT. VI.

Of pneumatics.

THIS ſcience treats of the nature, weight, and preſſure of the air, and the effects ariſing from it.

The properties of air.

The air is that thin tranſparent fluid body in which we live and breathe. It encompaſſes the whole earth to a conſiderable height; and, together with the clouds and vapours that float in it, is

is called the atmoſphere. The air is juſtly reckoned among the number of fluids, becauſe it has all the properties by which a fluid is diſtinguiſhed. For, it yields to the leaſt force impreſſed, its parts are eaſily moved among one another, it preſſes according to its perpendicular height, and its preſſure is every way equal.

That the air is a fluid, conſiſting of ſuch particles as have no coheſion betwixt them, but eaſily glide over one another, and yield to the ſlighteſt impreſſion, appears from that eaſe and freedom with which animals breathe in it, and move through it without any difficulty or ſenſible reſiſtance.

But it differs from all other fluids in the three following particulars. 1. It can be compreſſed into a leſs ſpace than what it naturally poſſeſſeth, which no other fluid can. 2. It cannot be congealed or fixed, as other fluids may. 3. It is of a different denſity in every part, upward from the earth's ſurface, decreaſing in its weight, bulk for bulk, the higher it riſes; and therefore muſt alſo decreaſe in denſity. 4. It is of an elaſtic or ſpringy nature, and the force of its ſpring is equal to its weight.

That air is a body, is evident from its excluding all other bodies out of the ſpace it poſſeſſes: for, if a glaſs jar be plunged with its mouth downward into a veſſel of water, there will but very little water get into the jar, becauſe the air of which it is full keeps the water out.

As air is a body, it muſt needs have gravity or weight: and that it is weighty, is demonſtrated by experiment. For, let the air be taken out of a veſſel

a veſſel by means of the air pump, then, having weighed the veſſel, let in the air again, and upon weighing it when re-filled with air, it will be found conſiderably heavier. Thus, a bottle that holds a wine quart, being emptied of air and weighed, is found to be about 17 grains lighter than when the air is let into it again; which ſhews that a quart of air weighs 17 grains. But a quart of water weighs 14625 grains, this divided by 17 quotes 860 in round numbers; which ſhews, that water is 860 times heavier than air near the ſurface of the earth.

As the air riſes above the earth's ſurface, it grows rarer, and conſequently lighter, bulk for bulk. For ſince it is of an elaſtic or ſpringy nature, and its lowermoſt parts are preſſed with the weight of all that is above them, it is plain that the air muſt be more denſe or compact at the earth's ſurface, than at any height above it; and gradually rarer the higher up. For, the denſity of the air will always be as the force that compreſſes it: and therefore, the air towards the upper part of the atmoſphere being leſs preſſed than that which is near the earth, it will expand itſelf, and thereby become thinner than at the ſurface of the earth.

Dr. *Cotes* has demonſtrated, that if altitudes in the air be taken in arithmetical proportion, the rarity of the air will be in geometrical proportion. For inſtance,

At

At the altitude of	Miles above the surface of the earth, the air is	times thinner and lighter than at the surface.
	7	4
	14	16
	21	64
	28	256
	35	1024
	42	4096
	49	16384
	56	65536
	63	262144
	70	1048576
	77	4194304
	84	16777216
	91	67108864
	98	268435456
	105	1073741824
	112	4294967296
	119	17179869184
	126	68719476736
	133	274877906944
	140	1099511627776

And hence it is eaſy to calculate, that a cubic inch of ſuch air as we breathe, would be ſo much rarefied at the altitude of 500 miles, that it would fill a ſphere equal in diameter to the orbit of Saturn.

The weight or preſſure of the air is exactly determined by the following experiment.

The *Torricellian* experiment.

Take a glaſs tube about three feet long, and open at one end; fill it with quickſilver, and putting your finger upon the open end, turn that end downward, and immerſe it into a ſmall veſſel of quickſilver, without letting in any air: then take

away

away your finger, and the quickſilver will remain ſuſpended in the tube $29\frac{1}{2}$ inches above its ſurface in the veſſel; ſometimes more, and at other times leſs, as the weight of the air is varied by winds and other cauſes. That the quickſilver is kept up in the tube by the preſſure of the atmoſphere upon that in the baſon, is evident; for, if the baſon and tube be put under a glaſs, and the air taken out of the glaſs, all the quickſilver in the tube falls down into the baſon; and when the air is let in again, the quickſilver riſes to the ſame height as before. Therefore the air's preſſure on the ſurface of the earth, is equal to the weight of $29\frac{1}{2}$ inches depth of quickſilver all over the earth's ſurface, at a mean rate.

A ſquare column of quickſilver, $29\frac{1}{2}$ inches high, and one inch thick, weighs juſt 15 pounds, which is equal to the preſſure of air upon every ſquare inch of the earth's ſurface; and 144 times as much, or 2160 pounds, upon every ſquare foot; becauſe a ſquare foot contains 144 ſquare inches. At this rate, a middle ſiz'd man, whoſe ſurface may be about 14 ſquare feet, ſuſtains a preſſure of 30240 pounds, or $13\frac{1}{2}$ tons, when the air is of a mean gravity: a preſſure which would be inſupportable, and even fatal to us, were it not equal on every part, and counterbalanced by the ſpring of the air within us, which is diffuſed through the whole body; and re-acts with an equal force againſt the outward preſſure.

Now, ſince the earth's ſurface contains (in round numbers) 200,000,000 ſquare miles, and every ſquare

ſquare mile 27878400 ſquare feet, there muſt be 5575680000000000 ſquare feet on the earth's ſurface; which multiplied by 2160 pounds (the preſſure on each ſquare foot) gives 12043468800000000000 pounds for the preſſure or weight of the whole atmoſphere.

When the air is taken out of a pipe, and the end of the pipe immerſed in water, the water will riſe in it to the height of 33 feet above the ſurface of the water in which it is immerſed; but will go no higher: for it is found, that a common pump will draw water no higher than 33 feet above the ſurface of the well: and unleſs the bucket goes within that diſtance from the well, the water will never get above it. Now, as it is the preſſure of the atmoſphere, on the ſurface of the water in the well, that cauſes the water to aſcend in the pump, and follow the piſton or bucket, as all the air above it is lifted up; it is evident, that a column of water 33 feet high, is equal in weight to a column of quickſilver of the ſame diameter, $29\frac{1}{2}$ inches high; and to as thick a column of air, reaching from the earth's ſurface to the top of the atmoſphere.

In ſerene calm weather, the air has weight enough to ſupport a column of quickſilver 31 inches high; but in tempeſtuous ſtormy weather, not above 28 inches. The quickſilver, thus ſupported in a glaſs tube, is found to be a nice counterbalance to the weight or preſſure of the air, and to ſhew its alterations at different times. And being now generally uſed to denote the changes in the weight of

The *barometer.*

the air, and of the weather conſequent upon them, it is called the *barometer*, or weather glaſs.

The preſſure of the air being equal on all ſides of a body expoſed to it, the ſofteſt bodies ſuſtain this preſſure without ſuffering any change in their figure; and ſo do the moſt brittle bodies without being broke.

The cauſe of *winds*.

The air is rarefied, or made to ſwell with heat; and of this property, *wind* is a neceſſary conſequence. For, when any part of the air is heated by the ſun, or otherwiſe, it will ſwell, and thereby affect the adjacent air: and ſo, by various degrees of heat in different places, there will ariſe various winds.

When the air is much heated, it will aſcend towards the upper part of the atmoſphere, and the adjacent air will ruſh in to ſupply its place; and therefore, there will be a ſtream or current of air from all parts towards the place where the heat is. And hence, we ſee the reaſon why the air ruſhes with ſuch force into a glaſs houſe; or towards any place where a great fire is made. And alſo, why ſmoke is carried up a chimney, and why the air ruſhes in at the key-hole of the door, or any ſmall chink, when there is a fire in the room. So we may take it in general, that the air will preſs towards that part of the world where it is moſt heated.

The *trade-winds*.

Upon this principle, we can eaſily account for the *trade-winds*, which blow conſtantly from eaſt to weſt about the equator. For, when the ſun ſhines perpendicularly on any part of the earth, it will

will heat the air very much in that part, which air will therefore riſe upward, and when the ſun withdraws, the adjacent air will ruſh in to fill its place; and conſequently cauſe a ſtream or current of air from all parts towards that which is moſt heated by the ſun. But, as the ſun, with reſpect to the earth, moves from eaſt to weſt, the common courſe of the air will be that way too; continually preſſing after the ſun: and therefore, at the equator, where the ſun ſhines ſtrongly, there will be a continual eaſterly wind; but, on the north-ſide, it will incline a little to the north, and on the ſouth-ſide, to the ſouth.

This general courſe of the wind about the equator is changed in ſeveral places, and upon ſeveral accounts; as, 1. By exhalations that riſe out of the earth at certain times, and from certain places; in earthquakes, and from volcano's. 2. By the falling of great quantities of rain, cauſing thereby a ſudden condenſation or contraction of the air. 3. By burning ſands, that often retain the ſolar heat to a degree incredible to thoſe who have not felt it, cauſing a more than ordinary rarefaction of the air contiguous to them. 4. By high mountains, that alter the direction of the winds in ſtriking againſt them. 5. By the declination of the ſun towards the north or ſouth, heating the air on the north or ſouth ſide of the equator.

To theſe and ſuch like cauſes is owing, 1. The irregularity and uncertainty of winds in climates diſtant from the equator, as in moſt parts of *Europe*. 2. Thoſe periodical winds called *monſoons*, which

The *monſoons.*

which in the *Indian* ſeas blow half a year one way, and the other half another. 3. Thoſe winds which on the coaſt of *Guinea*, and on the weſtern coaſts of *America*, blow always from weſt to eaſt. 4. The ſea-breezes, which, in hot countries, blow generally from ſea to land, in the day-time; and the land-breeze, which blows in the night; and, in ſhort, all thoſe ſtorms, hurricanes, whirlwinds, and irregularities, which happen at different times and places.

The *vivifying ſpirit* in air. All common air is impregnated with a *vivifying ſpirit*, which is neceſſary to continue the lives of animals: and this, in a gallon of air, is ſufficient for one man during the ſpace of a minute, and not much longer.

This ſpirit in air is deſtroyed by paſſing through the lungs of animals: and hence it is, that an animal dies ſoon, after being put under a veſſel which admits no freſh air to come to it. This ſpirit is alſo infuſed into water; for fiſh die when they are excluded from freſh air, as in a pond that is cloſely frozen over. And the little eggs of inſects, ſtopped up in a glaſs, do not produce their young, though aſſiſted by a kindly warmth. The ſeeds alſo of plants mixed with good earth, and incloſed in a glaſs, will not grow. So that freſh air is abſolutely neceſſary, both for the production and continuation of the lives of animals and plants.

This enlivening quality in air is alſo deſtroyed by the air's paſſing through fire; particularly charcoal fire, or the flame of ſulphur. Hence, ſmoaking chimneys muſt be very unwholſome, eſpecially if the rooms they are in be ſmall and cloſe.

Air

Air is alſo vitiated, by remaining cloſely pent up in any place for a conſiderable time; or perhaps, by being mixed with malignant ſteams and particles flowing from the neighbouring bodies; or laſtly, by the corruption of the vivifying ſpirit; as in the holds of ſhips, in oil-ciſterns, or wine cellars which have been ſhut up for a conſiderable time. The air in any of them is ſometimes ſo much vitiated, as to be immediate death to any animal that comes into it.

Air that has loſt its vivifying ſpirit is called *damp*, not only becauſe it is filled with humid or moiſt vapours, but becauſe it deadens fire, extinguiſhes flame, and deſtroys life.—The dreadful effects of damps are ſufficiently known to ſuch as work in mines. *Damps.*

If part of the vivifying ſpirit of air in any country begins to putrefy, the inhabitants of that country will be ſubject to an epidemical diſeaſe, which will continue until the putrefaction is over. And as the putrefying ſpirit occaſions the diſeaſe, ſo if the diſeaſed body contributes towards the putrefying of the air, then the diſeaſe will not only be epidemical, but peſtilential and contagious.

The atmoſphere is the common receptacle of all the effluvia or vapours ariſing from different bodies; of the ſteams and ſmoke of things burnt or melted; the fogs or vapours proceeding from damp watery places; and of the effluvia from ſulphureous, nitrous, acid and alkaline bodies. In ſhort, whatever may be called volatile, riſes in the air to greater or leſs heights, according to its ſpecific gravity.

Fermentations.

When the effluvia, which ariſe from acid and alkaline bodies, meet each other in the air, there will be a ſtrong conflict or *fermentation* between them; which will ſometimes be ſo great, as to produce a fire: then, if the effluvia be combuſtible, the fire will run from one part to another, juſt as the inflammable matter happens to lie.

Any one may be convinced of this, by mixing an acid and an alkaline fluid together, as the ſpirit of nitre and oil of cloves; upon the doing of which, a ſudden ferment, with a fine flame, will ariſe; and if the ingredients be very pure and ſtrong, there will be a ſudden exploſion.

Thunder and *lightning.*

Whoever conſiders the effects of fermentation, cannot be at a loſs to account for the dreadful effects of *thunder* and *lightning*: for the effluvia of ſulphureous and nitrous bodies, and others that may riſe into the atmoſphere, will ferment with each other, and take fire very often of themſelves; ſometimes by the aſſiſtance of the ſun's heat.

If the inflammable matter be thin and light, it will riſe to the upper part of the atmoſphere, where it will flaſh without doing any harm: but if it be denſe, it will lie nearer the ſurface of the earth, where taking fire, it will explode with a ſurpriſing force; and by its heat rarefy and drive away the air, kill men and cattle, ſplit trees, walls, rocks, &c. and be accompanied with terrible claps of thunder.

The heat of lightning appears to be quite different from that of other fires; for it has been known to run through wood, leather, cloth, &c.

without

without hurting them; while it has broken and melted iron, ſteel, ſilver, gold, and other hard bodies. Thus it has melted or burnt aſunder a ſword without hurting the ſcabbard, and money in a man's pocket without hurting his cloaths: the reaſon of this ſeems to be, that the particles of the fire are ſo fine, as to paſs through ſoft looſe bodies without diſſolving them; whilſt they ſpend their whole force upon the hard ones.

It is remarkable, that knives and forks which have been ſtruck with lightning have a very ſtrong magnetical virtue for ſeveral years after; and I have heard that lightning ſtriking upon the mariner's compaſs will ſometimes turn it round; and often make it ſtand the contrary way; or with the north pole towards the ſouth.

Much of the ſame kind with lightning are thoſe explosions called *fulminating* or *fire damps*, which ſometimes happen in mines; and are occaſioned by ſulphureous and nitrous, or rather oleaginous particles, riſing from the mine and mixing with the air, where they will take fire by the lights which the workmen are obliged to make uſe of. The fire being kindled, will run from one part of the mine to another, like a train of gunpowder, as the combuſtible matter happens to lie. And as the elaſticity of the air is increaſed by heat, that in the mine will conſequently ſwell very much, and ſo, for want of room, will explode with a greater or leſs degree of force, according to the denſity of the combuſtible vapours. It is ſometimes ſo ſtrong, as to blow up the mine; and at other times ſo *Fire damps.*

weak, that when it has taken fire at the flame of a candle, it is easily blown out.

Air that will take fire at the flame of a candle, may be produced thus. Having exhausted a receiver of the air-pump, let the air run into it through the flame of the oil of turpentine; then remove the cover of the receiver, and holding a candle to that air, it will take fire, and burn quicker or slower, according to the density of the oleaginous vapour.

Earth-quakes. When such combustible matter, as is above mentioned, kindles in the bowels of the earth, where there is little or no vent, it produces *earthquakes*, and violent storms or hurricanes of wind, when it breaks forth into the air.

An artificial earthquake may be made thus. Take 10 or 15 pounds of sulphur, and as much of the filings of iron, and knead them with common water into the consistence of a paste: this being buried in the ground, will, in 8 or 10 hours time, burst out in flames, and cause the earth to tremble all around to a considerable distance.

From this experiment we have a very natural account of the fire of mount *Ætna*, *Vesuvius*, and other volcano's, they being probably set on fire at first by the mixture of such metaline and sulphureous particles.

Plate XIV. Fig. 1. The *air-pump* being in effect the same as the water-pump, whoever understands the one, will be at no loss to understand the other.

The *air-pump.* Having put a wet leather on the plate *LL* of the air-pump, place the glass receiver *M* upon the leather,

PLATE XIV.

J. Ferguson delin.

J. Mynde sculp.

leather, ſo that the hole *i* in the plate may be within the glaſs. Then, turning the handle *F* backward and forward, the air will be pumped out of the receiver; which will then be held down to the plate by the preſſure of the external air, or atmoſphere. For, as the handle (Fig. 2.) is turned backwards, it riſes the piſton *de* in the barrel *BK*, by means of the wheel *F* and rack *Dd*: and as the piſton is leathered ſo tight, as to fit the barrel exactly, no air can get between the piſton and barrel; and therefore, all the air above *d* in the barrel is lifted up towards *B*, and a vacuum made in the barrel from *e* to *b*: upon which, part of the air in the receiver *M* (Fig. 1.), by its ſpring, ruſhes through the hole *i*, in the braſs plate *LL*, along the pipe *GG* (which communicates with both barrels by the hollow trunk *IHK*, Fig. 2.), and puſhing up the valve *b*, enters into the vacant place *be* of the barrel *BK*. For, wherever the reſiſtance or preſſure is taken off, the air will run to that place, if it can find a paſſage.—Then, as the handle *F* is turned forward, the piſton *de* is depreſſed in the barrel; and, as the air which had got into the barrel cannot be puſhed back through the valve *b*, it aſcends through a hole in the piſton, and eſcapes through a valve at *d*; and is hindered by that valve from returning into the barrel, when the piſton is again raiſed. At the next raiſing of the piſton, a vacuum is again made, in the ſame manner as before, between *b* and *e*; upon which, more of the air which was left in the receiver *M* gets out thence by its ſpring, and runs into the barrel *BK*, through the

the valve *b*. The ſame thing is to be underſtood with regard to the other barrel *AI*; and as the handle *F* is turned backwards and forwards, it alternately raiſes and depreſſes the piſtons in their barrels; always raiſing one whilſt it depreſſes the other. And, as there is a vacuum made in each barrel when its piſton is raiſed, every particle of air in the receiver *M* puſhes out another, by its ſpring or elaſticity, through the hole *i* and pipe *GG* into the barrels; until at laſt the air in the receiver comes to be ſo much dilated, and its ſpring ſo far weakened, that it can no longer get through the valves; and then, no more can be taken out. Hence, there is no ſuch thing as making a perfect vacuum in the receiver; for the quantity of air taken out at any one ſtroke, will always be as the denſity thereof in the receiver: and therefore it is impoſſible to take it all out, becauſe, ſuppoſing the receiver and barrels of equal capacity, there will be always as much left as was taken out at the laſt turn of the handle.

There is a cock *k* below the pump-plate, which being turned, lets the air into the receiver again; and then it becomes looſe, and may be taken off the plate. The barrels are fixed into the frame *ee* by two ſcrew-nuts *ff*, which preſs down the top-piece *E* upon the barrels: and the hollow trunk *H* (in Fig. 2.) is covered by a box, as *GH* in Fig. 1.

There is a glaſs tube *lmn* open at both ends, and about 34 inches long; the upper end communicating with the hole in the pump-plate, and the lower end immerſed in quickſilver at *n* in the veſſel

veſſel *N*. To this tube is fitted a wooden ruler *mm*, called the *gage*, which is divided into inches and parts of an inch, from the bottom at *n* (where it is even with the ſurface of the quickſilver) and continued up to the top, a little below *l*, to 30 or 31 inches.

As the air is pumped out of the receiver *M*, it is likewiſe pumped out of the glaſs tube *lmn*, becauſe that tube opens into the receiver through the pump-plate: and as the tube is gradually emptied of air, the quickſilver in the veſſel *N* is forced up into the tube by the preſſure of the atmoſphere. And if the receiver could be perfectly exhauſted of air, the quickſilver would ſtand as high in the tube, as it does at that time in the barometer: for it is ſupported by the ſame power or weight of the atmoſphere in both.

The quantity of air exhauſted out of the receiver on each turn of the handle, is always proportionable to the aſcent of the quickſilver on that turn: and the quantity of air remaining in the receiver, is proportionable to the defect of the height of the quickſilver in the gage, from what it is at that time in the barometer.

I ſhall now give an account of the experiments made with the air-pump, in my lectures; ſhewing the reſiſtance, weight, and elaſticity of the air.

I. *To ſhew the reſiſtance of the air.*

1. There is a little machine, conſiſting of two mills, *a* and *b*, which are of equal weights, inde- Fig. 3.

pendent

pendent of each other, and turn equally free on their axes in the frame. Each mill has four thin arms or ſails, fixed into the axis: thoſe of the mill *a* have their planes at right angles to its axis, and thoſe of *b* have their planes parallel to it. Therefore, as the mill *a* turns round in common air, it is but little reſiſted thereby, becauſe its ſails cut the air with their thin edges: but the mill *b* is much reſiſted, becauſe the broad ſides of its ſails move againſt the air when it turns round. In each axle is a pin near the middle of the frame, which goes quite through the axle, and ſtands out a little on each ſide of it: upon theſe pins, the ſlider *d* may be made to bear, and ſo hinder the mills from going, when the ſtrong ſpring *c* is ſet on bend againſt the oppoſite ends of the pins.

Having ſet this machine upon the pump-plate *LL* (Fig. 1.) draw up the ſlider *d* to the pins on one ſide, and ſet the ſpring *c* at bend upon the oppoſite ends of the pins: then puſh down the ſlider *d*; and the ſpring acting equally ſtrong on each mill, will ſet them both a going with equal forces and velocities: but the mill *a* will run much longer than the mill *b*, becauſe the air makes much leſs reſiſtance againſt the edges of its ſails, than againſt the ſides of the ſails of *b*.

Draw up the ſlider again, and ſet the ſpring upon the pins as before; then cover the machine with the receiver *M* upon the pump-plate, and having exhauſted the receiver of air, puſh down the wire *PP* (through the collar of leathers in the neck *q*) upon the ſlider; which will diſengage it from the pins,

pins, and allow the mills to turn round by the impulſe of the ſpring: and as there is no air in the receiver to make any ſenſible reſiſtance againſt them, they will both move a conſiderable time longer than they did in the open air; and the moment that one ſtops, the other will do ſo too.—This ſhews that air reſiſts bodies in motion, and that equal bodies may meet with different degrees of reſiſtance, according as they preſent greater or leſs ſurfaces to the air, in the planes of their motions.

2. Take off the receiver *M*, and the mills; and having put the guinea *a* and feather *b* upon the braſs flap *c*, turn up the flap, and ſhut it into the notch *d*. Then, putting a wet leather over the top of the tall receiver *AB* (it being open both at top and bottom) cover it with the plate *C*, from which the guinea-and-feather tongs *ed* will then hang within the receiver. This done, pump the air out of the receiver; and then draw up the wire *f* a little, which by a ſquare piece on its lower end will open the tongs *ed*; and the flap falling down, as at *c*, the guinea and feather will deſcend with equal velocities in the receiver; and both will fall upon the pump-plate at the ſame inſtant. *N. B.* In this experiment, the obſervers ought not to look at the top, but at the bottom of the receiver; in order to ſee the guinea and feather fall upon the plate: otherwiſe, on account of the quickneſs of their motion, they will eſcape the ſight of the beholders.

Fig. 4.

II. *To*

II. *To ſhew the weight of the air.*

1. Having fitted a braſs cap, with a valve tied over it, to the mouth of a thin bottle, or *Florence* flaſk, whoſe contents is exactly known, ſcrew the neck of this cap into the hole *i* of the pump-plate: then, having exhauſted the air out of the flaſk, and taken it off from the pump, let it be ſuſpended at one end of a balance, and nicely counterpoiſed by weights in the ſcale at the other end: this done, raiſe up the valve with a pin, and the air will ruſh into the flaſk with an audible noiſe; during which time, the flaſk will deſcend, and pull down that end of the beam. When the noiſe is over, put as many grains into the ſcale as will reſtore the equilibrium; and they will ſhew exactly the weight of the quantity of air which has got into the flaſk, and filled it. If the flaſk holds an exact quart, it will be found, that 17 grains will reſtore the equipoiſe of the balance, when the quickſilver ſtands at $29\frac{1}{2}$ inches in the barometer: which ſhews, that when the air is at a mean rate of denſity, a quart of it weighs 17 grains: it weighs more when the quickſilver ſtands higher; and leſs when it ſtands lower.

2. Place the ſmall receiver *O* (Fig. 1.) over the hole *i* in the pump-plate, and upon exhauſting the air, the receiver will be fixed down to the plate by the preſſure of the air on its outſide, which is left to act alone, without any air in the receiver to act againſt it: and this preſſure will be equal to as many

many times 15 pounds, as there are ſquare inches in that part of the plate which the receiver covers; which will hold down the receiver ſo faſt, that it cannot be got off, until the air be let into it by turning the cock *k*; and then it becomes looſe.

3. Set the little glaſs *AB* (which is open at both ends) over the hole *i* upon the pump-plate *L L*, and clap your hand cloſe upon the top of it at *B*: then, upon exhauſting the air out of the glaſs, you will find your hand preſſed down with a great weight upon it; ſo that you can hardly releaſe it, until the air be re-admitted into the glaſs by turning the cock *k*; which air, by acting as ſtrongly upward againſt the hand as the external air acted in preſſing it downward, will releaſe the hand from its confinement. Fig. 5.

4. Having tied a piece of wet bladder *b* over the open top of the glaſs *A* (which is alſo open at bottom) ſet it to dry, and then the bladder will be as tight as a drum. Then place the open end *A* upon the pump-plate, over the hole *i*, and begin to exhauſt the air out of the glaſs. As the air is exhauſting, its ſpring in the glaſs will be weakened, and give way to the preſſure of the outward air on the bladder, which, as it is preſſed down, will put on a ſpherical concave figure, which will grow deeper and deeper, until the ſtrength of the bladder be overcome by the weight of the air; and then it will break with a report as loud as that of a gun. —If a flat piece of glaſs be laid upon the open top of this receiver, and joined to it by a flat ring of wet leather between them; upon pumping the air Fig. 6.

air out of the receiver, the preſſure of the outward air upon the flat glaſs will break it all to pieces.

Fig. 7. 5. Immerſe the neck *cd* of the hollow glaſs ball *eb* in water, contained in the phial *aa*; then ſet it upon the pump-plate, and cover it and the hole *i* with the cloſe receiver *A*; and then begin to pump out the air. As the air goes out of the receiver by its ſpring, it will alſo by the ſame means go out of the hollow ball *eb*, through the neck *dc*, and riſe up in bubbles to the ſurface of the water in the phial; from whence it will make its way, with the reſt of the air in the receiver, through the air-pipe *GG* and valves *a* and *b*, into the open air. When it has done bubbling in the phial, the ball is ſufficiently exhauſted; and then, upon turning the cock *k*, the air will get into the receiver, and preſs ſo upon the ſurface of the water in the phial, as to force the water up into the ball in a jet, through the neck *cd*; and will fill the ball almoſt full of water. The reaſon why the ball is not quite filled, is becauſe all the air could not be taken out of it; and the ſmall quantity that was left in, and had expanded itſelf ſo as to fill the whole ball, is now condenſed into the ſame ſtate as the outward air, and remains in a ſmall bubble at the top of the ball; and ſo keeps the water from filling that part of it.

Fig. 8. 6. Pour ſome quickſilver into the jar *D* and ſet it on the pump-plate near the hole *i*; then ſet on the tall open receiver *AB*, ſo as to be over the jar and hole; and cover the receiver with the braſs plate *C*. Screw the open glaſs tube *fg* (which has a braſs

a brafs top on it at *h*) into the fyringe *H*, and putting the tube through a hole in the middle of the plate, fo as to immerfe the lower-end of the tube *e* in the quickfilver at *D*, fcrew the end *h* of the fyringe into the plate. This done, draw up the pifton in the fyringe by the ring *I*, which will make a vacuum in the fyringe below the pifton; and as the upper end of the tube opens into the fyringe, the air will be dilated in the tube, becaufe part of it, by its fpring, gets up into the fyringe; and the fpring of the undilated air in the receiver acting upon the furface of the quickfilver in the jar, will force part of it up into the tube: for the quickfilver will follow the pifton in the fyringe, in the fame way, and for the fame reafon, that water follows the pifton of a common pump when it is raifed in the pump barrel; and this, according to fome, is done by fuction. But to refute that erroneous notion, let the air be pumped out of the receiver *AB*, and then all the quickfilver in the tube will fall down by its own weight into the jar; and cannot be again raifed one hair's breadth in the tube by working the fyringe: which fhews that fuction had no hand in raifing the quickfilver; and, to prove that it is done by preffure, let the air into the receiver by the cock *k* (Fig. 1.) and its action upon the furface of the quickfilver in the jar will raife it up into the tube, although the pifton of the fyringe continues motionlefs.—If the tube be about 32 or 33 inches high, the quickfilver will rife in it very near as high as it ftands at

that time in the barometer. And, if the ſyringe has a ſmall hole, as *m*, near the top of it, and the piſton be drawn up above that hole, the air will ruſh through the hole into the ſyringe and tube, and the quickſilver will immediately fall down into the jar. If this part of the apparatus be air-tight, the quickſilver may be pumped up into the tube to the ſame height that it ſtands in the barometer; but it will go no higher, becauſe then the weight of the column in the tube is the ſame as the weight of a column of air of the ſame thickneſs with the quickſilver, and reaching from the earth to the top of the atmoſphere.

Fig. 9. 7. Having placed the jar *A*, with ſome quickſilver in it, on the pump-plate, as in the laſt experiment, cover it with the receiver *B*; then puſh the open end of the glaſs tube *de* through the collar of leathers in the braſs neck *C* (which it fits ſo as to be air-tight) almoſt down to the quickſilver in the jar. Then exhauſt the air out of the receiver, and it will alſo come out of the tube, becauſe the tube is cloſe at top. When the gauge *mm* ſhews that the receiver is well exhauſted, puſh down the tube, ſo as to immerſe its lower end into the quickſilver in the jar. Now, although the tube be exhauſted of air, none of the quickſilver will riſe into it, becauſe there is no air left in the receiver to preſs upon its ſurface in the jar. But let the air into the receiver by the cock *k*, and the quickſilver will immediately riſe in the tube; and ſtand as high in it, as it was pumped up in the laſt experiment.

Both

Both theſe experiments ſhew, that the quickſilver is ſupported in the barometer by the preſſure of the air on its ſurface in the box, in which the open end of the tube is placed. And that the more denſe and heavy the air is, the higher does the quickſilver riſe; and, on the contrary, the thinner and lighter the air is, the more will the quickſilver fall. For, if the handle *F* be turned ever ſo little, it takes ſome air out of the receiver, by raiſing one or other of the piſtons in its barrel; and conſequently, that which remains in the receiver is ſo much the rarer, and has ſo much the leſs ſpring and weight; and thereupon, the quickſilver falls a little in the tube: but upon turning the cock, and re-admitting the air into the receiver, it becomes as weighty as before, and the quickſilver riſes again to the ſame height.—Thus we ſee the reaſon why the quickſilver in the barometer falls before rain or ſnow, and riſes before fair weather; for, in the former caſe, the air is too thin and light to bear up the vapours, and in the latter, too denſe and heavy to let them fall.

N.B. In all mercurial experiments with the air-pump, a ſhort pipe muſt be ſcrewed into the hole *i*, ſo as to riſe about an inch above the plate, to prevent the quickſilver from getting into the air-pipe and barrels, in caſe any of it ſhould be accidentally ſpilt over the jar: for if it once gets into the pipes or barrels, it ſpoils them, by looſening the ſolder, and corroding the braſs.

8. Take the tube out of the receiver, and put one end of a bit of dry hazel branch, about an

 inch

inch long, tight into the hole, and the other end tight into a hole quite through the bottom of a ſmall wooden cup: then pour ſome quickſilver into the cup, and exhauſt the receiver of air, and the preſſure of the outward air, on the ſurface of the quickſilver, will force it through the pores of the hazel, from whence it will deſcend in a beautiful ſhower into a cup placed under the receiver to catch it.

9. Put a wire through the collar of leathers in the top of the receiver, and fix a bit of dry wood on the end of the wire within the receiver; then exhauſt the air, and puſh the wire down, ſo as to immerſe the wood into a jar of quickſilver on the pump-plate; this done, let in the air, and upon taking the wood out of the jar, and ſplitting it, its pores will be found full of quickſilver, which the force of the air, upon being let into the receiver, drove into the wood.

Fig. 10. 10. Join the two braſs hemiſpherical cups *A* and *B* together, with a wet leather between them, having a hole in the middle of it; then ſcrew the end *D* of the pipe *CD* into the plate of the pump at *i*, and turn the cock *E*, ſo as the pipe may be open all the way into the cavity of the hemiſpheres: then exhauſt the air out of them, and turn the cock a quarter round, which will ſhut the pipe *CD*, and keep out the air. This done, unſcrew the pipe at *D* from the pump; and ſcrew the piece *Fb* upon it at *D*; and let two ſtrong men try to pull the hemiſpheres aſunder by the rings *g* and *h*, which they will find hard to do: for if the diameter of the hemiſpheres

hemiſpheres be four inches, they will be preſſed together by the external air with a force equal to 188 pounds. And to ſhew that it is the preſſure of the air that keeps them together, hang them by either of the rings upon the hook *P* of the wire in the receiver *M* (Fig. 1.) and upon exhauſting the air out of the receiver, they will fall aſunder of themſelves.

11. Place a ſmall receiver *O* (Fig. 1.) near the hole *i* on the pump-plate, and cover both it and the hole with the receiver *M*; and turn the wire ſo by the top *P*, that its hook may take hold of the little receiver by a ring at its top, allowing that receiver to ſtand with its own weight on the plate. Then, upon working the pump, the air will come out of both receivers; but the large one *M*, will be forcibly held down to the pump by the preſſure of the external air; whilſt the ſmall one *O*, having no air to preſs upon it, will continue looſe, and may be drawn up and let down at pleaſure, by the wire *P P*. But, upon letting it quite down to the plate, and admitting the air into the receiver *M*, by the cock *k*, the air will preſs ſo upon the ſmall receiver *O*, as to fix it down to the plate; and at the ſame time, by counter-balancing the outward preſſure on the large receiver *M*, it will become looſe. This experiment evidently ſhews, that the receivers are held down by preſſure, and not by ſuction, for the internal receiver continued looſe whilſt the operator was pumping, and the external one was held down; but the former became faſt immediately by letting in the air upon it.

Fig. 11. 12. Screw the end *A* of the braſs pipe *A B F* into the hole of the pump-plate, and turn the cock *e* until the pipe be open; then put a wet leather upon the plate *cd*, which is fixed on the pipe, and cover it with the tall receiver *G H*, which is cloſe at top: then exhauſt the air out of the receiver, and turn the cock *e* to keep it out; which done, unſcrew the pipe from the pump, and ſet its end *A* into a baſon of water, and turn the cock *e* to open the pipe; on which, as there is no air in the receiver, the preſſure of the atmoſphere on the water in the baſon will drive the water forcibly through the pipe, and make it play up in a jet to the top of the receiver.

13. Set the ſquare phial *A* (Fig. 14.) upon the pump-plate, and having covered it with the wire cage *B*, put a cloſe receiver over it, and exhauſt the air out of the receiver; in doing of which, the air will alſo make its way out of the phial through a ſmall hole in its neck under the valve *b*. When the air is exhauſted, turn the cock below the plate, to re-admit the air into the receiver; and as it cannot get into the phial again, becauſe of the valve, the phial will be broke in a thouſand pieces by the preſſure of the air upon it. Had it been of a round form, it would have ſuſtained this preſſure like an arch, without breaking; but as its ſides are flat, it cannot.

To

To shew the elasticity or spring of the air.

14. Tie up a very small quantity of air in a bladder, and put it under a receiver; then exhaust the air out of the receiver, and the small quantity which is confined in the bladder (having nothing to act against it) will expand itself so by the force of its spring, as to fill the bladder as full as it could be blown of common air. But upon letting the air into the receiver again, it will overpower the air in the bladder, and press its sides almost close together.

15. If the bladder so tied up be put into a wooden box, and have 20 or 30 pounds weight of lead put upon it in the box, and the box be covered with a close receiver; upon exhausting the air out of the receiver, that air which is confined in the bladder will expand itself so, as to raise up all the lead by the force of its spring.

16. Take the glass ball mentioned in the fifth experiment, which was left full of water all but a small bubble of air at top, and having set it with its neck downward into the empty phial *a a*, and covered it with a close receiver, exhaust the air out of the receiver, and the small bubble of air in the top of the ball will expand itself, so as to force all the water out of the ball into the phial. Fig. 7.

17. Screw the pipe *AB* into the pump-plate, place the tall receiver *GH* upon the plate *cd*, as in the twelfth experiment, and exhaust the air out of the receiver; then, turn the cock *e* to keep out the Fig. 11.

air, unſcrew the pipe from the pump, and ſcrew it into the mouth of the copper veſſel *CC* (Fig. 15.) the veſſel having firſt been about half filled with water. Then turn the cock *e* (Fig. 11.) and the ſpring of the air which is confined in the copper veſſel will force the water up through the pipe *AB* in a jet into the exhauſted receiver, as ſtrongly as it did by its preſſure on the ſurface of the water in a baſon, in the twelfth experiment.

18. If a fowl, a cat, rat, mouſe, or bird, be put under a receiver, and the air be exhauſted, the animal is at firſt oppreſſed as with a great weight, then grows convulſed, and at laſt expires in all the agonies of a moſt bitter and cruel death. But as this experiment is too ſhocking to every ſpectator who has the leaſt degree of humanity, we ſubſtitute a machine called the *lungs-glaſs* in place of the animal; which, by a bladder within it, ſhews how the lungs of animals are contracted into a ſmall compaſs when the air is taken out of them.

19. If a butterfly be ſuſpended in a receiver, by a fine thread tied to one of its horns, it will fly about in the receiver, as long as the receiver continues full of air; but if the air be exhauſted, though the animal will not die, and will continue to flutter its wings, it cannot remove itſelf from the place where it hangs in the middle of the receiver, until the air be let in again, and then the animal will fly about as before.

Fig. 12. 20. Pour ſome quickſilver into the ſmall bottle *A*, and ſcrew the braſs collar *c* of the tube *BC* into the braſs neck *b* of the bottle, and the lower end

end of the tube will be immerfed into the quick-filver, fo that the air above the quickfilver in the bottle will be confined there, becaufe it cannot get out about the joinings, nor can it be drawn out through the quickfilver into the tube. This tube is alfo open at top, and is to be covered with the receiver *G* and large tube *EF*, which tube is fixed by brafs collars to the receiver, and is clofe at the top. This preparation being made, exhauft the air both out of the receiver and its tube; and the air will by the fame means be exhaufted out of the inner tube *BC*, through its open top at *C*; and as the receiver and tubes are exhaufting, the air that is confined in the glafs bottle *A* will prefs fo by its fpring upon the furface of the quickfilver, as to force it up in the inner tube as high as it was raifed in the ninth experiment by the preffure of the atmo-fphere: which demonftrates that the fpring of the air is equivalent to its weight.

21. Screw the end *C* of the pipe *CD* into the hole of the pump-plate, and turn all the three cocks *d*, *G*, and *H*, fo as to open the communications between all the three pipes *E*, *F*, *DC*, and the hollow trunk *AB*. Then, cover the plates *g* and *h* with wet leathers, which have holes in their middle where the pipes open into the plates; and place the clofe receiver *I* upon the plate *g*: this done, fhut the pipe *F* by turning the cock *H*, and exhauft the air out of the receiver *I*. Then, turn the cock *d* to fhut out the air, unfcrew the machine from the pump, and having fcrewed it to the wooden foot *L*, put the receiver *K* upon the plate Fig. 13.

h;

b; this receiver will continue looſe on the plate as long as it keeps full of air; which it will do until the cock *H* be turned to open the communication between the pipes *F* and *G*, through the trunk *AB*; and then the air in the receiver *K*, having nothing to act againſt its ſpring, will run from *K* into *I*, until it be ſo divided between theſe receivers, as to be of equal denſity in both; and then they will be held down with equal forces to their plates by the preſſure of the atmoſphere; though each receiver will then be kept down but with one half of preſſure upon it, that the receiver *I* had, when it was exhauſted of air; becauſe it has now one half of the common air in it which filled the receiver *K* when it was ſet upon the plate; and therefore, a force equal to half the force of the ſpring of common air, will act within the receivers againſt the whole preſſure of the common air upon their outſides. This is called transferring the air out of one veſſel into another.

Fig. 14. 22. Put a cork into the ſquare phial *A*, and fix it in with wax or cement; put the phial upon the pump-plate, with the wire cage *B* over it, and cover the cage with a cloſe receiver. Then, exhauſt the air out of the receiver, and the air that was corked up in the phial will, by the force of its ſpring, break the phial outwards, becauſe there is no air left on the outſide of the phial to act againſt the air within it.

22. Put a ſhrivelled apple under a cloſe receiver, and exhauſt the air; then the ſpring of the air within the apple will plump it out, ſo as to cauſe all

the wrinkles difappear; but upon letting the air into the receiver again, to prefs upon the apple, it will inftantly return to its former decayed and fhrivelled ftate.

23. Take a frefh egg, and cut off a little of the fhell and film from its fmalleft end, then put the egg under a receiver, and pump out the air; upon which, all the contents in the egg will be forced out into the receiver, by the expanfion of a fmall bubble of air contained in the great end, between the fhell and film.

24. Put fome warm beer in a glafs, and having fet it on the pump, cover it with a clofe receiver, and then exhauft the air. Whilft this is doing, and thereby the preffure more and more taken off from the beer in the glafs, the air therein will expand itfelf, and rife up in innumerable bubbles to the furface of the beer; and from thence it is taken away with the other air in the receiver. When the receiver is near exhaufted, the air in the beer, which could not difentangle itfelf quick enough to get off with the reft, will now expand itfelf fo, as to caufe the beer to have all the appearance of boiling; and the greater part of it will go over the glafs.

25. Put fome warm water in a glafs, and put a bit of dry wainfcot or other wood into the water. Then, cover the glafs with a receiver, and exhauft the air; upon which, the air in the wood having liberty to expand itfelf, will come out plentifully, and make all the water to bubble about the wood, efpecially about the ends, becaufe the pores lie lengthwife.

lengthwiſe. A cubic inch of dry wainſcot has ſo much air in it, that it will continue bubbling for near half an hour together.

Miſcellaneous experiments.

25. Screw the ſyringe *H* (Fig. 8.) to a piece of lead that weighs one pound at leaſt; and, holding the lead in one hand, pull up the piſton in the ſyringe with the other; then, quitting hold of the lead, the air will puſh it upward, and drive back the ſyringe upon the piſton. The reaſon of this is, that the drawing up of the piſton made a vacuum in the ſyringe, and the air, which preſſes every way equally, having nothing to reſiſt its preſſure upward, the lead and ſyringe were preſſed up, contrary to their natural tendency by gravity. If the ſyringe ſo loaded, be hung in a receiver, and the air be exhauſted, the ſyringe and lead will deſcend upon the piſton-rod by their natural gravity; and, upon admitting the air into the receiver, they will be drove upward again, until the piſton be at the very bottom of the ſyringe.

26. Let a large piece of cork be ſuſpended by a thread at one end of a balance, and counterpoiſed by a leaden weight, ſuſpended in the ſame manner, at the other. Let this balance be hung to the inſide of the top of a large receiver; which being ſet on the pump, and the air exhauſted, the cork will preponderate, and ſhew itſelf to be heavier than the lead; but upon letting in the air again, the equilibrium will be reſtored. The reaſon

of

of this is, that ſince the air is a fluid, and all bodies loſe as much of their abſolute weight in it, as is equal to the weight of their bulk of the fluid, the cork being the larger body, loſes more of its real weight than the lead does; and therefore muſt in fact be heavier, to balance it under the diſadvantage of loſing ſome of its weight: which diſadvantage being taken off by removing the air, the bodies then gravitate according to their real quantities of matter, and the cork, which balanced the lead in air, ſhews itſelf to be heavier when in *vacuo*.

27. Set a lighted candle upon the pump, and cover it with a tall receiver. If the receiver holds a gallon, the candle will burn a minute, and then, after having gradually decayed from the firſt inſtant, it will go out: which ſhews, that a conſtant ſupply of freſh air is neceſſary to feed flame; and ſo it alſo is for animal life. For a bird kept under a cloſe receiver will ſoon die, although no air be pumped out; and it is found that a gallon of air is ſufficient only for one minute for a man to breathe in.

The moment that the candle goes out, the ſmoke will be ſeen to aſcend to the top of the receiver, and there it will form a ſort of cloud: but upon exhauſting the air, the ſmoke will fall down to the bottom of the receiver, and leave it as clear at top as it was before it was ſet upon the pump. This ſhews, that ſmoke does not aſcend on account of its being poſitively light, but becauſe it is lighter than air; and its falling to the bottom when the

air was taken away, ſhews, that it is not deſtitute of weight. So moſt ſorts of wood aſcend or ſwim in water; and yet there are none who doubt of the wood's having gravity or weight.

28. Set a receiver, which is open at top, upon the air-pump, and cover it with a braſs-plate, and wet leather; and having exhauſted it of air, let the air in again at top through an iron pipe, making it paſs through a charcoal flame at the end of the pipe; and when the receiver is full of that air, lift up the cover and let down a mouſe or bird into the receiver, and the burnt air will immediately kill it. If a candle be let down into the air, it will go out directly, but will purify the air ſo far as it was let down; and ſo, by lighting it and letting it down over and over again, all the air in the receiver will be purified.

29. Set a bell upon a cuſhion on the pump-plate, and cover it with a receiver; then ſhake the pump to make the clapper ſtrike againſt the bell, and the ſound will be very well heard: but, exhauſt the receiver of air, and then, if the clapper be made to ſtrike ever ſo hard againſt the bell, it will make no ſound at all; which ſhews, that air is abſolutely neceſſary for the propagation of ſound.

30. Let a candle be placed on one ſide of a receiver, and viewed through the receiver at ſome diſtance; then, as ſoon as the air begins to be exhauſted, the receiver will be filled with vapours which riſe from the wet leather, by the ſpring of the air in it; and the light of the candle being refracted

refracted through that medium of vapours, will have the appearance of circles of various colours, ſuch as are ſeen about the moon in a hazy air at night.

The air-pump was invented by *Otho Guerick* of *Magdeburg*, but was much improved by Mr. *Boyle*, to whom we are indebted for our greateſt part of the knowledge of the wonderful properties of the air, demonſtrated in the above experiments.

The elaſtic air which is contained in many bodies, and is kept in them by the weight of the atmoſphere, may be got out of them either by boiling, or by the air-pump, as ſhewn in the 25th experiment: but the fixed air, which is by much the greater quantity, cannot be got out but by diſtillation, fermentation, or putrefaction.

If fixed air did not come out of bodies with difficulty, and ſpend ſome time in extricating itſelf from them, it would tear them to-pieces. Trees would be rent by the change of air from a fixt to an elaſtic ſtate, and animals would be burſt in pieces by the exploſion of air in their food.

Dr. *Hales* found by experiment, that the air in apples is ſo much condenſed, that if it were let out into the common air, it would fill a ſpace 48 times as great as the bulk of the apples themſelves: ſo that its preſſure outwards was equal to 11776 lb. and, in a cubic inch of oak to 19860 lb. againſt its ſides. So that if the air was let looſe at once in theſe ſubſtances, they would tear every thing to-

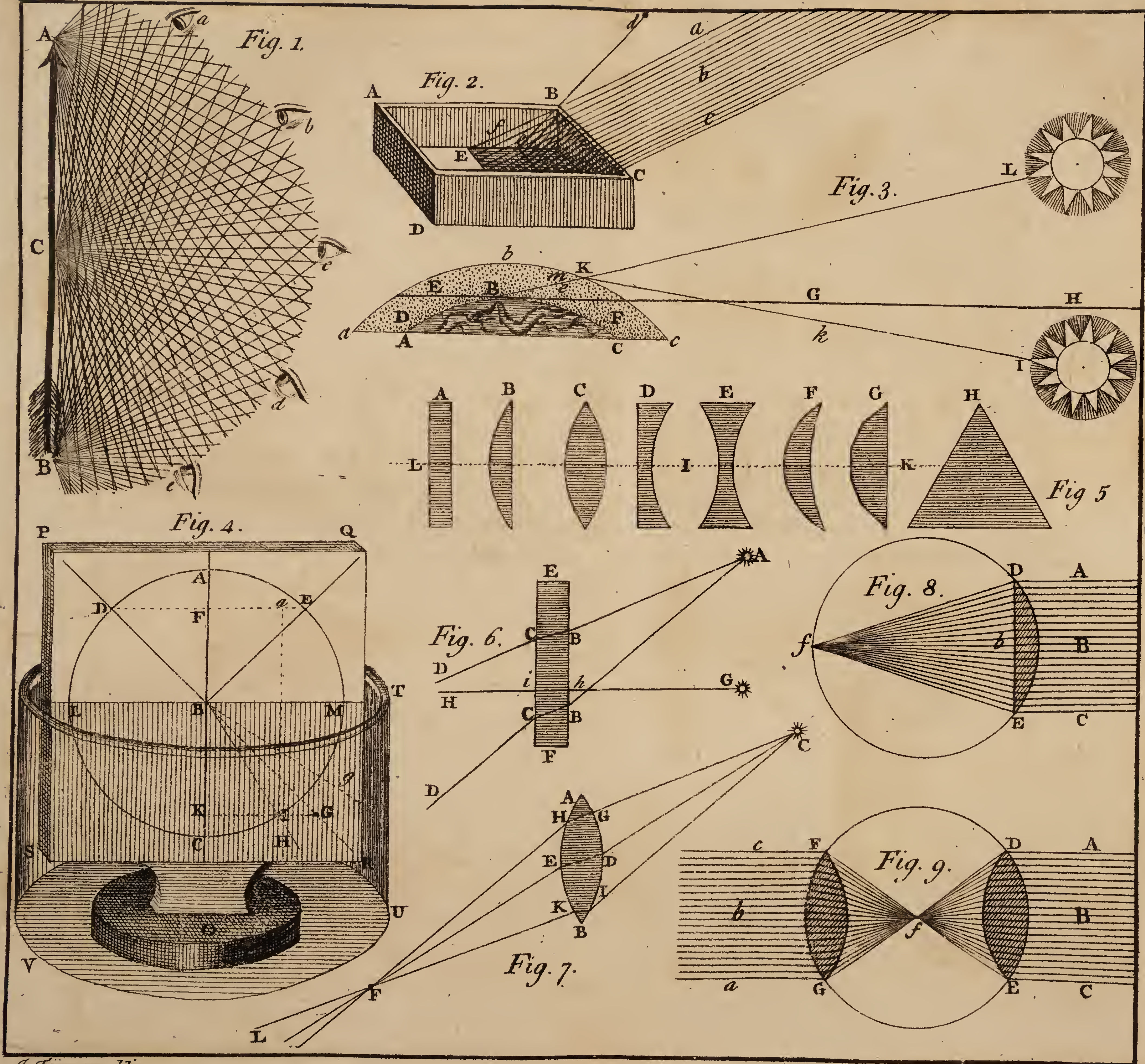
pieces

pieces about them with a force much superior to gunpowder. Hence, in eating apples, it is well that they part with the air by degrees, as they ferment in the ſtomach, otherwiſe, an apple would be immediate death to him who eats it.

The mixing of ſome ſubſtances with others will releaſe the air from them, all of a ſudden, which may be attended with very great danger. Of this we have a remarkable inſtance in an experiment made by Dr. *Slare*; who having put half a dram of oil of carraway-ſeed into one glaſs, and a dram of compound ſpirit of nitre in another, covered them both on the air-pump with a receiver ſix inches wide, and eight inches deep, and then exhauſted the air, and continued pumping until all that could poſſibly be got both out of the receiver, and out of the two fluids, was extricated: then, by a particular contrivance from the top of the receiver, he mixed the fluids together; upon which, they produced ſuch a prodigious quantity of air, as inſtantly blew up the receiver; although it was preſſed down by the atmoſphere with upwards of 400 pound weight.

LECT.

PLATE XV.



J. Ferguson delin.   J. Mynde sc.

L E C T. VIII.

Of optics.

LIGHT consists of an inconceivably great number of particles flowing from a luminous body in all manner of directions; and these particles are so small as to surpass all human comprehension.

That the number of particles of light is inconceivably great, appears from hence, that if a candle be lighted, and there be no obstacle in the way to obstruct the passage of its rays, it will fill all the space within two miles of it every way with luminous particles, before it has lost the least sensible part of its substance.

A ray of light is a continued stream of these particles, flowing from any visible body in a straight line: and that the particles themselves are incomprehensibly small, appears from the following experiment. Make a small pin-hole in a piece of black paper, and hold the paper upright on a table facing a row of candles standing by one another; then place a sheet of pasteboard at a little distance behind the paper; and some of the rays which flow from all the candles through the hole in the paper, will form as many specks of light on the pasteboard as there are candles on the table before the plate: each speck being as distinct and clear, as if there was only one speck from one single candle: which shews, that the particles of light are exceed-

The amazing smallness of the particles of light.

exceedingly ſmall, otherwiſe they could not paſs through the hole from ſo many different candles without confuſion.—Dr. *Niewenlyt* has computed, that there flows more than 6,000,000,000,000 times as many particles of light from a candle in one ſecond of time, as there are grains of ſand in the whole earth; ſuppoſing each cubic inch of it to contain 1,000,000.

Theſe particles, by falling directly upon our eyes, excite in our minds the idea of light. And when they fall upon bodies, and are thereby reflected to our eyes, they excite in us the ideas of theſe bodies. And as every point of a viſible body reflects the rays of light in all manner of directions, every point will be viſible in every part to which the light is reflected from it. Thus the

Plate XV. Fig. 1. Reflected light.

object *ABC* is viſible to an eye in any part where the rays *Aa*, *Ab*, *Ac*, *Ad*, *Ae*, *Ba*, *Bb*, *Bc*, *Bd*, *Be*, and *Ca*, *Cb*, *Cc*, *Cd*, *Ce*, come. Here we have ſhewn the rays as if they were only reflected from the ends *A* and *B*, and the middle point *C* of the object; every other point being ſuppoſed to reflect rays in the ſame manner. So that, wherever a ſpectator is placed with regard to the body, every point of that part of the ſurface which is towards him will be viſible, when no intervening object ſtops the paſſage of the light.

Since no object can be ſeen through the bore of a bended pipe, it is evident that the rays of light move in ſtraight lines, whilſt there is nothing to refract or turn them out of their rectilineal courſe.

Whilſt

Whilſt the rays of light continue in any * medium of an uniform denſity, they are ſtraight; but when they paſs obliquely out of one medium into another, which is either more denſe or more rare, they are refracted towards the denſer medium: and this refraction is more or leſs, as the rays fall more or leſs obliquely on the refracting ſurface of the medium.

To prove this by experiment, ſet the empty veſſel *ABCD* into any place where the ſun ſhines obliquely, and obſerve the part where the ſhadow of the edge *BC* falls on the bottom of the veſſel at *E*; then fill the veſſel with water, and the ſhadow will reach no farther than *e*; which ſhews, that the ray *a B E* which came ſtraight in the open air, juſt over the edge of the veſſel at *B* to its bottom at *E*, is refracted by falling obliquely on the ſurface of the water at *B*; and inſtead of going on in the rectilineal direction *ABE*, it is bent downward in the water from *B* to *e*; the whole bend being at the ſurface of the water: and ſo of all the other rays *a b c*. Fig. 2. Refracted light.

If a ſtick be laid over the veſſel, and the ſun's rays be reflected from a glaſs perpendicularly into the veſſel, the ſhadow of the ſtick will fall upon the ſame part of the bottom, whether the veſſel be empty or full; which ſhews, that the rays of light are not refracted when they fall perpendicularly on the ſurface of any medium.

* Any thing through which the rays of light can paſs is called a medium; as air, water, glaſs, diamond, or even a vacuum.

 The

The rays of light are as much refracted by paſſing out of water into air, as by paſſing out of air into water. Thus, if a ray of light flows from the point *e*, under water, in the direction *e B*; when it comes to the ſurface of the water at *B*, it will not go on thence in the rectilineal courſe *B d*, but will be refracted into the line *B a*. Therefore,

To an eye at *e* looking through a plane glaſs in the bottom of the empty veſſel, the point *a* cannot be ſeen, becauſe the ſide *B c* of the veſſel interpoſes; and the point *d* will juſt be ſeen over the edge of the veſſel at *B*. But if the veſſel be filled with water, the point *a* will be ſeen from *e*; and will appear as at *d*, elevated in the direction of the ray *e b* *.

The days are made longer by the refraction of the ſun's rays.

The time of ſun-riſing or ſetting, ſuppoſing its rays ſuffered no refraction, is eaſily found by calculation. But obſervation proves that the ſun riſes ſooner, and ſets later every day than the calculated time; the reaſon of which is plain from what was ſaid immediately above. For, though the ſun's rays do not come part of their way to us through water, yet they do through the air or atmoſphere, which being a groſſer medium than the free ſpace between the ſun and the top of the at-

* Hence a piece of money lying at *e*, in the bottom of an empty veſſel, cannot be ſeen by an eye at *a*, becauſe the edge of the veſſel intervenes; but let the veſſel be filled with water, and the ray *e b* being then refracted at *B*, will ſtrike the eye at *a*, and ſo render the money viſible, which will appear as if it were raiſed up to *f* in the line *a B f*.

mosphere, the rays, by entering obliquely into the atmosphere are there refracted, and thence bent down to the earth. And although there are many places of the earth to which the sun is vertical at noon, and consequently his rays can suffer no refraction at that time, because they come perpendicularly through the atmosphere; yet there is no place to which the sun's rays do not fall obliquely on the top of the atmosphere, at his rising and setting; and consequently, no clear day in which the sun will not be visible before he rises in the horizon, and after he sets in it: and the longer or shorter, as the atmosphere is more or less replete with vapours. For, let *ABC* be part of the earth's surface, *DEF* the atmosphere that covers it, and *EBGH* the horizon of an observer at *B*. As every point of the sun's surface sends out rays of light in all manner of directions, some of his rays will constantly fall upon, and enlighten, one half of our atmosphere; and therefore, when the sun is at *I*, below the horizon *H*, those rays which go on in the free space *IkK* preserve a rectilineal course until they fall upon the top of the atmosphere; and those which fall so about *K*, are refracted at their entrance into the atmosphere, and bent down in the line *KeB*, to the observer's place at *B*: and therefore, the sun will appear at *L*, in the direction of the ray *BeK*, above the horizon *BGH*, when he is really below it at *I*. Fig. 3.

The angle contained between a ray of light, and a perpendicular to the refracting surface, is called *the angle of incidence*; and the angle con- *Angle of incidence.*

 tained

tained between the ſame perpendicular, and the ſame ray after refraction, is called *the angle of refraction.* Thus, let *LBM* be the refracting ſurface of a medium (ſuppoſe water) and *ABC* a perpendicular to that ſurface; let *DB* be a ray of light, going out of air into water at *B*, and therein refracted in the line *BH*; the angle *ABD*, is the angle of incidence, of which *DF* is the ſine; and the angle *KBH*, is the angle of refraction, whoſe ſine is *KI*.

Fig. 4. *Angle of refraction.*

When the refracting medium is water, the ſine of the angle of incidence is to the ſine of the angle of refraction, as 4 to 3; which is confirmed by the following experiment, taken from Doctor SMITH's optics.

Deſcribe the circle *DAEC* on a plane ſquare board, and croſs it at right angles with the ſtraight lines *ABC*, and *LBM*; then, from the interſection *A*, with any opening of the compaſſes, ſet off the equal arcs *AD* and *AE*, and draw the right line *DFE*: then, taking *Fa*, which is three quarters of the length *FE*, from the point *a*, draw *aI* parallel to *ABK*, and join *KI*, parallel to *BM*: ſo *KI* will be equal to three quarters of *FE* or of *DF*. This done, fix the board upright upon the leaden pedeſtal *O*, and ſtick three pins perpendicularly into the board, at the points *D*, *B*, and *I*: then ſet the board upright into the veſſel *TUV*, and fill up the veſſel with water to the line *LBM*. When the water has ſettled, look along the line *DB*, ſo as you may ſee the head of the pin *B* over the head of the pin *D*; and the pin *I* will

will appear in the ſame right line produced to *G*, for its head will be ſeen juſt over the head of the pin at *B*: which ſhews, that the ray *IB*, coming from the pin at *I*, is ſo refracted at *B*, as to proceed from thence in the line *BD* to the eye of the obſerver; the ſame as it would do from any point *G* in the right line *DBG*, if there were no water in the veſſel: and alſo ſhews, that *KI*, the ſine of refraction in water, is to *DF*, the ſine of incidence in air, as 3 to 4*.

Hence, if *DBH* were a crooked ſtick put obliquely into the water, it would appear a ſtraight one, as *DBG*. Therefore, as the line *BH* appears at *BG*, ſo the line *BG* will appear at *Bg*; and conſequently, a ſtraight ſtick *DBG* put obliquely into water, will ſeem bent at the ſurface of the water in *B*, and crooked as *DBg*.

When a ray of light paſſes out of air into glaſs, the ſine of incidence is to the ſine of refraction, as 3 to 2; and when out of air into a diamond, as 5 to 2.

Glaſs may be ground into eight different ſhapes at leaſt, for optical purpoſes, *viz.*

1. A *plane-glaſs*, which is flat on both ſides, and of equal thickneſs in all its parts, as *A*. Fig. 5.

2. A *plano-convex*, which is flat on one ſide and convex on the other, as *B*.

3. A *double-convex*, which is convex on both ſides, as *C*.

* This is ſtrictly true of the red rays only, for the other coloured rays are differently refracted; but the difference is ſo ſmall, that it need not be conſidered in this place.

Lenses. 4. A *plano-concave*, which is flat on one side and concave on the other, as *D*.

5. A *double-concave*, which is concave on both sides, as *E*.

6. A *meniscus*, which is concave on one side and convex on the other, as *F*.

7. A *flat-plano-convex*, whose convex side is ground into several little flat surfaces, as *G*.

8. A *prism*, which has three flat sides, and when viewed endwise, appears like an equilateral triangle, as *H*.

Glasses ground into any of the above shapes *B*, *C*, *D*, *E*, *F*, are generally called *lenses*.

A right line *LIK*, going perpendicularly through the middle of a lens, is called *the axis of the lens*.

Fig. 6. A ray of light *Gh*, falling perpendicularly on a plain glass *EF*, will pass through the glass in the same direction *hi*, and go out of it into the air in the same right course *iH*.

A ray of light *AB*, falling obliquely on a plain glass, will go out of the glass in the same direction, but not in the same right line; for in touching the glass, it will be refracted in the line *BC*, and in leaving the glass, it will be refracted in the line *CD*.

Fig. 7. A ray of light *CD*, falling obliquely on the middle of a convex glass, will go forward in the same direction *DE*, as if it had fallen with the same degree of obliquity on a plane glass; and will go out of the glass in the same direction with which it entered: for it will be equally refracted at the points *D* and *E*, as if it had passed through a plane surface. But the rays *CG* and *CI*, will be

be ſo refracted, as to meet again at the point *F*. Therefore, all the rays which flow from the point *C*, ſo as to touch the glaſs, will meet again at *F*; and if they go farther onward, as to *L*, they croſs at *F*, and go forward on the oppoſite ſides of the middle ray *CDEF*, to what they were in approaching it in the directions *HF* and *KF*.

When parallel rays, as *ABC*, fall directly upon a plano-convex glaſs *DE*, and paſs through it, they will be ſo refracted, as to unite in a point *f* behind it; and this point is called the *principal focus*; the diſtance of which, from the middle of the glaſs, is called the *focal diſtance*; which is equal to twice the radius of the ſphere of the glaſs's convexity. And, Fig. 8. The properties of different lenſes.

When parallel rays, as *ABC*, fall directly upon a glaſs *DE*, which is equally convex on both ſides, and paſs through it; they will be ſo refracted, as to meet in a point or principal focus *f*, whoſe diſtance is equal to the radius or ſemidiameter of the ſphere of the glaſs's convexity. But, if a glaſs be more convex on one ſide than on the other, the rule for finding the focal diſtance is this; as the ſum of the ſemidiameters of both convexities is to the ſemidiameter of either, ſo is double the ſemidiameter of the other to the diſtance of the focus. Or, divide the double product of the radii by their ſum, and the quotient will be the diſtance ſought. Fig. 9.

Since all thoſe rays of the ſun which paſs through a convex glaſs are collected together in its focus, the force of all their heat is collected into that

part;

part; and is in proportion to the common heat of the ſun, as the area of the glaſs is to the area of the focus. Hence we ſee the reaſon why a convex glaſs cauſes the ſun's rays to burn, after paſſing through it.

All theſe rays croſs the middle ray in the focus *f*, and then diverge from it in the ſame manner *FfG*, as they converged in the ſpace *DfE* in coming to it.

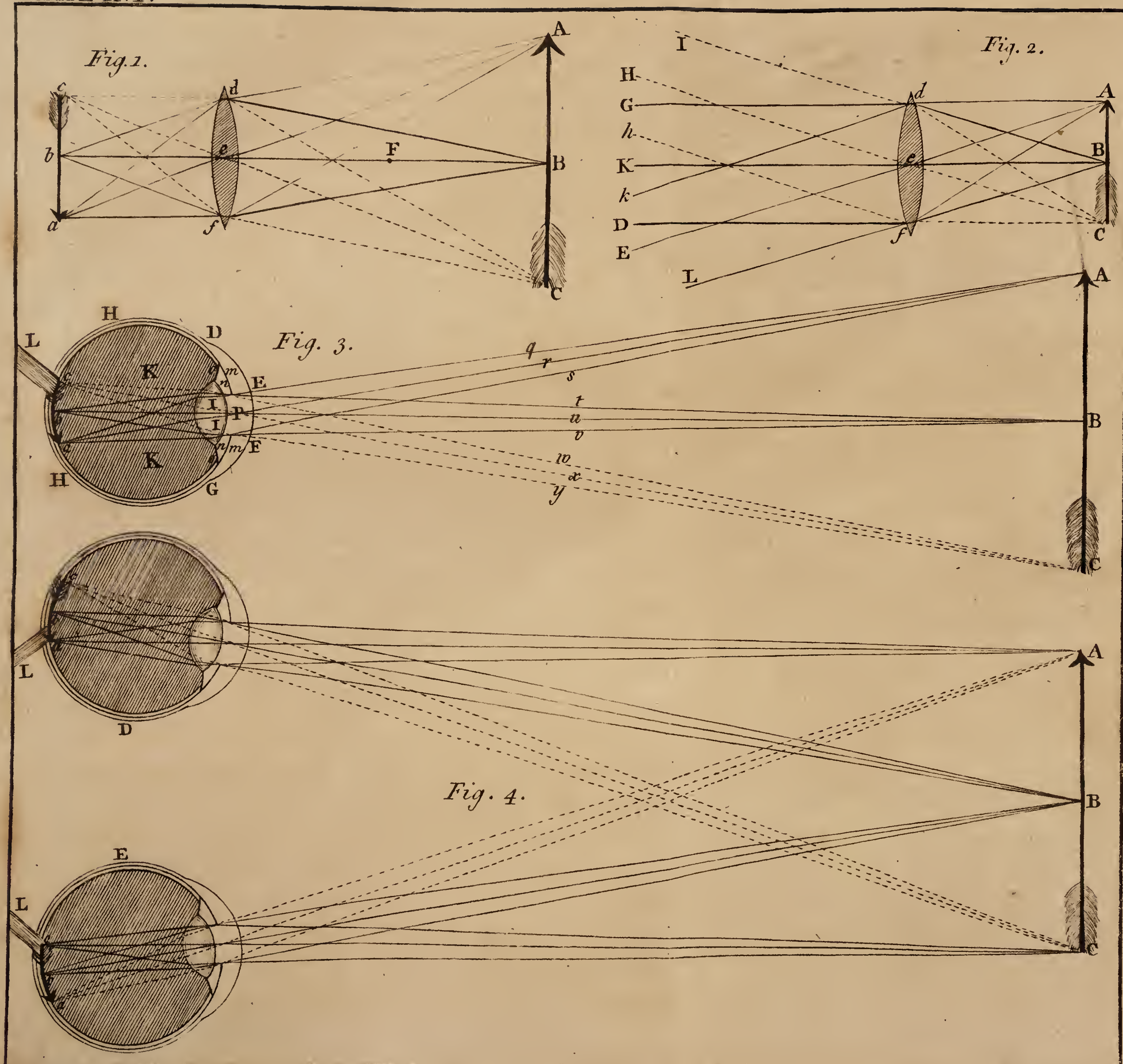
If another glaſs *FG*, of the ſame convexity as *DE*, be placed in the rays at the ſame diſtance from the focus, it will refract them ſo, as that after going out of it, they will be all parallel, as *a*, *b*, *c*; and go on in the ſame manner as they came to the firſt glaſs *DE*, through the ſpace *ABC*; but on the contrary ſides of the middle ray *Bfb*: for the ray *ADf* will go on from *f* in the direction *fGa*, and the ray *CEf* in the direction *fFc*; and ſo of the reſt.

The rays diverge from any radiant point, as from a principal focus: therefore, if a candle be placed at *f*, in the focus of the convex glaſs *FG*, the diverging rays in the ſpace *FfG*, will be ſo refracted by the glaſs, as, that after going out of it, they will all become parallel, as ſhewn in the ſpace *cba*.

If the candle be placed nearer the glaſs than its focal diſtance, the rays will diverge after paſſing through the glaſs, more or leſs, as the candle is more or leſs diſtant from the focus.

If the candle be placed farther from the glaſs than its focal diſtance, the rays will converge after paſſing

PLATE XVI.



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paſſing through the glaſs, and meet in a point, which will be more or leſs diſtant from the glaſs, as the candle is nearer to, or farther from, its focus: and where the rays meet, they will form an inverted image of the flame of the candle; which may be ſeen on a paper placed in the meeting of the rays.

Hence, if any object *ABc*, be placed beyond the focus *F*, of the convex glaſs *def*, ſome of the rays which flow from every point of the object, on the ſide next the glaſs, will fall upon it, and after paſſing through it, they will be converged into as many points on the oppoſite ſide of the glaſs, where the image of every point will be formed; and conſequently, the image of the whole object, which will be inverted. Thus, the rays *Ad*, *Ae*, *Af*, flowing from the point *A*, will converge in the ſpace *daf*, and by meeting at *a*, will there form the image of the point *A*. The rays *Bd*, *Be*, *Bf*, flowing from the point *B*, will be united at *b* by the refraction of the glaſs, and will there form the image of the point *B*. And the rays *Cd*, *Ce*, *Cf*, flowing from the point *C*, will be united at *c*, where they will form the image of the point *C*. And ſo of all the other intermediate points between *A* and *C*. The rays which flow from every particular point of the object, and are united again by the glaſs, are called *a pencil of rays*. Plate XVI. Fig. 1.

If the object *ABC* be brought nearer to the glaſs, the picture *abc* will be removed to a greater diſtance. For then, more rays flowing from every

ſingle

ſingle point, will fall more diverging upon the glaſs; and therefore cannot be ſo ſoon collected into the correſponding points behind it. Conſequently, if

Fig. 2. the diſtance of the object ABC, be equal to the diſtance eB of the focus of the glaſs, the rays of each pencil will be ſo refracted by paſſing through the glaſs, that they will go out of it parallel to each other; as I, H, b, from the point C; G, K, D, from the point B; and k, E, L, from the point A: and therefore, there will be no picture formed behind the glaſs.

If the focal diſtance of the glaſs, and the diſtance of the object from the glaſs, be known; the diſtance of the picture from the glaſs may be found by this rule, *viz.* multiply the diſtance of the focus by the diſtance of the object, and divide the product by their difference; the quotient will be the diſtance of the picture.

Fig. 1. The picture will be as much bigger or leſs than the object, as its diſtance from the glaſs is greater or leſs than the diſtance of the object. For, as Be is to eb, ſo is AC to ac. So that, if ABC be the object, cab will be the picture; or, if cab be the object, ABC will be the picture.

The manner of viſion. Having deſcribed how the rays of light, flowing from objects and paſſing through convex glaſſes, are collected into points, and form the images of the objects; it will be eaſy to underſtand how the rays are effected by paſſing through the humours of the eye, and are thereby collected into innumerable points on the bottom of the eye, and thereon form the images of the objects which they

flow

flow from. For, the different humours of the eye, and particularly the chryſtalline humour, are to be conſidered as a convex glaſs; and the rays in paſſing through them, to be effected in the ſame manner as in paſſing through a convex glaſs.

The eye is nearly globular. It conſiſts of three coats and three humours. The part *DHHG* of the outer coat, is called the *ſclerotica*, the reſt *GFED* the *cornea*. Next within this coat is that called the *choroides*, which ſerves as it were for a lining to the other, and joins with the *iris mn*, *mn*. The *iris* is compoſed of two ſets of muſcular fibres; the one of a circular form, which contract the hole in the middle, called the *pupil*, when the light would otherwiſe be too ſtrong for the eye; and the other of radial fibres, tending every where from the circumference of the iris towards the middle of the pupil; which fibres, by their contraction, dilate and enlarge the pupil when the light is weak, in order to take in the more of its rays. The third coat is only a fine expanſion of the optic nerve *L*, which ſpreads like net-work all over the inſide of the choroides, and is therefore called the *retina*; upon which are painted (as it were) the images of all viſible objects, by the rays of light which either flow or are reflected from them.

Fig. 3.

The eye deſcribed.

Under the cornea is a fine tranſparent fluid like water, which is therefore called the *aqueous humour*. It gives a protuberant figure to the cornea, fills the two cavities *mm* and *nn*, which communicate by the pupil *P*, and has the ſame limpidity, ſpecific gravity, and refractive power, as water.—At the

the back of this lies the *chryſtalline humour II*, which is ſhaped like a double convex glaſs; and is a little more convex on the back than the fore part. It converges the rays, which paſs through it, from every viſible object, to its focus at the bottom of the eye. This humour is tranſparent like chryſtal, is of the conſiſtence of hard jelly, and exceeds the ſpecific gravity of water in the proportion of 11 to 10. It is incloſed in a fine tranſparent membrane, from which proceed radial fibres *oo*, called the *ligamentum ciliare*, all around its edge; and join to the circumference of the iris. Theſe fibres have a power of contracting and dilating occaſionally, by which means they alter the ſhape or convexity of the chryſtalline humour, and alſo ſhift it a little backward or forward in the eye, ſo as to adapt its focal diſtance at the bottom of the eye to the different diſtances of objects; without which proviſion, we could ſee only theſe objects diſtinctly, that were all at one diſtance from the eye.

At the back of the chryſtalline, lies the *vitreous humour KK*, which is tranſparent like glaſs, and is largeſt of all in quantity, filling the whole orb of the eye, and giving it a globular ſhape. It is much of a conſiſtence with the white of an egg, and very little exceeds the ſpecific gravity and refractive power of water.

As every point of an object *ABC* ſends out rays in all directions, ſome rays from every point on the ſide next the eye will fall upon the cornea, between *E* and *F*; and by paſſing on, through the humours and

and pupil of the eye, they will be converged to as many points on the retina or bottom of the eye, and will thereon form a diſtinct inverted picture *a b c* of the object. Thus, the pencil of rays *q r s*, that flows from the point *A* of the object, will be converged to the point *a* on the retina; thoſe from the point *B* will be converged to the point *b*; thoſe from the point *C* will be converged to the point *c*, and ſo of all the intermediate points; by which means the whole image *a b c* is formed, and the object made viſible; although it muſt be owned, that the method by which this ſenſation is carried from the eye by the optic nerve to the common ſenſory in the brain, and there diſcerned, is above the reach of our comprehenſion.

But, that viſion is effected in this manner, may be demonſtrated experimentally. Take a bullock's eye whilſt it is freſh, and having cut off the three coats from the back part, quite to the vitreous humour, put a piece of white paper over that part, and hold the eye towards any bright object, and you will ſee an inverted picture of it upon the paper.

Since the image is inverted, many have wondered why the object appears upright. But we are to conſider, 1. that *inverted* is only a relative term; and 2. that there is a very great difference between the real object and the means or image by which we perceive it. When all the parts of a diſtant proſpect are painted upon the retina, they are all right with reſpect to one another, as well as the parts of the proſpect itſelf; and we can only judge of an object's being inverted, when it is turned reverſe to its natural poſition with reſpect to other objects which we ſee

and

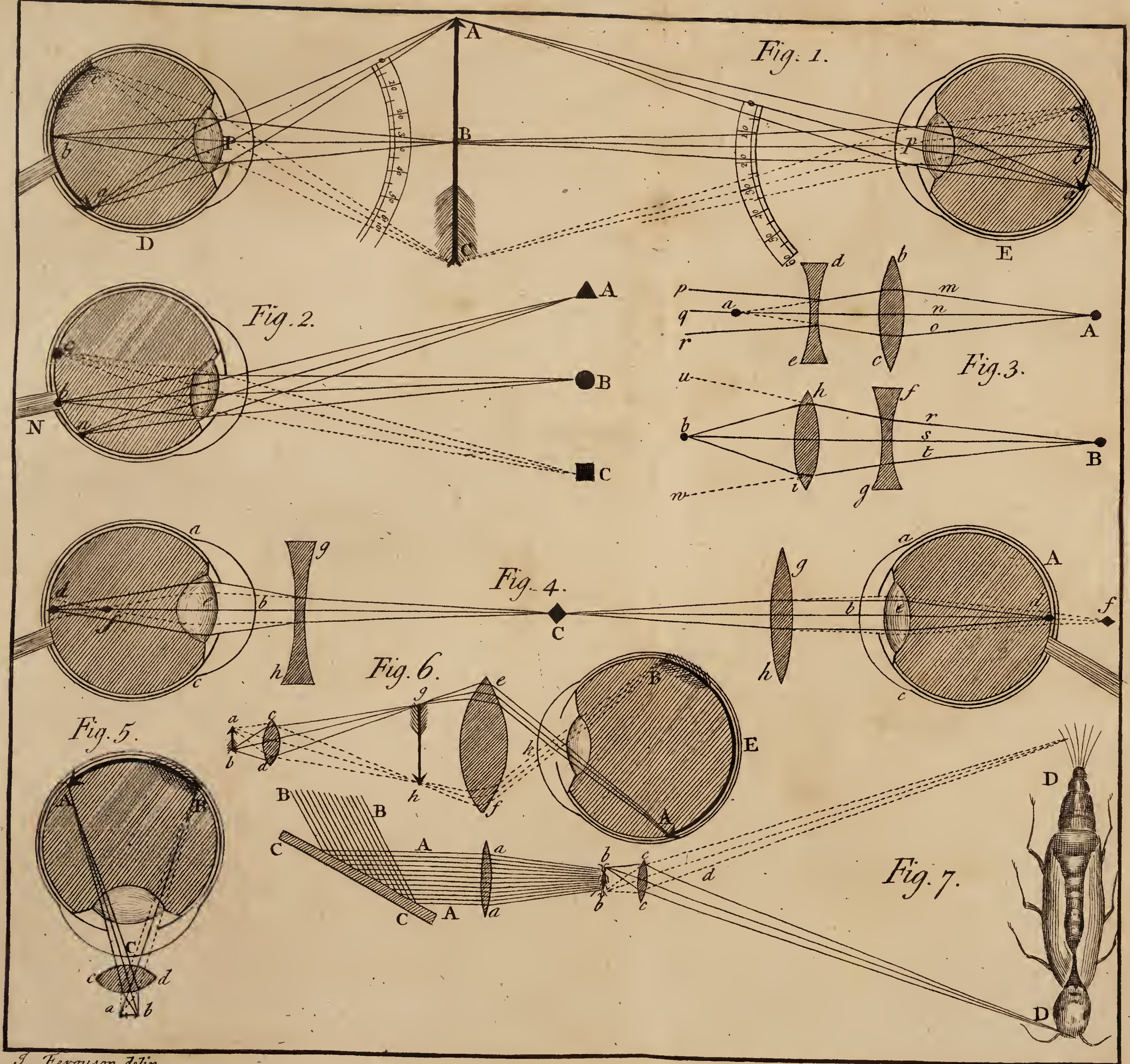
and compare it with.—If we lay hold of an upright ſtick in the dark, we can tell which is the upper or lower part of it, by moving our hand upward or downward; and know very well that we cannot feel the upper end by moving our hand downward. Juſt ſo we find by experience, that upon directing our eyes towards a tall object, we cannot ſee its top by turning our eyes downward, nor its foot by turning our eyes upward; but muſt trace the object the ſame way by the eye to ſee it from head to foot, as we do by the hand to feel it: and as the judgment is informed by the motion of the hand in one caſe, ſo it is alſo by the motion of the eye in the other.

Fig. 4. In Fig. 4. is exhibited the manner of ſeeing the ſame object *A B C*, by both the eyes *D* and *E* at once.

When any part of the image *a b c* falls upon the optic nerve *L*, the correſponding part of the object becomes inviſible. On which account, nature has wiſely placed the optic nerve of each eye not in the middle of the bottom of the eye, but towards the ſide next the noſe: ſo that, whatever part of the image falls upon the optic nerve of one eye, may not fall upon the optic nerve of the other. Thus, the point *a* of the image *a b c*, falls upon the optic nerve of the eye *D*, but not of the eye *E*; and the point *c* falls upon the optic nerve of the eye *E*, but not of the eye *D*: and therefore, to both eyes taken together, the whole object *A B C* is viſible.

The nearer that any object is to the eye, the larger is the angle under which it is ſeen, and the magnitude

PLATE XVII.



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magnitude under which it appears. Thus to the eye *D*, the object *ABC* is seen under the angle *APC*; and its image *abc* is very large upon the retina: But to the eye *E* at a double distance, the same object is seen under the angle *ApC*, which is equal only to half the angle *APC*, as is evident by the figure. The image *abc* is likewise twice as large in the eye *D*, as the image *abc* is in the eye *E*. In both these representations, a part of the image falls on the optic nerve, and the object in the corresponding part is invisible. Plate XVII. Fig. I.

As the sense of seeing is allowed to be occasioned by the impulse of the rays from the visible object upon the retina of the eye, and forming the image of the object thereon, and that the retina is only the expansion of the optic nerve all over the choroides; it should seem surprising, that the part of the image which falls on the nerve should render the like part of the object invisible; especially, as the nerve is allowed to be the instrument by which the impulse and image are conveyed to the common sensory in the brain. But this difficulty vanishes, when we consider that there is an artery within the trunk of the optic nerve, which entirely obscures the image in that part, and conveys no sensation to the brain.

That the part of the image which falls upon the middle of the optic nerve is lost, and consequently the corresponding part of the object rendered invisible, is plain by experiment. For, if a person fixes three patches *A*, *B*, *C*, upon a white wall, at the height of the eye, and the distance of about a foot Fig. 2.

 from

from each other, and places himſelf before them, ſhutting the right eye, and directing the left towards the patch *C*, he will ſee the patches *A* and *C*, but the middle patch *B* will diſappear. Or, if he ſhuts his left eye, and directs the right towards *A*, he will ſee both *A* and *C*, but *B* will diſappear; and if he directs his eye towards *B*, he will ſee both *B* and *A*, but not *C*. For whatever patch is directly oppoſite to the optic nerve *N* vaniſhes. This requires a little practice, after which he will find it eaſy to direct his eye ſo, as to loſe the ſight of whatever patch he pleaſes.

We are not commonly ſenſible of this diſappearance, becauſe the motions of the eye are ſo quick and inſtantaneous, that we no ſooner loſe the ſight of any part of an object, than we recover it again; much the ſame as in the twinkling of our eyes, for at each twinkling we are blinded; but it is ſo ſoon over, that we are ſcarce ever ſenſible of it.

Fig. 4.

Why ſome eyes require ſpectacles.

Some eyes require the aſſiſtance of convex glaſſes to make them ſee objects diſtinctly, and others of concave. If either the cornea *a b c* or chryſtalline humour *e*, or both of them, be too flat, as in the eye *A*, their focus will not be on the retina, as at *d*, where it ought to be, in order to render viſion diſtinct; but beyond the eye, as at *f*. And therefore, thoſe rays which flow from the object *C*, and paſs through the humours of the eye, are not converged enough to unite at *d*; and therefore the obſerver can have but a very indiſtinct view of the object. This is remedied by placing a convex glaſs

glaſs *g h* before the eye, which makes the rays converge ſooner, and imprints the image duly on the retina.

If either the cornea, or chryſtalline humour, or both of them, be too convex, as in the eye *B*, the rays that enter it from the object *C*, will be converged to a focus in the vitreous humour, as at *f*; and by diverging from thence to the retina, will form a very confuſed image thereon: and, ſo of courſe, the obſerver will have as confuſed a view of the object as if his eye had been too flat. This inconvenience is remedied by placing a concave glaſs *g h* before the eye; which glaſs, by cauſing the rays to diverge between it and the eye, lengthens the focal diſtance ſo; that if the glaſs be properly choſen, the rays will unite at the retina, and form a diſtinct picture of the object upon it.

Such eyes as have their humours of a due convexity, cannot ſee any object diſtinctly at a leſs diſtance than ſix inches; and there are numberleſs objects too ſmall to be ſeen at that diſtance, becauſe they cannot appear under any ſenſible angle. The method of viewing ſuch minute objects is by a *microſcope*, of which there are three ſorts, *viz.* the *ſingle*, the *double*, and the *ſolar*.

The *ſingle microſcope* is only a ſmall convex glaſs, Fig. 5.
as *c d*, having the object *a b* placed in its focus, and the eye at the ſame diſtance on the other ſide; that the rays of each pencil, flowing from every point of the object on the ſide next the glaſs, may go on parallel to the eye, after paſſing through the glaſs; The *ſingle micro-ſcope.*
and then, by entering the eye at *C*, they will be

 converged

converged to as many different points on the retina, and form a large inverted picture *A B* upon it, as in the figure.

To find how much this glaſs magnifies, divide the leaſt diſtance (which is about ſix inches) at which an object can be ſeen diſtinctly with the bare eye, by the focal diſtance of the glaſs; and the quotient will ſhew how much the glaſs magnifies the diameter of the object.

Fig. 6. The *double* or *compound microſcope*, conſiſts of an object-glaſs *cd*, and an eye-glaſs *ef*. The ſmall object *a b* is placed at a little greater diſtance from the glaſs *c d* than its principal focus, ſo that the pencils of rays flowing from the different points of the object, and paſſing through the glaſs, may be made to converge, and unite in as many points between *g* and *h*, where the image of the object will be formed: which image is viewed by the eye through the eye-glaſs *e f*. For the eye-glaſs being ſo placed, that the image *g h* may be in its focus, and the eye much about the ſame diſtance on the other ſide, the rays of each pencil will be parallel, after going out of the eye-glaſs, as at *e* and *f*, till they come to the eye at *k*, where they will begin to converge by the refractive power of the humours; and after having croſſed each other in the pupil, and paſſed through the chryſtalline and vitreous humours, they will be collected into points on the retina, and form the large inverted image *A B* thereon.

The *double microſcope.*

The magnifying power of this microſcope is as follows. Suppoſe the image *g h*, to be ſix times the diſtance of the object *a b* from the object-glaſs *c d*; then

then will the image be ſix times the length of the object: but ſince the image could not be ſeen diſtinctly by the bare eye at a leſs diſtance than ſix inches, if it be viewed by an eye-glaſs *e f*, of one inch focus, it will thereby be brought ſix times nearer the eye; and conſequently viewed under an angle ſix times as large as before; ſo that it will be again magnified ſix times; that is, ſix times by the object glaſs, and ſix times by the eye-glaſs; which multiplied into one another make 36 times: and ſo much is the object magnified in diameter, more than what it appears to the bare eye; and conſequently 36 times 36, or 1296 times in ſurface.

But, becauſe the extent or field of view is very ſmall in this microſcope, there are generally two eye-glaſſes, placed ſometimes cloſe together, and ſometimes an inch aſunder; by which means, although the object appears leſs magnified, yet the viſible area is much enlarged by the interpoſition of a ſecond eye-glaſs; and conſequently a much pleaſanter view is obtained.

The *ſolar microſcope*, invented by Dr. *Lieberkhun*, is conſtructed in the following manner. Having procured a very dark room, let a round hole be made in the window-ſhutter, about three inches diameter, through which the ſun may caſt a cylinder of rays *A A* into the room. In this hole, place the end of a tube containing two convex glaſſes and an object, *viz.* 1. A convex glaſs *a a*, of about two inches diameter, and three inches focal diſtance, is to be placed in that end of the tube which is put into

Fig. 7.

The *ſolar micro-ſcope.*

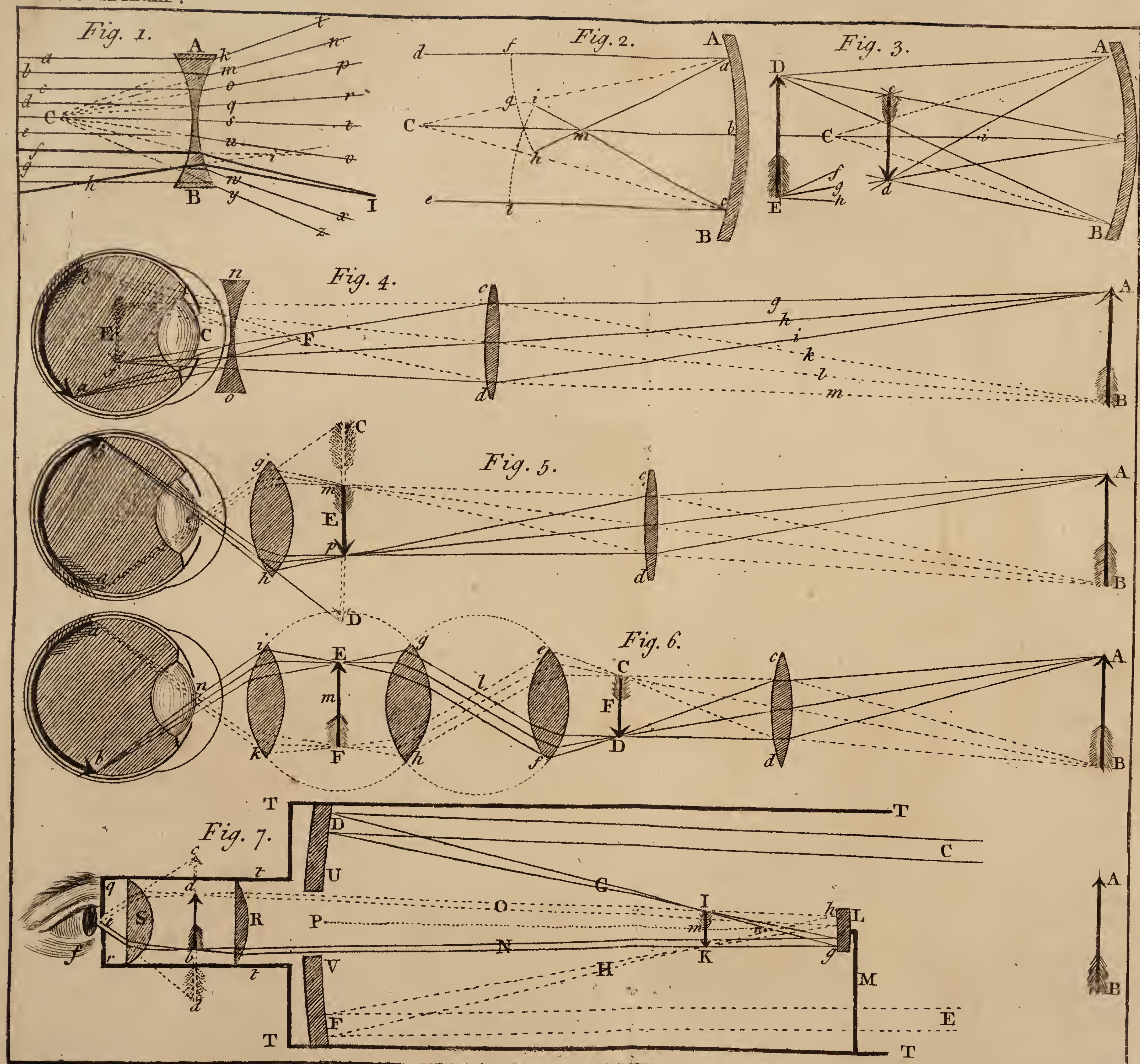
the hole. 2. The object *bb*, being put between two glasses (which must be concave to hold it at liberty) is placed about two inches and an half from the glass *aa*. 3. A little more than a quarter of an inch from the object, is placed the small convex glass *cc*, whose focal distance is a quarter of an inch.

The tube may be so placed, when the sun is low, that his rays *AA* may enter directly into it: but when he is high, his rays *BB* must be reflected into the tube by the plane mirrour or looking-glass *CC*.

Things being thus prepared, the rays that enter the tube will be conveyed by the glass *aa* towards the object *bb*, by which means it will be strongly illuminated; and the rays *d* which flow from it, through the convex glass *cc*, will make a large inverted picture of the object at *DD*, which, being received on a white paper, will represent the object magnified in length, in proportion of the distance of the picture from the glass *cc*, to the distance of the object from the same glass. Thus, suppose the distance of the object from the glass to be $\frac{3}{10}$ parts of an inch, and the distance of the distinct picture to be 12 feet or 144 inches, in which there are 1440 tenths of an inch; and this number divided by 3 tenths, gives 480; which is the number of times the picture is longer or broader than the object: and the length multiplied by the breadth shews how much the whole surface is magnified.

Before

PLATE XVIII.



J. Ferguson delin.

J. Mynde sc.

Before we enter upon the defcription of tele- *Telefcopes.* fcopes, it will be proper to fhew how the rays of light are affected by paffing through concave glaffes, and by falling upon concave mirrours.

When parallel rays, as *abcdefgh*, pafs directly Pl. XVIII. Fig. 1. through a glafs *AB*, which is equally concave on both fides, they will diverge after paffing through the glafs, as if they had come from a radiant point *C*, in the center of the glafs's concavity; which point, is called the negative or virtual focus of the glafs. Thus, the ray *a*, after paffing through the glafs *AB*, will go on in the direction *kl*, as if it had proceeded from the point *C*, and no glafs in the way. The ray *b* will go on in the direction *mn*; the ray *c* in the direction *op*, *&c.*—The ray *C*, that falls directly upon the middle of the glafs, fuffers no refraction in paffing through it; but goes on in the fame rectilineal direction, as if no glafs had been in its way.

If the glafs had been concave only on one fide, and the other fide quite plane, the rays would have diverged, after paffing through it, as if they had come from a radiant point at double the diftance of *C* from the glafs; that is, as if the radiant had been at the diftance of a whole diameter of the glafs's concavity.

If rays come more converging to fuch a glafs, than parallel rays diverge after paffing through it, they will continue to converge after paffing through it; but will not meet fo foon as if no glafs had been in the way; and will incline towards the fame fide to which they would have diverged, if they

 had

had come parallel to the glaſs. Thus, the rays *f* and *h*, going in a converging ſtate towards the edge of the glaſs at *B*, and converging more in their way to it than the parallel rays diverge after paſſing through it, they will go on converging after they paſs through it; though in a leſs degree than they did before, and will meet at *I*: but if no glaſs had been in their way, they would have met at *i*.

Fig. 2. When parallel rays, as *dfa*, *Cmb*, *elc*, fall upon a concave mirrour *AB* (which is not tranſparent, but has only the ſurface *AbB* of a clear poliſh) they will be reflected back from that mirrour; and meet in a point *m*, at half the diſtance of the ſurface of the mirrour from *C*, the center of its concavity: for they will be reflected at as great an angle from a perpendicular to the ſurface of the mirrour, as they fell upon it, with regard to that perpendicular; but on the other ſide thereof. Thus, let *C* be the center of concavity of the mirrour *AbB*, and let the parallel rays *dfa*, *Cmb*, and *elc*, fall upon it at the points *a*, *b*, and *c*. Draw the lines *Cia*, *Cmb*, and *Chc*, from the center *C* to theſe points; all which will be perpendicular to the ſurface of the mirrour, becauſe they proceed thereto like ſo many *radii* or ſpokes from its center. Make the angle *Cah* equal to the angle *daC*, and draw the line *amh*, which will be the direction of the ray *dfa*, after it is reflected from the point *a* of the mirrour: ſo that the angle of incidence *daC*, is equal to the angle of reflection *Cah*; the rays making equal angles with the perpendicular *Cia* on its oppoſite ſides.

Draw

Draw alſo the perpendicular *Chc* to the point *c*, where the ray *elc* touches the mirrour; and, having made the angle *Cci* equal to the angle *Cce*, draw the line *cmi*, which will be the courſe of the ray *elc*, after it is reflected from the mirrour.

The ray *Cmb* paſſing through the center of concavity of the mirrour, and falling upon it at *b*, is perpendicular to it; and is therefore reflected back from it in the ſame line *bmC*.

All theſe reflected rays meet in the point *m*; and in that point the image of the body which emits the parallel rays *da*, *Cb*, and *ec*, will be formed: which point is diſtant from the mirrour equal to half the radius *bmC* of its concavity.

The rays which proceed from any celeſtial object may be eſteemed parallel at the earth; and therefore, the images of that object will be formed at *m*, when the reflecting ſurface of the concave mirrour is turned directly towards the object. Hence, the focus *m* of parallel rays is not in the center of the mirrour's concavity, but half way between the mirrour and that center.

The rays which proceed from any remote terreſtrial object, are nearly parallel at the mirrour; not ſtrictly ſo, but come diverging to it, in ſeparate pencils, or, as it were, bundles of rays, from each point of the ſide of the object next the mirrour: and therefore they will not be converged to a point, at the diſtance of half the radius of the mirrour's concavity from its reflecting ſurface; but into ſeparate points at a little greater diſtance from the mirrour. And the nearer the object is to the mirrour,

rour, the farther these points will be from it; and in them an inverted image of the object will be formed, which will seem to hang pendent in the air; and will be seen by an eye placed beyond it (with regard to the mirrour) in all respects like the object, and as distinct as the object itself.

Fig. 3. Let *AcB* be the reflecting surface of a mirrour, whose center of concavity is at *C*; and let the upright object *DE* be placed beyond the center *C*, and send out a conical pencil or bundle of diverging rays from its upper extremity *D*, to every point of the concave surface of the mirrour *AcB*. But to avoid confusion, we only draw three rays of that pencil; as *DA*, *Dc*, *DB*.

From the center of concavity *C*, draw the three right lines *CA*, *Cc*, *CB*, touching the mirrour in the same points where the foresaid rays touch it; and all these lines will be perpendicular to the surface of the mirrour. Make the angle *CAd*, equal to the angle *DAC*, and draw the right line *Ad* for the course of the reflected ray *DA*: make the angle *Ccd* equal to the angle *DcC*, and draw the right line *cd* for the course of the reflected ray *Dd*: make also the angle *CBd* equal to the angle *DBC*, and draw the right line *Bd* for the course of the reflected ray *DB*. All these reflected rays will meet in the point *d*, where they will form the extremity *d* of the inverted image *ed*, similar to the extremity *D* of the upright object *DE*.

If the pencil of rays *Efgh* be also continued to the mirrour, and their angles of reflection from it be made equal to their angles of incidence upon

it,

it, as in the former pencil from *D*, they will all meet at *e* by reflection, and form the extremity *e* of the image *e d*, similar to the extremity *E* of the object *DE*.

And as each intermediate point of the object, between *D* and *E*, sends out a pencil of rays in like manner to every point of the mirrour, the rays of each pencil will be reflected back from it, and meet in as many intermediate points between the extremities *e* and *d* of the image; and so the whole image will be formed, not at *i*, half the distance of the mirrour from its center of concavity *C*; but at a greater distance, between *i* and the object *DE*: and the image will be inverted with respect to the object.

This being well understood, the reader will easily see how the image is formed by the large concave mirrour of the reflecting telescope, when he comes to the description of that instrument.

When the object is more remote from the mirrour than its center of concavity *C*, the image will be less than the object, and between the object and mirrour: when the object is nearer than the center of concavity, the image will be more remote and bigger than the object: thus, if *DE* be the object, *ed* will be its image; but if *ed* be the object, *DE* will be its image; for, as the object recedes from the mirrour, the image approaches nearer to it; and as the object approaches nearer to the mirrour, the image recedes farther from it; on account of the lesser or greater divergency of the pencils of rays which proceed from the object: for, the

less

leſs they diverge, the ſooner they are converged to points by reflection; and the more they diverge, the farther they muſt be reflected before they meet.

If the radius of the mirrour's concavity and the diſtance of the object from it be known, the diſtance of the image from the mirrour is found by this rule; divide the product of the diſtance and radius, by double the diſtance made leſs by the radius, and the quotient is the diſtance required.

If the object be in the center of the mirrour's concavity, the image and object will be coincident, and equal in bulk.

If a man places himſelf directly before a large concave mirrour, but farther from it than its center of concavity, he will ſee an inverted image of himſelf in the air, between him and the mirrour, of a leſs ſize than himſelf. And if he holds out his hand towards the mirrour, the hand of the image will come out towards his hand, and coincide with it, of an equal bulk, when his hand is in the center of concavity; and he will imagine he may ſhake hands with his image. If he reaches his hand farther, the hand of the image will paſs by his hand, and come between his hand and his body: and if he moves his hand towards either ſide, the hand of the image will move towards the other; ſo that whatever way the object moves, the image will move the contrary.

All the while a by-ſtander will ſee nothing of the image, becauſe none of the reflected rays that form it enter his eyes.

If

If a fire be made in a large room, and a ſmooth mahogony table be placed at a good diſtance near the wall, before a large concave mirrour, ſo placed that the light of the fire may be reflected from the mirrour to its focus upon the table; if a perſon ſtands by the table, he will ſee nothing upon it but a longiſh beam of light: but if he ſtands at a diſtance towards the fire, not directly between the fire and mirrour, he will ſee an image of the fire upon the table, large and erect. And if another perſon, who knows nothing of this matter before-hand, ſhould chance to come into the room, and ſhould look from the fire towards the table, he would be ſtartled at the appearance; for the table would ſeem to be on fire, and by being near the wainſcot, to endanger the whole houſe. In this experiment, there ſhould be no light in the room but what proceeds from the fire; and the mirrour ought to be at leaſt fifteen inches in diameter.

If the fire be darkened by a ſcreen, and a large candle be placed at the back of the ſcreen; a perſon ſtanding by the candle will ſee the appearance of a fine large ſtar, or rather planet, upon the table as bright as Venus or Jupiter. And if a ſmall wax taper (whoſe flame is much leſs than the flame of the candle) be placed near the candle, a ſatellite to the planet will appear on the table: and if the taper be moved round the candle, the ſatellite will go round the planet.

For theſe two pleaſing experiments, I am indebted to the reverend Dr. LONG, *Lowndes*'s profeſſor of aſtronomy at Cambridge, who fa-

voured

voured me with the ſight of them, and many more of his curious inventions.

The refracting telescope. In a *refracting teleſcope*, the glaſs which is neareſt the object in viewing it, is called *the object-glaſs*; and that which is neareſt the eye, is called *the eye-glaſs*. The object-glaſs muſt be convex, but the eye-glaſs may be either convex or concave: and generally in looking through a teleſcope, the eye is in the focus of the eye-glaſs; though that is not very material: for the diſtance of the eye, as to diſtinct viſion, is indifferent, provided the rays of the pencils fall upon it parallel: only, the nearer the eye is to the end of the teleſcope, the larger is the ſcope or area of the field of view.

Fig. 4. Let *cd* be a convex glaſs fixed in a long tube, and have its focus at *E*. Then, a pencil of rays *ghi*, flowing from the upper extremity *A* of the remote object *AB*, will be ſo refracted by paſſing through the glaſs, as to converge and meet in the point *f*; whilſt the pencil of rays *klm*, flowing from the lower extremity *B*, of the ſame object *AB*, and paſſing through the glaſs, will converge and meet in the point *e*: and in theſe points *f* and *e*, the images of the points *A* and *B* will be formed. And as all the intermediate points of the object, between *A* and *B*, ſend out pencils of rays in the ſame manner, a ſufficient number of theſe pencils will paſs through the object-glaſs *cd*, and converge to as many intermediate points between *e* and *f*; and ſo will form the whole inverted image *eEf*, of the diſtinct object. But becauſe this image is ſmall, a concave glaſs *no*, is ſo placed in the end of

of the tube next the eye, that its virtual focus may be at *F*. And as the pencils of rays paſs converging through the concave glaſs, but converge leſs after paſſing through it than before, they go on further, as to *b* and *a*, before they meet; and the pencils themſelves being made to diverge by paſſing through the concave glaſs, they enter the eye, and form the large picture *a b* upon the retina, whereon it is magnified under the angle *b F a*.

But this teleſcope has one inconveniency which renders it unfit for moſt purpoſes, which is, that the pencils of rays being made to diverge by paſſing through the concave glaſs *no*, very few of them can enter the pupil of the eye; and therefore the field of view is but very ſmall, as is evident by the figure. For none of the pencils which flow either from the top or bottom of the object *AB* can enter the pupil of the eye at *C*, but are all ſtopt by falling upon the iris above and below the pupil: and therefore, only the middle part of the object can be ſeen when the teleſcope lies directly towards it, by means of thoſe rays which proceed from the middle of the object. So that to ſee the whole of it, the teleſcope muſt be moved upwards and downwards, unleſs the object be very remote; and then it is never ſeen diſtinctly.

This inconvenience is remedied by ſubſtituting a convex eye-glaſs, as *g h*, in place of the concave one; and fixing it ſo in the tube, that its focus may be coincident with the focus of the object-glaſs *c d*, as at *E*. For then, the rays of the pencils flowing from the object *AB*, and paſſing Fig. 5.

through

through the object-glaſs *cd*, will meet in its focus, and form the inverted image *mEp*: and as the image is formed in the focus of the eye-glaſs *gh*, the rays of each pencil will be parallel, after paſſing through that glaſs; but the pencils themſelves will croſs in its focus on the other ſide, as at *e*: and the pupil of the eye being in this focus, the image will be viewed through the glaſs, under the angle *geh*; and being at *E*, it will appear magnified, ſo as to fill the whole ſpace *CmepD*.

But, as this teleſcope inverts the image with reſpect to the object, it gives an unpleaſant view of terreſtrial objects; and is only fit for viewing the heavenly bodies, in which we regard not their poſition, becauſe their being inverted does not appear, on account of their being round. But whatever way the object ſeems to move, this teleſcope muſt be moved the contrary way, in order to keep ſight of it; for, ſince the object is inverted, its motion will be ſo too.

The magnifying power of this teleſcope is, as the focal diſtance of the object-glaſs to the focal diſtance of the eye-glaſs. Therefore, if the former be divided by the latter, the quotient will expreſs the magnifying power.

When we ſpeak of the magnifying of a teleſcope or microſcope, it is only meant with regard to the diameter, not the area nor ſolidity of the object. But as the inſtrument magnifies the vertical diameter, as much as it does the horizontal, it is eaſy to find how much the whole viſible area or ſurface is magnified: for, if the diameters be multiplied into one

one another, the product will expreſs the magnification of the whole viſible area. Thus, ſuppoſe the focal diſtance of the object-glaſs be ten times as great as the focal diſtance of the eye-glaſs; then, the object will be magnified ten times, both in length and breadth: and 10 multiplied by 10, produces 100; which ſhews, that the area of the object will appear 100 times as big when ſeen through ſuch a teleſcope, as it does to the bare eye.

Hence it appears, that if the focal diſtance of the eye-glaſs were equal to the focal diſtance of the object-glaſs, the magnifying power of the teleſcope would be nothing.

This teleſcope may be made to magnify in any given degree, provided it be of a ſufficient length. For, the greater the focal diſtance of the object-glaſs, the leſs may be the focal diſtance of the eye-glaſs; though not directly in proportion. Thus, an object-glaſs, of 10 feet focal diſtance, will admit of an eye-glaſs, whoſe focal diſtance is little more than $2\frac{1}{2}$ inches; which will magnify near 48 times: but an object-glaſs, of 100 feet focus, will require an eye-glaſs ſomewhat more than 6 inches; and will therefore magnify almoſt 200 times.

A teleſcope for viewing terreſtrial objects, ſhould be ſo conſtructed, as to ſhew them in their natural poſture. And this is done by one object-glaſs *cd*, and three eye-glaſſes *ef*, *gh*, *ik*, ſo placed, that the diſtance between any two, which are neareſt to each other, may be equal to the ſum of their focal diſtances; as in the figure, where the focus of the glaſſes *cd* and *ef* meet at *F*, thoſe of the glaſſes *ef* Fig. 6.

and *gh*, meet at *l*, and of *gh* and *ik*, at *m*; the eye being at *n*, in or near the focus of the eye-glaſs *ik*, on the other ſide. Then, it is plain, that theſe pencils of rays, which flow from the object *AB*, and paſs through the object-glaſs *cd*, will meet and form an inverted image *CFD* in the focus of that glaſs; and the image being alſo in the focus of the glaſs *ef*, the rays of the pencils will become parallel, after paſſing through that glaſs, and croſs in its oppoſite focus at *l*; from whence they paſs on to the next glaſs *gh*, and by going through it they are converged to points in its other focus, where they form an erect image *EmF*, of the object *AB*: and as this image is alſo in the focus of the eye-glaſs *ik*, and the eye on the oppoſite ſide of the ſame glaſs; the image is viewed through the eye-glaſs in this teleſcope, in the ſame manner as through the eye-glaſs in the former one; only in a contrary poſition, that is, in the ſame poſition with the object.

The three glaſſes next the eye, have all their focal diſtances equal: and the magnifying power of this teleſcope, is found the ſame way as that of the laſt above; *viz.* by dividing the focal diſtance of the object-glaſs *cd*, by the focal diſtance of the eye-glaſs *ik*, or *gh*, or *ef*, ſince all theſe three are equal.

Why the object appears coloured when ſeen through a teleſcope.

When the rays of light are ſeparated by refraction, they become coloured, and if they be united again, they will be a perfect white. But thoſe rays which paſs through a convex glaſs, near its edges, are more unequally refracted than thoſe which

which are nearer the middle of the glaſs. And when the rays of any pencil are unequally refracted by the glaſs, they do not all meet again in one and the ſame point, but in ſeparate points; which makes the image indiſtinct, and coloured, about its edges. The remedy is, to have a plate with a ſmall round hole in its middle, fixed in the tube at *m*, parallel to the glaſſes. For, the wandering rays about the edges of the glaſſes will be ſtopt, by the plate, from coming to the eye; and none admitted but thoſe which come through the middle of the glaſs, or at leaſt at a good diſtance from its edges, and paſs through the hole in the middle of the plate. But this circumſcribes the image, and leſſens the field of view, which would be much larger if the plate could be diſpenſed with.

The great inconvenience attending the management of long teleſcopes of this kind, has brought them much into diſuſe ever ſince the *reflecting teleſcope* was invented. For one of this ſort, ſix feet in length, magnifies as much as one of the other an hundred. It was invented by Sir *Iſaac Newton*, but has received conſiderable improvements ſince his time; and is now generally conſtructed in the following manner, which was firſt propoſed by Dr. *Gregory*.

The *reflecting teleſcope.*

At the bottom of the great tube *TTTT*, is placed the large concave mirrour *DUVF*, whoſe principal focus is at *m*; and in its middle is a round hole *P*, oppoſite to which is placed the ſmall mirrour *L*, concave toward the great one; and ſo fixed to a ſtrong wire *M*, that it may be moved

Fig. 7.

farther from the great mirrour, or nearer to it, by means of a long ſcrew on the outſide of the tube, keeping its axis ſtill in the ſame line *Pmn* with that of the great one.—Now, ſince in viewing a very remote object, we can ſcarce ſee a point of it but what is at leaſt as broad as the great mirrour, we may conſider the rays of each pencil, which flow from every point of the object, to be parallel to each other, and to cover the whole reflecting ſurface *DUVF*. But to avoid confuſion in the figure, we ſhall only draw two rays of a pencil flowing from each extremity of the object into the great tube, and trace their progreſs, through all their reflections and refractions, to the eye *f*, at the end of the ſmall tube *tt*, which is joined to the great one.

Let us then ſuppoſe the object *AB* to be at ſuch a diſtance, that the rays *C* may flow from its lower extremity *B*, and the rays *E* from its upper extremity *A*. Then, the rays *C* falling parallel upon the great mirrour at *D*, will be thence reflected converging, in the direction *DG*; and by croſſing at *I* in the principal focus of the mirrour, they will form the upper extremity *I* of the inverted image *IK*, ſimilar to the lower extremity *B* of the object *AB*: and paſſing on to the concave mirrour *L* (whoſe focus is at *n*) they will fall upon it at *g*, and be thence reflected converging, in the direction *gN*, becauſe *gm* is ſhorter than *gn*; and paſſing through the hole *P* in the large mirrour, they would meet ſomewhere about *r*, and form the lower extremity *b* of the erect image *ab*, ſimilar to the lower

lower extremity *B* of the object *AB*. But by paſſing through the plano-convex glaſs *R* in their way, they form that extremity of the image at *b*. In like manner, the rays *E*, which come from the top of the object *AB*, and fall parallel upon the great mirrour at *F*, are thence reflected converging to its focus, where they form the lower extremity *K* of the inverted image *IK*, ſimilar to the upper extremity *A* of the object *AB*; and thence paſſing on to the ſmall mirrour *L*, and falling upon it at *h*, they are thence reflected in the converging ſtate *hO*; and going on through the hole *P* of the great mirrour, they would meet ſomewhere about *q*, and form there the upper extremity *a* of the erect image *ab*, ſimilar to the upper extremity *A* of the object *AB*: but by paſſing through the convex-glaſs *R* in their way, they meet and croſs ſooner, as at *a*, where that point of the erect image is formed.— The like being underſtood of all thoſe rays which flow from the intermediate points of the object, between *A* and *B*, and enter the tube *TT*; all the intermediate points of the image between *a* and *b* will be formed: and the rays paſſing on from the image, through the eye-glaſs *S*, and through a ſmall hole *e* in the end of the leſſer tube *tt*, they enter the eye *f*, which ſees the image *ab* (by means of the eye-glaſs) under the large angle *ced*, and magnified in length, under that angle, from *c* to *d*.

In the beſt reflecting teleſcopes, the focus of the ſmall mirrour is never coincident with the focus *m* of the great one, where the firſt image *IK* is formed, but a little beyond it (with reſpect to the

 eye)

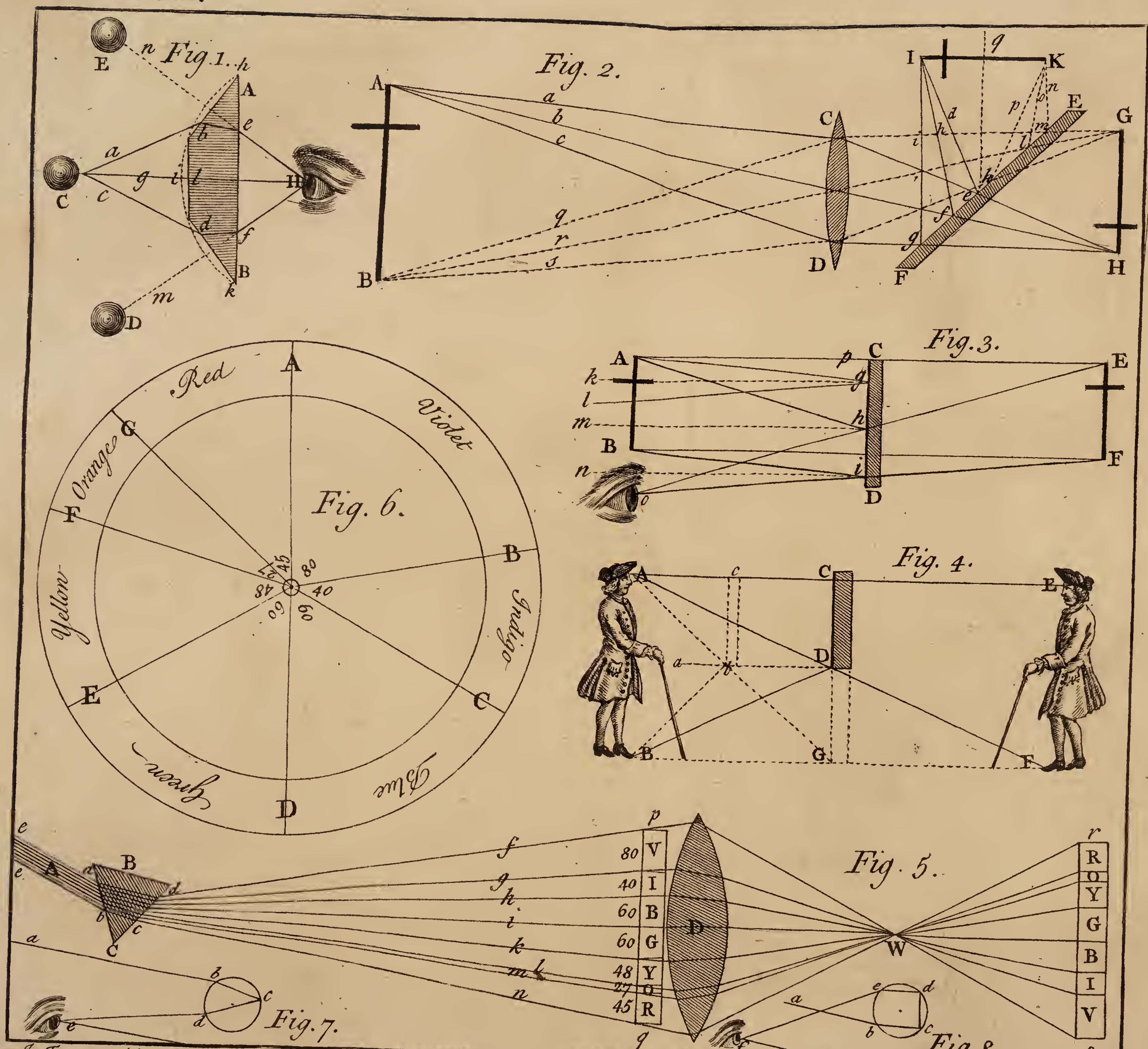
eye) as at *n*: the conſequence of which is, that the rays of the pencils will not be parallel after reflection from the ſmall mirrour, but converge ſo as to meet in points about *q*, *e*, *r*; where they would form a larger upright image than *ab*, if the glaſs *R* was not in their way: and this image might be viewed by means of a ſingle eye-glaſs properly placed between the image and the eye: but then the field of view would be leſs, and conſequently not ſo pleaſant; for which reaſon, the glaſs *R* is ſtill retained, to enlarge the ſcope or area of the field.

To find the magnifying power of this teleſcope, multiply the focal diſtance of the great mirrour by the diſtance of the ſmall mirrour from the image next the eye, and multiply the focal diſtance of the ſmall mirrour by the focal diſtance of the eye-glaſs: then, divide the product of the former multiplication by the product of the latter, and the quotient will expreſs the magnifying power.

I ſhall here ſet down the dimenſions of one of Mr. *Short*'s reflecting teleſcopes, as it is mentioned in Dr. *Smith*'s optics.

The focal diſtance of the great mirrour 9.6 inches, its breadth 2.3; the focal diſtance of the ſmall mirrour 1.5, its breadth 0.6: the breadth of the hole in the great mirrour 0.5; the diſtance between the ſmall mirrour and the next eye-glaſs 14.2; the diſtance between the two eye-glaſſes 2.4; the focal diſtance of the eye-glaſs next the metals

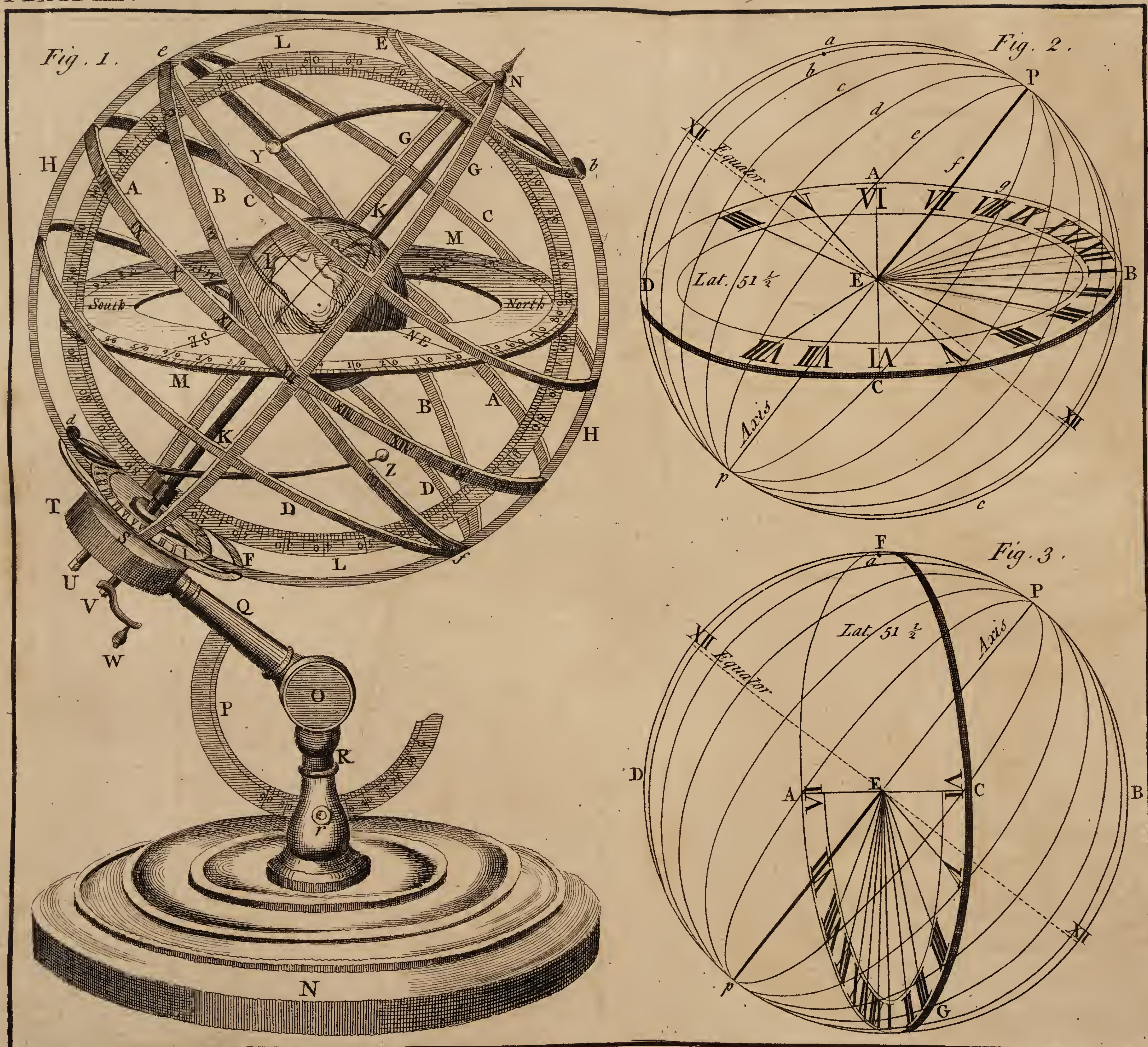
3.8;



J. Ferguson delin.

J. Mynde sc.

PLATE XX.



J. Ferguson delin. J. Mynde sculp.

3.8; and the focal diſtance of the eye-glaſs next the eye 1.1.

One great advantage of the reflecting teleſcope is, that it will admit of an eye-glaſs of a much ſhorter focal diſtance than a refracting teleſcope will; and, conſequently, it will magnify ſo much the more: for the rays are not coloured by reflection from a concave mirrour, if it be ground to a true figure, as they are by paſſing through a convex-glaſs, let it be ground ever ſo true.

The adjuſting ſcrew on the outſide of the great tube fits this teleſcope to all ſorts of eyes, by bringing the ſmall mirrour either nearer to the eye, or removing it farther: by which means, the rays are made to diverge a little for ſhort ſighted eyes, or to converge for thoſe of a long ſight.

The nearer an object is to the teleſcope, the more its pencils of rays will diverge before they fall upon the great mirrour, and therefore they will be the longer of meeting in points after reflection; ſo that the firſt image *IK* will be formed at a greater diſtance from the large mirrour, when the object is near the teleſcope, than when it is very remote. But as this image muſt be formed farther from the ſmall mirrour than its principal focus *n*, this mirrour muſt be always ſet at a greater diſtance from the large one, in viewing near objects, than in viewing remote ones. And this is done by turning the ſcrew on the outſide of the tube, until the ſmall mirrour be ſo adjuſted, that the object (or rather its image) appears perfect.

In looking through any telefcope towards an object, we never fee the object itfelf, but only that image of it which is formed next the eye in the telefcope. For, if a man holds his finger or a ftick between his bare eye and an object, it will hide part (if not the whole) of the object from his view. But if he ties a ftick acrofs the mouth of a telefcope, before the object-glafs, it will hide no part of the imaginary object he faw through the telefcope before, unlefs it covers the whole mouth of the tube: the only effect it will have, is, to make the object appear dimmer, becaufe it intercepts part of the rays. Whereas, if he puts only a piece of wire acrofs the infide of the tube, between the eye-glafs and his eye, it will hide part of the object which he thinks he fees: which proves that he fees not the real object, but its image. This is alfo confirmed by means of the fmall mirrour *L*, in the reflecting telefcope, which is made of opake metal, and ftands directly between the eye and the object towards which the telefcope is turned: and will hide the whole object from the eye at *e*, if the two glaffes *R* and *S* are taken out of the tube.

Plate XIX. Fig. 1. *The multiplying glafs.*

The multiplying glafs is made by grinding down the round fide *hik* of a convex glafs *AB*, into feveral flat furfaces, as *hb*, *bld*, *dk*. An object *C* will not appear magnified, when feen through this glafs, by the eye at *H*; but it will appear multiplied into as many different objects as the glafs contains plane furfaces. For, fince rays will flow from the object *C* to all parts of the glafs,

glafs, and each plane furface will refract thefe rays to the eye, the fame object will appear to the eye, in the direction of the rays which enter it through each furface. Thus, a ray *giH*, falling perpendicularly on the middle furface, will go through the glafs to the eye without fuffering any refraction; and will therefore fhew the object in its true place at *C*: whilft a ray *ab* flowing from the fame object, and falling obliquely on the plane furface *bh*, will be refracted in the direction *be*, by paffing through the glafs; and upon leaving it, will go on to the eye in the direction *eH*; which will caufe the fame object *C* to appear alfo at *E*, in the direction of the ray *He*, produced in the right line *Hen*. And the ray *cd*, flowing from the object *C*, and falling obliquely on the plane furface *dk*, will be refracted (by paffing through the glafs and leaving it at *f*) to the eye at *H*; which will caufe the fame object to appear at *D*, in the direction *Hfm*.—If the glafs be turned round the line *glH*, as an axis, the object *C* will keep its place, becaufe the furface *bld* is not removed; but all the other objects will feem to go round *C*, becaufe the oblique planes, on which the rays *ab*, *cd* fall, will go round by the turning of the glafs.

The *camera obfcura* is made by a convex glafs *CD*, placed in a hole of a window-fhutter. Then, if the room be darkened fo, as no light can enter but what comes through the glafs, the pictures of all the objects (as fields, trees, buildings, men, cattle, &c.) on the outfide, will be fhewn in an inverted order, on a white paper placed at *GH* in the

Fig. 2. The *camera obfcura*.

the focus of the glaſs: and will afford a moſt beautiful and perfect piece of perſpective or landſcape of whatever is before the glaſs; eſpecially if the ſun ſhines upon the objects.

If the convex glaſs *CD* be placed in a tube in the ſide of a ſquare box, within which is the plane mirrour *EF*, reclining backwards in an angle of 45 degrees from the perpendicular *kq*, the pencils of rays flowing from the outward objects, and paſſing through the convex-glaſs to the plane mirrour, will be reflected upwards from it, and meet in points as *I* and *K* (at the ſame diſtance that they would have met at *H* and *G*, if no mirrour had been in the way) and will form the aforeſaid images on an oiled paper ſtretched horizontally in the direction *IK*; on which paper, the out-lines of the images may be eaſily drawn with a black lead pencil; and then copied on a clean ſheet, and coloured by art, as the objects themſelves are by nature.—In this machine, it is uſual to place a plane glaſs unpoliſhed, in the horizontal ſituation *IK*, which glaſs receives the images of the outward objects; and their out-lines may be traced upon it by a black lead pencil.

N. B. The tube in which the convex-glaſs *CD* is fixed, muſt be made to draw out, or puſh in, ſo as to adjuſt the diſtance of that glaſs from the plane mirrour, in proportion to the diſtance of the outward objects; which the operator does, until he ſees their images diſtinctly painted on the horizontal glaſs at *IK*.

The

The forming a horizontal image, as *IK*, of an upright object *AB*, depends upon the angles of incidence of the rays upon the plane mirrour *EF*, being equal to their angles of reflection from it. For, if a perpendicular be ſuppoſed to be drawn to the ſurface of the plane mirrour at *e*, where the ray *AaCe* falls upon it, that ray will be reflected upwards in an equal angle with the other ſide of the perpendicular, in the line *edI*. Again, if a perpendicular be drawn to the mirrour from the point *f*, where the ray *Abf* falls upon it, that ray will be reflected in an equal angle from the other ſide of the perpendicular, in the line *fhI*. And if a perpendicular be drawn from the point *g*, where the ray *Acg* falls upon the mirrour, that ray will be reflected in an equal angle from the other ſide of the perpendicular, in the line *giI*. So that all the rays of the pencil *abc*, flowing from the upper extremity of the object *AB*, and paſſing through the convex-glaſs *CD*, to the plane mirrour *EF*, will be reflected from the mirrour, and meet at *I*, where they will form the extremity *I* of the image *IK*, ſimilar to the extremity *A* of the object *AB*. The like is to be underſtood of the pencil *qrs*, flowing from the lower extremity of the object *AB*, and meeting at *K* (after reflection from the plane mirrour) the rays form the extremity *K* of the image, ſimilar to the extremity *B* of the object: and ſo of all the pencils that flow from the intermediate points of the object to the mirrour, through the convex glaſs.

If

The *opera-glaſs.* If a convex-glaſs, of a ſhort focal diſtance, be placed near the plane mirrour, in a ſhort tube, and a convex-glaſs be placed in a hole in the ſide of the tube, ſo as the image may be formed between the laſt mentioned convex-glaſs, and the plane mirrour; the image being viewed through this glaſs, will appear magnified.—In this manner, the *opera-glaſſes* are conſtructed; with which a gentleman may look at any lady at a diſtance in the company, and the lady know nothing of it.

The common *looking glaſs.* The image of any object that is placed before a plane mirrour, appears as big to the eye as the object itſelf; and is erect, diſtinct, and ſeemingly as far behind the mirrour, as the object is before it: and that part of the mirrour, which reflects the image of the object to the eye, (the eye being ſuppoſed equally diſtant from the glaſs with the object) is juſt half as long and half as broad as the object itſelf. Fig. 3. Let *AB* be an object placed before the reflecting ſurface *gbi* of the plane mirrour *CD*; and let the eye be at *o*. Let *Ah* be a ray of light flowing from the top *A* of the object, and falling upon the mirrour at *h*; and *hm* be a perpendicular to the ſurface of the mirrour at *h*: the ray *Ah* will be reflected from the mirrour to the eye at *o*, making an angle *mho* equal to the angle *Ahm*: then will the top of the image *E* appear to the eye in the direction of the reflected ray *oh* produced to *E*, where the right line *ApE*, from the top of the object, cuts the right line *ohE*, at *E*. Let *Bi* be a ray of light proceeding from the foot of the object at *B* to the mirrour at *i*; and

ni

ni a perpendicular to the mirrour from the point *i*, where the ray *Bi* falls upon it: this ray will be reflected in the line *io*, making an angle *nio*, equal to the angle *Bin*, with that perpendicular, and entering the eye at *o*: then will the foot *F* of the image appear in the direction of the reflected ray *oi*, produced to *F*, where the right line *BF* cuts the reflected ray produced to *F*. All the other rays that flow from the intermediate points of the object *AB*, and fall upon the mirrour between *h* and *i*, will be reflected to the eye at *o*; and all the intermediate points of the image *EF* will appear to the eye in the direction of these reflected rays produced. But all the rays that flow from the object, and fall upon the mirrour above *h*, will be reflected back above the eye at *o*; and all the rays that flow from the object, and fall upon the mirrour below *i*, will be reflected back below the eye at *o*: so that none of the rays that fall above *h*, or below *i*, can be reflected to the eye at *o*; and the distance between *h* and *i* is equal to half the length of the object *AB*.

A man will see his image in a plane looking-glass, that is but half his height.

Hence it appears, that if a man sees his whole image in a plane looking-glass, the part of the glass that reflects his image must be just half as long and half as broad as himself; let him stand at any distance from it whatever: and that his image must appear just as far behind the glass as he is before it. Thus, the man *AB* viewing himself in the plane mirrour *CD*, which is just half as long as himself, sees his whole image as at *EF*, behind the glass, exactly equal to his own size. Fig. 4.

For,

For, a ray *AC*, proceeding from his eye at *A*, and falling perpendicularly upon the ſurface of the glaſs at *C*, is reflected back to his eye in the ſame line *CA*; and the eye of his image will appear at *E*, in the ſame line produced to *E*, beyond the glaſs. And a ray *BD*, flowing from his foot, and falling obliquely on the glaſs at *D*, will be reflected as obliquely on the other ſide of the perpendicular *abD*, in the direction *DA*; and the foot of his image will appear at *F*, in the direction of the reflected ray *AD*, produced to *F*, where it is cut by the right line *BGF*, drawn parallel to the right line *ACE*. Juſt the ſame as if the glaſs were taken away, and a real man ſtood at *F*, equal in ſize to the man ſtanding at *B*: for to his eye at *A*, the eye of the other man at *E* would be ſeen in the direction of the line *ACE*; and the foot of the man at *F* would be ſeen by the eye *A*, in the direction of the line *ADF*.

If the glaſs be brought nearer the man *AB*, as ſuppoſe to *cb*, he will ſee his image as at *CDG*: for the reflected ray *CA* (being perpendicular to the glaſs) will ſhew the eye of the image as at *C*; and the incident ray *Bb*, being reflected in the line *bA*, will ſhew the foot of his image as at *G*; the angle of reflection *abA* being always equal to the angle of incidence *Bba*: and ſo of all the intermediate rays from *A* to *B*. Hence, if the man *AB* advances towards the glaſs *CD*, his image will approach towards it; and if he recedes from the glaſs, his image will alſo recede from it.

Having

Having already ſhewn, that the rays of light are refracted when they paſs obliquely through different mediums, we come now to prove that ſome rays are more refracted than others; and that, as they are differently refracted, they excite in our minds the ideas of different colours. This will account for the colours ſeen about the edges of the images of thoſe objects which are viewed through teleſcopes.

Let the ſun ſhine into a dark room through a ſmall hole, as at *ee*, in a window-ſhutter; and place a triangular priſm *BC*, in the beam of rays *A*, in ſuch a manner, that the beam may fall obliquely on one of the ſides *abC* of the priſm. The rays will ſuffer different refractions by paſſing through the priſm, ſo that inſtead of going all out of it on the ſide *dcC*, in one direction, they will go on from it in the different directions repreſented by the lines *f*, *g*, *h*, *i*, *k*, *l*, *m*, *n*; and falling upon the oppoſite ſide of the room, or on a white paper placed as at *pq*, to receive them, they will paint upon it a ſeries of moſt beautiful lively colours (not to be equalled by art) in this order, *viz.* thoſe which are leaſt refracted by the priſm, and will therefore go on between the lines *n* and *m*, will be of a very bright intenſe red at *n*, degenerating from thence gradually into an orange colour, as they are nearer the line *m*: the next will be of a fine orange colour at *m*, and from thence degenerate into a yellow colour towards *l*: at *l* they will be of a fine yellow, which will incline towards a green, more and more, as they are nearer and nearer

Fig. 5.

The *priſm.*

The *colours of the light.*

nearer *k*: at *k* they will be a pure green, but from thence towards *i* they will incline gradually to a blue: at *i* they will be a perfect blue, inclining to an indigo-colour, from thence towards *h*: at *h* they will be quite the colour of *indigo*, which will gradually change towards a violet, the nearer they are to *g*: and at *g* they will be of a fine violet colour, which will incline gradually to a red, the nearer they are to *f*, where the coloured image ends.

There is not an equal quantity of rays in each of these colours; for, if the oblong image *pq* be divided into 360 equal parts, the red space *R* will take up 45 of these parts; the orange *O*, 27; the yellow *Y*, 48; the green *G*, 60; the blue *B*, 60; the indigo *I*, 40; and the violet *V*, 80: all which spaces are as nearly proportioned in the figure as the small space *qp* would admit of.

If all these colours be blended together again, they will make a pure white; as is proved thus. Take away the paper on which the colours *pq* fell, and place a large convex-glass *D* in the rays *f*,*g*,*h*, &c. which will refract them so, as to make them unite and cross each other at *W*: and if a white paper be placed there to receive them, they will excite the idea of a strong lively white. But if the paper be placed farther from the glass, as at *rs*, the different colours will appear again upon it, in an inverted order, occasioned by the rays crossing at *W*.

As white is a composition of all colours, so black is a privation of them all, and, therefore, properly no colour. Let

Let two concentric circles be drawn on a ſmooth round board *ABCDEFG*, and the outermoſt of them divided into 360 equal parts or degrees: then, draw ſeven right lines, as ⊙ *A*, ⊙ *B*, &c. from the center to the outermoſt circle, making the lines ⊙ *A* and ⊙ *B* include 80 degrees of that circle; the lines ⊙ *B* and ⊙ *C* 40 degrees; ⊙ *C* and ⊙ *D* 60; ⊙ *D* and ⊙ *E* 60; ⊙ *E* and ⊙ *F* 48; ⊙ *F* and ⊙ *G* 27; ⊙ *G* and ⊙ *A* 45. Then, between theſe two circles, paint the ſpace *AG* red, inclining to orange near *G*; *GF* orange, inclining to yellow near *F*; *FE* yellow, inclining to green near *E*; *ED* green, inclining to blue near *D*; *DC* blue, inclining to indigo near *C*; *CB* indigo, inclining to violet near *B*; and *BA* violet, inclining to a ſoft red near *A*. This done, paint all that part of the board black which lies within the inner circle; and putting an axis through the center of the board, let it be turned very ſwiftly round that axis, ſo as the rays proceeding from the above colours, may be all blended and mixt together in coming to the eye; and then, the whole coloured part will appear like a white ring, a little greyiſh; not perfectly white, becauſe no colours prepared by art are perfect.

Fig. 6.

All the priſmatic colours blended together, make a white.

Any of theſe colours, except red and violet, may be made by mixing together the two contiguous priſmatic colours. Thus, yellow is made by mixing together a due proportion of orange and green; and green may be made by a mixture of yellow and blue.

All bodies appear of that colour, whoſe rays they reflect moſt; as a body appears red when it reflects moſt of the red-making rays, and abſorbs the reſt.

Tranſparent colours become opake if put together.

Any two or more colours that are quite tranſparent by themſelves, become opake when put together. Thus, if water or ſpirits of wine be tinged red, and put in a phial, every object ſeen through it will appear red; becauſe it lets only the red rays paſs through it, and ſtops all the reſt. If water or ſpirits be tinged blue, and put in a phial, all objects ſeen through it will appear blue, becauſe it tranſmits only the blue rays, and ſtops all the reſt. But if theſe two phials are held cloſe together, ſo as both of them may be between the eye and object, the object will no more be ſeen through them than through a plate of metal; for whatever rays are tranſmitted through the fluid in the phial next the object, are ſtopped by that in the phial next the eye. In this experiment, the phials ought not to be round, but ſquare; becauſe nothing but the light itſelf can be ſeen through a round tranſparent body, at any diſtance.

As the rays of light ſuffer different degrees of refraction by paſſing obliquely through a priſm, or through a convex-glaſs, and are thereby ſeparated into all the ſeven original or primary colours; ſo they alſo ſuffer different degrees of refraction by paſſing through drops of falling rain; and then, being reflected towards the eye, from the ſides of theſe drops which are fartheſt from the eye, and again refracted by paſſing out of theſe drops into the

the air, in which refracted directions they come to the eye; they make all the colours to appear in the form of a fine arch in the heavens, which is called the *rain-bow*. The *rain-bow*.

There are always two rain-bows seen together, the interior of which is formed by the rays *ab*, which falling upon the upper part *b*, of the drop *bcd*, are refracted into the line *bc* as they enter the drop, and are reflected from the back of it at *c*, in the line *cd*, and then, by passing out of the drop into air, they are again refracted at *d*; and from thence they pass on to the eye at *e*: so that to form the interior bow, the rays suffer two refractions, as at *b* and *d*; and one reflection, as at *c*. Fig. 7.

The exterior bow is formed by rays which suffer two reflections, and two refractions; which is the occasion of its being less vivid than the interior, and also of its colours being inverted with respect to those of the interior. For, when a ray *ab*, falls upon the lower part of the drop *bcde*, it is refracted into the direction *bc* by entering the drop; and passing on to the back of the drop at *c*, it is thence reflected in the line *cd*, in which direction it is impossible for it to enter the eye at *f*: but by being again reflected from the point *d* of the drop, it goes on in the drop to *e*, where it passes out of the drop into the air, and is there refracted downward to the eye, in the direction *ef*. Fig. 8.

L E C T. VIII.

The description and use of the globes, and armillary sphere.

The terrestrial globe.

IF a map of the world be accurately delineated on a spherical ball, it will be a true representation of the earth: for the highest hills are so inconsiderable with respect to the bulk of the earth, that they take off no more from its roundness, than grains of sand do from the roundness of a common globe; for the diameter of the earth is 8000 miles, in round numbers, and no known hill upon it is three miles in perpendicular height.

Proof of the earth's being globular.

That the earth is spherical, or round like a globe, appears, 1. from its casting a round shadow upon the moon, whatever side be turned towards her when she is eclipsed. 2. From its having been sailed round by several persons. 3. From our seeing the farther, the higher we stand. 4. From our seeing the masts of a ship, whilst the hull is hid by the convexity of the water.

And that it may be peopled on all sides without any one's being in danger of falling away from it.

The attractive power of the earth draws all terrestrial bodies towards its center; as is evident from the descent of heavy bodies in lines perpendicular to the earth's surface, at the places whereon they fall; even when they are thrown off from the earth on opposite sides, and consequently, in opposite directions. So that the earth may be compared to a great magnet rolled in filings of steel, which attracts and keeps them equally fast to its surface

ſurface on all ſides. Hence, as all terreſtrial bodies are attracted toward the earth's center, they can be in no danger of falling from the earth on any one part, or ſide of the earth, more than from any other.

Up and *down*, what?

The heaven or ſky ſurrounds the whole earth: and when we ſpeak of *up* or *down*, we mean only with regard to ourſelves; for no point, either in the heaven, or on the ſurface of the earth, is *above* or *below*, but only with reſpect to us. And let us be upon what part of the earth we will, we ſtand with our feet toward its center, and our heads toward the ſky: and ſo we ſay, it is *up* toward the ſky, and *down* toward the center of the earth.

All objects in the heaven appear equally diſtant.

To an obſerver placed any where in an indefinite ſpace, where there is nothing to limit his view, all objects that are very remote appear equally diſtant from him; and ſeem to be placed in a vaſt concave ſphere, of which his eye is the center. Every aſtronomer can demonſtrate, that the moon is much nearer to us than the ſun is; that ſome of the planets are ſometimes nearer to us, and ſometimes farther from us, than the ſun; that others of them never come ſo near us as the ſun always is; that the remoteſt planet in our ſyſtem, is beyond compariſon nearer to us than any of the fixed ſtars are; and that it is highly probable ſome ſtars are, in a manner, infinitely more diſtant from us than others. And yet, all theſe celeſtial objects appear equally diſtant from us. Therefore, if we imagine a large hollow ſphere of glaſs, to have as many golden ſtuds fixed to its inſide, as there are

The face of the heaven and earth reprefented in a machine.

ftars vifible in the heavens, and thefe ftuds to be of different magnitudes, and placed at the fame angular diftances from each other as the ftars are; it will be a juft reprefentation of the ftarry heavens, to an eye fuppofed to be in its center, and viewing it all around. And if a fmall globe, with a map of the earth upon it, be placed on an axis in the center of this ftarry fphere, and the fphere be made to turn round on this axis, it will reprefent the apparent motion of the heavens round the earth.

The *equinoctial.*

If a great circle be fo drawn upon this fphere, as to divide it into two equal parts, or hemifpheres, and the plane of the circle be perpendicular to the axis of the fphere, this will reprefent the *equinoctial circle*, which divides the heaven into two equal parts, called the *northern* and the *fouthern hemifpheres*; and every point of that circle will be equally diftant from the *poles*, or ends of the axis in the fphere. That pole which is in the middle of the northern hemifphere, will be called the *north pole of the fphere*, and that which is in the middle of the fouthern hemifphere, the *fouth pole.*

The *poles.*

The *ecliptic.*

If another great circle be drawn upon the fphere, in fuch a manner as to cut the equinoctial at an angle of $23\frac{1}{2}$ degrees; in two oppofite points, it will reprefent the *ecliptic*, or circle of the fun's apparent annual motion: one half of which is on the north fide of the equinoctial, and the other half on the fouth.

If a large ftud be made to move eaftward in this ecliptic, in fuch a manner as to go quite round it, in

in the time that the ſphere is turned round weſtward, 366 times upon its axis; this ſtud will repreſent the *ſun*, changing his place every day a 365th part of the ecliptic; and going round weſtward, the ſame way as the ſtars do; but with a motion ſo much ſlower than the motion of the ſtars, that they will make 366 revolutions about the axis of the ſphere, in the time that the ſun makes only 365. During one half of theſe revolutions, the ſun will be on the north ſide of the equinoctial; during the other half, on the ſouth; and at the end of each half, in the equinoctial. The *ſun*.

If we ſuppoſe the terreſtrial globe, in this machine, to be about one inch in diameter, and the diameter of the ſtarry ſphere to be about five or ſix feet; a ſmall inſect on the globe would ſee only a very little portion of its ſurface; but would ſee one half of the ſtarry ſphere; the convexity of the globe hiding the other half from his view. If the ſphere be turned weſtward round the globe, and the inſect could judge of the appearances which ariſe from that motion, he would ſee ſome ſtars riſing to his view in the eaſtern ſide of the ſphere, whilſt others were ſetting from his view on the weſtern: but as all the ſtars are fixed to the ſphere, the ſame ſtars would always riſe in the ſame points of his view on the eaſt ſide, and ſet in the ſame points of his view on the weſt ſide. With the ſun it would be otherwiſe, becauſe the ſun is not fixed to any point of the ſphere, but moves ſlowly along an oblique circle in it. And if the inſect ſhould look towards the ſouth, and call that point of the The *earth*. The apparent motion of the heavens.

globe, where the equinoctial in the ſphere ſeems to cut it on the left ſide, the *eaſt point*; and where it cuts the globe on the right ſide, the *weſt point*; the little animal would ſee the ſun riſe north of the eaſt, and ſet north of the weſt, for $182\frac{1}{2}$ revolutions; after which, for as many more, the ſun would riſe ſouth of the eaſt, and ſet ſouth of the weſt. And in the whole 365 revolutions, the ſun would riſe only twice in the eaſt point, and ſet twice in the weſt. All theſe appearances would be the ſame, if the ſtarry ſphere ſtood ſtill, (the ſun only moving in the ecliptic) and the earthly globe were turned round the axis of the ſphere eaſtward. For, as the inſect would be carried round with the globe, he would be quite inſenſible of its motion; and the ſun and ſtars would appear to move weſtward.

We are but very ſmall beings compared with our earthly globe, and *it* is but a dimenſionleſs point compared with the magnitude of the ſtarry heavens. Whether the earth be at reſt, and the heaven turns round it, or the heaven be at reſt, and the earth turns round, the appearance to us will be exactly the ſame. And, becauſe the heaven is ſo immenſely large, in compariſon of the earth, we ſee one half of the heaven as well from the earth's ſurface, as we could do from its center.

Circles of the ſphere. We may imagine as many circles deſcribed upon the earth as we pleaſe; and we may imagine the plane of any circle, deſcribed upon the earth, to be continued until it marks a circle in the concave ſphere of the heavens.

The

The *horizon* is either *ſenſible* or *rational.* The *ſenſible* horizon is that circle which a man, ſtanding upon a large plain, obſerves to terminate his view all around, where the heaven and earth ſeem to meet. The plane of our ſenſible horizon continued to the heaven, divides it into two hemiſpheres; one viſible to us, the other hid by the convexity of the earth. The *hori-zon.*

The plane of the *rational horizon*, is ſuppoſed parallel to the plane of the ſenſible; to paſs through the center of the earth, and to be continued to the heavens. And although the plane of the ſenſible horizon touches the earth in the plane of the obſerver, yet its plane, and that of the rational horizon, will ſeem to coincide in the heavens, becauſe the whole earth is but a point compared to the ſphere of the heavens.

The earth being a ſpherical body, the horizon, or limit of our view, muſt change as we change our place.

The *poles of the earth*, are thoſe two points on its ſurface in which its axis terminates. The one is called the *north pole*, and the other the *ſouth pole*. *Poles.*

The *poles of the heaven*, are the two points in which the earth's axis produced terminates in the heaven: ſo that the *north pole* of the heaven is directly over the north pole of the earth; and the *ſouth pole* of the heaven is directly over the ſouth pole of the earth.

The *equator* is a great circle upon the earth, every part of which is equally diſtant from either of the poles. It divides the earth into two equal parts, *Equator.*

parts, called the *northern* and *ſouthern hemiſpheres.* If we ſuppoſe the plane of this circle extended to the heaven, it will mark the *equinoctial* therein; and divide the heaven into two equal parts, called the *northern* and *ſouthern* hemiſpheres of the heavens.

Meridian. The *meridian* of any place is a great circle paſſing through that place and the poles of the earth. We may imagine as many ſuch meridians as we pleaſe, becauſe any place that is ever ſo little to the eaſt or weſt of any other place, muſt have a different meridian from it: for no one circle can paſs through any two ſuch places and the poles of the earth.

The meridian of any place is divided by the poles, into two ſemicircles: that which paſſes through the place is called the *geographical*, or *upper meridian*: and that which paſſes through the oppoſite place, is called the *lower meridian.*

Noon, and midnight. When the rotation of the earth brings the plane of the geographical meridian to the ſun, it is *noon* or *mid-day* to that place; and when the lower meridian comes to the ſun, it is *midnight.*

All places lying under the ſame geographical meridian, have their noon at the ſame time, and conſequently all the other hours. All thoſe places are ſaid to have the ſame *longitude*, becauſe no one of them lies either eaſtward or weſtward from any of the reſt.

Hour circles. If we imagine 24 ſemicircles, one of which is the geographical meridian of a given place, to meet at the poles, and to divide the equator into

24

24 equal parts; each of these will come round to the sun in 24 hours, by the earth's equable motion round its axis in that time. And, as the equator contains 360 degrees, there will be 15 degrees contained between any two of these semicircles which are nearest to one another: for 24 times 15 is 360. And as the earth's motion is eastward, the sun's apparent motion will be westward, at the rate of 15 degrees each hour. Therefore,

They whose geographical meridian is 15 degrees eastward from us, have noon, and every other hour, an hour sooner than we have. They whose meridian is 15 degrees westward from us, have noon, and every other hour, an hour later than we have: and so on in proportion, reckoning one hour for every 15 degrees. ***Longitude.***

As the earth turns round its axis once in 24 hours, and shews itself all round to the sun in that time; so it goes round the sun once a year, in a great circle called the *ecliptic*, which crosses the equinoctial in two opposite points, making an angle of $23\frac{1}{2}$ degrees with the equinoctial on each side. So that one half of the ecliptic is in the northern hemisphere, and the other in the southern. It contains 360 equal parts, called degrees, (as all other circles do, whether great or small) and as the earth goes once round it every year, the sun will appear to do the same, changing his place almost a degree, at a mean rate, every 24 hours. So that whatever place or degree of the ecliptic the earth is in, the sun will appear in the opposite. And as one half of the ecliptic is on the north side of the *Ecliptic.*

the

the equinoctial, and the other half on the south; the sun, as seen from the earth, will be half a year on the south side of the equinoctial, and half a year on the north: and twice a year in the equinoctial itself.

Signs and *degrees.*

The ecliptic is divided by astronomers into 12 equal parts, called *signs*, each sign into 30 *degrees*, and each degree into 60 *minutes*: but in using the globes, we seldom want the sun's place nearer than half a degree of the truth.

The names and characters of the 12 signs, are as follow; beginning at that point of the ecliptic where it crosses the equinoctial to the northward, and reckoning eastward round to the same point again. And the days of the months on which the sun now enters the signs, are set down below them.

Aries,	*Taurus,*	*Gemini,*	*Cancer,*	*Leo,*	*Virgo,*
♈	♉	♊	♋	♌	♍
March,	April,	May,	June,	July,	August,
20	20	21	21	23	23
Libra,	*Scorpio,*	*Sagittarius,*	*Capricorn,*	*Aquarius,*	*Pisces,*
♎	♏	♐	♑	♒	♓
Septemb.	Octob.	Novemb.	Decemb.	January,	Feb.
23	23	22	21	20	18

By remembering on what day the sun enters any particular sign, we may easily find his place any day afterward, whilst he is in that sign, by reckoning a degree for a day: which will occasion no error of consequence in using the globes.

When the sun is at the beginning of *aries*, he is in the equinoctial; and from that time he declines northward

northward every day, until he comes to the beginning of *cancer*, which is 23½ degrees from the equinoctial. From thence he recedes ſouthward every day, for half a year; in the middle of which half, he croſſes the equinoctial at the beginning of *libra*; and at the end of that half year, he is at his greateſt ſouth declination, in the beginning of *capricorn*, which is alſo 23½ degrees from the equinoctial. Then, he recedes northward from capricorn every day, for half a year; in the middle of which half, he croſſes the equinoctial at the beginning of *aries*; and at the end of it he arrives at *cancer*.

The ſun's motion in the ecliptic is not perfectly equable, for he continues eight days longer in the northern half of the ecliptic, than in the ſouthern: ſo that the ſummer half year, in the northern hemiſphere, is eight days longer than the winter half year, and the contrary in the ſouthern hemiſphere.

The *tropics* are leſſer circles in the heavens, parallel to the equinoctial; one on each ſide of it, touching the ecliptic in the points of its greateſt declination; ſo that each tropic is 23½ degrees from the equinoctial, one on the north ſide of it, and the other on the ſouth. The northern tropic touches the ecliptic at the beginning of cancer, the ſouthern, at the beginning of capricorn; for which reaſon, the former is called the *tropic of cancer*, and the latter the *tropic of capricorn*. *Tropics.*

The *polar circles* in the heavens, are each 23½ degrees from the poles, all around. That which goes *Polar circles.*

goes round the north pole, is called the *arctic circle*, from ἄρκτος, which signifies a *bear*; and there is a conſtellation or groupe of ſtars near the north pole, which goes by that name. The ſouth polar circle, is called the *antarctic circle*, from its being oppoſite to the arctic.

The ecliptic, tropics, and polar circles, are drawn upon the terreſtrial globe, as well as upon the celeſtial. But the ecliptic, being a great fixed circle in the heavens, cannot properly be ſaid to belong to the terreſtrial globe; and is laid down upon it only for the conveniency of ſolving ſome problems.

In order to form a true idea of the earth's motion round its axis every 24 hours, which is the cauſe of day and night; and of its motion in the ecliptic, round the ſun every year, which is the cauſe of the different lengths of days and nights, and viciſſitude of ſeaſons; take the following method, which will be both eaſy and pleaſant.

An idea of the ſeaſons.

Let a ſmall terreſtrial globe, of about 3 inches diameter, be ſuſpended by a long thread of twiſted ſilk, fixt to its north pole: then, having placed a lighted candle on a table, to repreſent the ſun, in the center of a hoop of a large caſk, which may repreſent the ecliptic, the hoop making an angle of $23\frac{1}{2}$ degrees with the plane of the table; hang the globe within the hoop, near to it; and if the table be level, the equator of the globe will be parallel to the table, and the plane of the hoop will cut the equator at an angle of $23\frac{1}{2}$ degrees; ſo that one half of the equator will be above the

hoop,

hoop, and the other half below it: and the candle will enlighten one half of the globe, as the ſun enlightens one half of the earth, whilſt the other half is in the dark.

Things being thus prepared, twiſt the thread towards the left hand, that it may turn the globe the ſame way by untwiſting; that is from weſt, by ſouth, to eaſt. As the globe turns round its axis or thread, the different places on its ſurface will go regularly through the light and dark; and have, as it were, an alternate return of day and night in each rotation. As the globe continues to turn round, and ſhew itſelf all around to the candle, carry it ſlowly round the hoop by the thread, from weſt, by ſouth, to eaſt; which is the way that the earth moves round the ſun, once a year, in the ecliptic: and you will ſee, that whilſt the globe continues in the lower part of the hoop, the candle (being then north of the equator) will conſtantly ſhine round the north pole; and all the northern places, which go through any part of the dark, will go through a leſs portion of it than they do of the light; and the more ſo, the farther they are from the equator: conſequently, their days are then longer than their nights. When the globe comes to a point in the hoop, mid-way between the higheſt and loweſt points, the candle will be directly over the equator, and enlighten the globe juſt from pole to pole; and then, every place on the globe will go through equal portions of light and darkneſs, as it turns round its axis; and conſequently, the day and night will be of

equal length at all places upon it. As the globe advances thence, toward the higheſt part of the hoop, the candle will be on the ſouth ſide of the equator, ſhining farther and farther round the ſouth pole, as the globe riſes higher and higher in the hoop; leaving the north pole as much in darkneſs as the ſouth pole is in the light, and making long days and ſhort nights on the ſouth ſide of the equator, and the contrary on the north ſide, whilſt the globe continues in the northern or higher ſide of the hoop: and when it comes to the higheſt point, the days will be at the longeſt, and nights at the ſhorteſt, in the ſouthern hemiſphere; and the reverſe in the northern. As the globe advances and deſcends in the hoop, the light will gradually recede from the ſouth pole, and approach towards the north pole, which will cauſe the northern days to lengthen, and the ſouthern days to ſhorten in the ſame proportion. When the globe comes to the middle point, between the higheſt and loweſt points of the hoop, the candle will be over the equator, enlightening the globe juſt from pole to pole, when every place of the earth (except the poles) will go through equal portions of light and darkneſs; and conſequently, the day and night will be then equal, all over the globe.

And thus, at a very ſmall expence, one may have a delightful and demonſtrative view of the cauſe of days and nights, with their gradual increaſe and decreaſe in length, through the whole year; together with the viciſſitudes of ſpring,

ſummer,

ſummer, autumn, and winter, in each annual courſe of the earth round the ſun.

If the hoop be divided into 12 equal parts, and the ſigns be marked in order upon it, beginning with *cancer* at the higheſt point of the hoop, and reckoning eaſtward (or contrary to the apparent motion of the ſun) you will ſee how the ſun appears to change his place every day in the ecliptic, as the globe advances eaſtward along the hoop, and turns round its own axis: and that when the earth is in a low ſign, as at *capricorn*, the ſun muſt appear in a high ſign, as at *cancer*, oppoſite to the earth's real place: and that whilſt the earth is in the ſouthern half of the ecliptic, the ſun appears in the northern half, and *vice verſâ*: that the farther any place is from the equator, between it and the polar circle, the greater is the difference between the longeſt and ſhorteſt day at that place; and that the poles have but one day and one night in the whole year.

Theſe things premiſed, we ſhall proceed to the deſcription and uſe of the terreſtrial globe, and explain the geographical terms as they occur in the problems.

The *terreſtrial globe* deſcribed.

This globe has the boundaries of land and water laid down upon it, the countries and kingdoms divided by dots, and coloured to diſtinguiſh them, the iſlands properly ſituated, the rivers and principal towns inſerted, as truly as they have been aſcertained upon the earth by meaſurement and obſervation.

The equator, ecliptic, tropics, polar circles, and meridians, are laid down upon the globe in the manner already described. The ecliptic is divided into 12 signs, and each sign into 30 degrees, which are generally subdivided into halves, and into quarters if the globe is large. Each tropic is $23\frac{1}{2}$ degrees from the equator, and each polar circle $23\frac{1}{2}$ degrees from its respective pole. Circles are drawn parallel to the equator, at every ten degrees distance from it on each side, to the poles: these circles are called *parallels of latitude.* On large globes there are circles drawn perpendicularly through every tenth degree of the equator, intersecting each other at the poles: but on globes of or under a foot diameter, they are only drawn through every fifteenth degree of the equator: these circles are generally called *meridians*, sometimes *circles of longitude*, and at other times *hour-circles.*

The globe is hung in a brass ring, called the *brasen meridian*; and turns upon a wire in each pole, sunk half its thickness into one side of the meridian-ring; by which means, that side of the ring divides the globe into two equal parts, called the *eastern* and *western hemispheres*; as the equator divides it into two equal parts, called the *northern* and *southern hemispheres.* This ring is divided into 360 equal parts or degrees, on the side wherein the axis of the globe turns. One half of these degrees are numbered, and reckoned, from the equator to the poles, where they end at 90: their use is to shew the latitudes of places. The degrees on the other half of the meridian-ring, are numbered

from

from the poles to the equator, where they end at 90: their uſe is to ſhew how to elevate either the north or ſouth pole above the horizon, according to the latitude of any given place, as it is north or ſouth of the equator.

The braſen meridian is let into two notches made in a broad flat ring, called the *wooden horizon*, the upper ſurface of which divides the globe into two equal parts, called the *upper* and *lower hemiſpheres*. One notch is in the north point of the horizon, and the other in the ſouth. On this horizon, are ſeveral concentric circles, which contain the months and days of the year, the ſigns and degrees anſwering to the ſun's place for each month and day, and the 32 points of the compaſs.—The graduated ſide of the braſs meridian lies toward the eaſt ſide of the horizon, and ſhould be generally kept toward the perſon who works problems by the globes.

There is a ſmall *horary circle*, ſo fixed to the north part of the braſen meridian, that the wire in the north pole of the globe is in the center of that circle; and on the wire is an *index*, which goes over all the 24 hours of the circle, as the globe is turned round its axis. Sometimes there are two horary circles, one between each pole of the globe and the braſen meridian; which is the contrivance of the ingenious Mr. *Joſeph Harris*, maſter of the aſſay-office in the Tower of London; and makes it very convenient for putting the poles of the globe through the horizon, and for elevating the pole to ſmall latitudes and declinations of

the sun; which cannot be done where there is only one horary circle fixed to the outer edge of the brasen meridian.

There is a thin slip of brass, called the *quadrant of altitude*, which is divided into 90 equal parts or degrees, answering exactly to so many degrees of the equator. It is occasionally fixed to the uppermost point of the brasen meridian, by a nut and screw. The divisions end at the nut, and the quadrant is turned round upon it.

The globe being a machine which has been seen by most people, and upon the figure of which, in a plate, neither the circles, nor countries, can be properly expressed, we judge it would signify very little to refer to a figure of it; and shall therefore only give some directions how to choose a globe, and then describe its use.

Directions for choosing of globes.

1. See that the papers be well and neatly pasted on the globes, which you may know, if the lines and circles thereon meet exactly, and continue all the way even and whole; the circles not breaking into several arches, nor the papers either coming short, or lapping over one another.

2. See that the colours be transparent, and not laid too thick upon the globe to hide the names of places.

3. See that the globe hang evenly between the brasen meridian and the wooden horizon; not inclining either to one side or to the other.

4. See that the globe be as close to the horizon and meridian, as it conveniently may; otherwise you will be too much puzzled to find against what part

part of the globe any degree of the meridian or horizon is.

5. See that the equinoctial line be even with the horizon all around, when the north or south pole is elevated 90 degrees above the horizon.

6. See that the equinoctial line cuts the horizon in the east and west points, in all elevations of the pole, from 0 to 90 degrees.

7. See that the degrees of the brasen meridian, marked with 0 and 90, be exactly over the equinoctial line of the globe.

8. See that there be exactly half of the brasen meridian above the horizon; which you may know, if you bring any of the decimal divisions on the meridian to the north point of the horizon, and find their complement to 90 in the south point.

9. See that when the quadrant of altitude is placed at the highest point of the brasen meridian, the beginning of the degrees of the quadrant reaches just to the plane surface of the horizon.

10. See that whilst the index of the hour-circle (by the motion of the globe) passes from one hour to another, 15 degrees of the equator pass under the graduated edge of the brasen meridian.

11. See that the wooden horizon be made substantial and strong: it being generally observed, that in most globes, the horizon is the first part that fails, on account of its having been made too slight.

In using the globes, keep the east side of the horizon towards you (unless your problem requires the turning of it) which side you may know by the — Directions for using them.

the word EAST, upon the horizon: for then you have the graduated side of the meridian towards you, the quadrant of altitude before you, and the globe divided exactly into two equal parts, by the graduated side of the meridian.

In working some problems, it will be necessary to turn the whole globe and horizon about, that you may look on the west side thereof; which turning will be apt to jog the ball so, as to shift away that degree of the globe which was before set to the horizon or meridian: to avoid which inconvenience, you may thrust the feather-end of a quill in between the ball of the globe and the brasen meridian; which, without hurting the ball, will keep it from turning in the meridian, whilst you turn the west side of the horizon towards you.

PROBLEM I.

*To find the * latitude and † longitude of any given place upon the globe.*

Turn the globe upon its axis, until the given place comes exactly under that graduated side of the brasen meridian, on which the degrees are numbered

* The latitude of a place is its distance from the equator, and is north or south, as the place is north or south of the equator. Those who live at the equator have no latitude, because it is there that the latitude begins.

† The longitude of a place is the number of degrees (reckoned upon the equator) that the meridian of the said place

numbered from the equator; and observe what degree of the meridian the place then lies under; which is its latitude, north or south, as the place is north or south of the equator.

The globe remaining in this position, the degree of the equator, which is under the brasen meridian, is the longitude of the place; which is east or west, as the place lies on the east or west side of the first meridian of the globe.—All the *Atlantic Ocean*, and *America*, is on the west side of the meridian of *London*; and the greatest part of *Europe*, and of *Africa*, together with all *Asia*, is on the east side of the meridian of *London*, which is reckoned the *first meridian* of the globe by the *English* geographers and astronomers.

PROBLEM II.

The longitude and latitude of a place being given, to find that place on the globe.

Look for the given longitude in the equator, (counted eastward or westward from the first meridian, as it is mentioned east or west;) and bring

place is distant from the meridian of any other place from which we reckon, either eastward or westward, for 180 degrees, or half round the globe. The English reckon the longitude from the meridian of London, and the French now reckon it from the meridian of Paris. The meridian of that place, from which the longitude is reckoned, is called the *first meridian*. The places upon this meridian have no longitude, because it is there that the longitude begins.

the point of longitude in the equator to the brasen meridian, on that side which is above the south point of the horizon: then count from the equator, on the brasen meridian, to the degree of latitude given, towards the north or south pole, according as the latitude is north or south; and under that degree of latitude on the meridian, lies the place required.

PROBLEM III.

To find the difference of longitude, or difference of latitude, between any two given places.

Bring each place to the brasen meridian, and see what its latitude is: the lesser latitude subtracted from the greater, if both places are on the same side of the equator, or both latitudes added together, if they are on different sides of it, is the difference of latitude required. And the number of degrees contained between these places, reckoned on the equator, when they are brought separately under the brasen meridian, is their difference of longitude; if it be less than 180: but if more, let it be subtracted from 360, and the remainder is the difference of longitude required. Or,

Having brought one of the places to the brasen meridian, and set the hour index to XII, turn the globe until the other place comes to the meridian, and the number of hours and parts of an hour, past over by the index, will give the longitude in time; which may be easily reduced to degrees, by

allowing

allowing 15 degrees for every hour, and one degree for every four minutes.

N. B. When we speak of bringing any place to the brasen meridian, the graduated side of the meridian is meant.

PROBLEM IV.

Any place being given, to find all those places that have the same longitude or latitude with it.

Bring the given place to the brasen meridian, then all those places which lie under that side of the meridian, from pole to pole, have the same longitude with the given place. Turn the globe round its axis, and all those places which pass under the same degree of the meridian that the given place does, have the same latitude with that place.

Since all latitudes are reckoned from the equator, and all longitudes are reckoned from the first meridian, it is evident, that the point of the equator which is cut by the first meridian, has neither latitude nor longitude.—The greatest latitude cannot exceed 90 degrees, because no place is more than 90 degrees from the equator. And the greatest longitude cannot exceed 180 degrees, because no place is more than 180 degrees from the first meridian.

PROBLEM V.

To find the * antœci, † periœci, *and* ‡ antipodes, *of any given place.*

Bring the given place to the brasen meridian, and having found its latitude, keep the globe in that

* The *antœci* are those people who live on the same meridian, and in equal latitudes, on different sides of the equator. Being on the same meridian, they have the same hours; that is, when it is noon to the one, it is also noon to the other, and when it is midnight to the one, it is also midnight to the other, &c. Being on different sides of the equator, they have different or opposite seasons at the same time; the length of any day to the one, is equal to the length of the night of that day to the other; and they have equal elevations of the different poles.

† The *periœci* are those people who live on the same parallel of latitude, but on opposite meridians: so that though their latitude be the same, their longitude differs 180 degrees. By being in the same latitude, they have equal elevations of the same pole (for the elevation of the pole, is always equal to the latitude of the place), the same length of days or nights, and the same seasons. But being on opposite meridians, when it is noon to the one, it is midnight to the other.

‡ The *antipodes* are those who live diametrically opposite to one another upon the globe, standing with feet towards feet, on opposite meridians and parallels. Being on opposite sides of the equator, they have opposite seasons, winter to one, when it is summer to the other; being equally distant from the equator, they have the contrary poles equally elevated above the horizon; being on opposite meridians, when

that situation, and count the same number of degrees of latitude from the equator towards the contrary pole, and where the reckoning ends, you have the *antœci* of the given place, upon the globe. Those who live at the equator have no *antœci*.

The globe remaining in the same position, set the hour index to the upper XII on the horary circle, and turn the globe until the index comes to the lower XII; then, the place which lies under the meridian, in the same latitude with the given place, is the *periœci* required. Those who live at the poles have no *periœci*.

As the globe now stands, (with the index at the lower XII,) the *antipodes* of the given place will be under the same point of the brasen meridian where its *antœci* stood before. Every place upon the globe has its *antipodes*.

PROBLEM VI.

To find the distance between any two places on the globe.

Lay the graduated edge of the quadrant of altitude over both the places, and count the number of degrees intercepted between them on the quadrant; then multiply these degrees by 60, and the

when it is noon to the one, it must be midnight to the other; and as the sun recedes from the one when he approaches to the other, the length of the day to one must be equal to the length of the night at the same time to the other.

product

product will give the distance in geographical miles: but to find the distance in English miles, multiply the degrees by $69\frac{1}{2}$, and the product will be the number of miles required. Or, take the distance betwixt any two places with a pair of compasses, and apply that extent to the equator; the number of degrees, intercepted between the points of the compasses, is the distance in degrees of a great circle*; which may be reduced either to geographical miles, or to English miles, as above.

Great circle. * Any circle that divides the globe into two equal parts, is called a *great circle*, as the equator or meridian. Any circle that divides the globe into two unequal parts, (which *Lesser circle.* every parallel of latitude does) is called a *lesser circle*. Now, as every circle, whether great or small, contains 360 degrees, and a degree upon the equator or meridian contains 60 geographical miles, it is evident, that a degree of longitude upon the equator, is longer than a degree of longitude upon any parallel of latitude, and must therefore contain a greater number of miles. So that, although all the degrees of latitude are equally long upon an artificial globe, (though not precisely so upon the earth itself) yet the degrees of longitude decrease in length, as the latitude increases, but not in the same proportion. The following table shews the length of a degree of longitude, in geographical miles, and hundredth parts of a mile, for every degree of latitude, from the equator to the poles: a degree on the equator being 60 geographical miles.

The number of miles in a degree of longitude, in all degrees of latitude.

Deg.	Miles. Parts.	Deg.	Miles. Parts.	Deg.	Miles. Parts.	Deg.	Miles. Parts.	Deg.	Miles. Parts.
1	59.99	19	56.73	37	47.92	55	34.41	73	17.54
2	59.96	20	56.38	38	47.28	56	33.55	74	16.53
3	59.92	21	56.02	39	46.63	57	32.68	75	15.52
4	59.85	22	55.63	40	45.97	58	31.79	76	14.31
5	59.77	23	55.23	41	45.28	59	30.90	77	13.50
6	59.67	24	54.81	42	44.59	60	30.00	78	12.48
7	59.56	25	54.38	43	43.88	61	29.09	79	11.45
8	59.42	26	53.93	44	43.16	62	28.17	80	10.42
9	59.26	27	53.46	45	42.43	63	27.24	81	9.38
10	59.09	28	52.96	46	41.68	64	26.30	82	8.35
11	58.89	29	52.47	47	40.92	65	25.36	83	7.32
12	58.69	30	51.96	48	40.15	66	24.41	84	6.28
13	58.46	31	51.43	49	39.36	67	23.44	85	5.24
14	58.22	32	50.88	50	38.57	68	22.48	86	4.20
15	58.95	33	50.32	51	37.76	69	21.50	87	3.15
16	57.67	34	49.74	52	36.94	70	20.52	88	2.10
17	57.38	35	49.15	53	36.11	71	19.53	89	1.05
18	57.06	36	48.54	54	35.27	72	18.54	90	0.00

PROBLEM VII.

A place on the globe being given, and its true distance from any other place, to find all the other places upon the earth which are at the same distance from the given place.

Bring the given place to the brasen meridian, and screw the quadrant of altitude to the meridian, directly over that place; then keeping the globe in that position, turn the quadrant quite round

round upon it, and the degree of the quadrant that touches the ſecond place, will paſs over all the other places which are equally diſtant with it from the given place.

This is the ſame as if one foot of a pair of compaſſes was ſet in the given place, and the other foot extended to the ſecond place, whoſe diſtance is known; for if the compaſſes be then turned round the firſt place as a center, the moving foot will go over all thoſe places which are at the ſame diſtance with the ſecond from it.

PROBLEM VIII.

The hour of the day at any place being given, to find all thoſe places where it is noon at that time.

Bring the given place to the braſen meridian, and ſet the index to the given hour; this done, turn the globe until the index points to the upper XII, and then, all the places that lie under the braſen meridian have noon at that time.

N. B. The upper XII always ſtands for noon; and when the bringing of any place to the braſen meridian is mentioned, the ſide of that meridian on which the degrees are reckoned from the equator is meant, unleſs the contrary ſide be mentioned.

PROBLEM IX.

The hour of the day at any place being given, to find what o'clock it is at that time at any other place.

Bring the given place to the brasen meridian, and set the index to the given hour; then turn the globe, until the place where the hour is required comes to the meridian, and the index will point out the hour at that place.

PROBLEM X.

*To find the sun's place in the ecliptic, and his * declination, for any given day of the year.*

Look on the horizon for the given day, and right against it thereon, you have the degree of the sign in which the sun is (or his place) on that day at noon. Find the same degree of that sign in the ecliptic line upon the globe, and having brought it to the brasen meridian, observe what degree of the meridian stands over it; for that is the sun's declination, reckoned from the equator.

* The sun's declination is his distance from the equinoctial in degrees, and is north or south, as the sun is between the equinoctial and the north or south pole.

PROBLEM

PROBLEM XI.

The day of the month being given, to find all those places of the earth over which the sun will pass vertically on that day.

Find the sun's place in the ecliptic for the given day, and having brought it to the brasen meridian, observe what degree of the meridian is over it; then, turning the globe round its axis, all those places which pass under that degree of the meridian, are the places required: for as their latitude is equal in degrees to the sun's declination, he must be directly over head to each of them at its respective noon.

PROBLEM XII.

*A place being given in the * torrid zone, to find those two days of the year, on which the sun shall be vertical to that place.*

Bring the given place to the brasen meridian, and mark the degree of latitude that is exactly over it

* The globe is divided into five zones; one torrid, two temperate, and two frigid. The *torrid zone* lies between the two tropics, and is 47 degrees in breadth, or $23\frac{1}{2}$ on each side of the equator: the *temperate zones* lie between the tropics and polar circles, or from $23\frac{1}{2}$ degrees of latitude,

it on the meridian; then turn the globe round its axis, and obferve the two points of the ecliptic which pafs exactly under that degree of latitude: laftly, find on the wooden horizon, the two days of the year in which the fun is in thofe points or degrees of the ecliptic, and they are the days required: for on them, and none elfe, the fun's declination is equal to the latitude of the given place.

PROBLEM XIII.

To find all thofe places of the north frigid zone, where the fun begins to fhine conftantly without fetting, on any given day, from the 21ft of March to the 23d of September.

On thefe two days, the fun is in the equinoctial, and enlightens the globe exactly from pole to pole: therefore, as the earth turns round its axis, which terminates in the poles, every place upon it will go equally through the light and the dark, and fo make equal day and night to all places of the earth. But as the fun declines from the equator, towards either pole, he will fhine juft as many degrees round that pole, as are equal to his decli-

tude, to $66\frac{1}{2}$, on each fide of the equator; and are each 43 degrees in breadth: the *frigid zones* are the fpaces included within the polar circles, which being each $23\frac{1}{2}$ degrees from their refpective poles, the breadth of each of thefe zones is 47 degrees. As the fun never goes without the tropics, he muft every moment be vertical to fome place or other in the torrid zone.

nation from the equator; ſo that no place within that diſtance of the pole will then go through any part of the dark, and conſequently the ſun will not ſet to it. Now, as the ſun's declination is northward, from the 21ſt of March to the 23d of September, he muſt conſtantly ſhine round the north pole all that time; and on the day that he is in the northern tropic, he ſhines upon the whole north frigid zone; ſo that no place within the north polar circle goes through any part of the dark on that day. Therefore,

Having brought the ſun's place for the given day to the braſen meridian, and found his declination, (by Prob. 9.) count as many degrees on the meridian, from the north pole, as are equal to the ſun's declination from the equator, and mark that degree from the pole where the reckoning ends: then, turning the globe round its axis, obſerve what places in the north frigid zone paſs directly under that mark; for they are the places required.

The like may be done for the ſouth frigid zone, from the 23d of September to the 21ſt of March, during which time the ſun ſhines conſtantly on the ſouth pole.

PROBLEM XIV.

To find the place over which the ſun is vertical, at any hour of a given day.

Having found the ſun's declination for the given day, (by Prob. 9.) mark it with a chalk on the braſen

brasen meridian: then bring the place where you are (suppose London) to the brasen meridian, and set the index to the given hour; which done, turn the globe on its axis, until the index points to XII at noon; and the place on the globe, which is then under the point of the sun's declination marked upon the meridian, has the sun that moment in the zenith, or directly over head.

PROBLEM XV.

The day and hour at any place being given, to find all those places where the sun is then rising, or setting, or on the meridian: consequently, all those places which are enlightened at that time, and those which are in the dark.

This problem cannot be solved by any globe fitted up in the common way, with the hour circle fixed upon the brass meridian; unless the sun be on or near some of the tropics on the given day. But with a globe fitted up according to Mr. *Joseph Harris*'s invention, (already mentioned, page 275) where the hour circle lies on the surface of the globe, below the meridian, it may be solved for any day of the year, according to his method; which is as follows.

Having found the place to which the sun is vertical at the given hour, if the place be in the northern hemisphere, elevate the north pole as many degrees above the horizon, as are equal to the latitude of that place; if the place be in the southern

hemisphere, elevate the south pole accordingly; and bring the said place to the brasen meridian. Then, all those places which are in the western semicircle of the horizon, have the sun rising to them at that time; and those in the eastern semicircle have it setting: to those under the upper semicircle of the brass meridian, it is noon; and to those under the lower semicircle, it is midnight. All those places which are above the horizon, are enlightened by the sun, and have the sun just as many degrees above them, as they themselves are above the horizon: and this height may be known, by fixing the quadrant of altitude on the brasen meridian over the place to which the sun is vertical; and then, laying it over any other place, observe what number of degrees on the quadrant are intercepted between the said place and the horizon. In all those places that are 18 degrees below the western semicircle of the horizon, the morning twilight is just beginning; in all those places that are 18 degrees below the eastern semicircle of the horizon, the evening twilight is ending; and all those places that are lower than 18 degrees, have dark night.

If any place be brought to the upper semicircle of the brasen meridian, and the hour index be set to the upper XII or noon, and then the globe be turned round its axis; when any place comes to the western semicircle of the horizon, the index will shew the time of sun-rising at that place; and if the place be brought to the eastern semicircle of the horizon, the index will shew the time of sun-set.

To

To those places which do not go under the horizon, the sun sets not on that day; and to those which do not come above it, he does not rise.

PROBLEM XVI.

The day and hour of a lunar eclipse being given; to find all those places of the earth to which it will be visible.

The moon is never eclipsed but when she is full, and so directly opposite to the sun, that the earth's shadow falls upon her. Therefore, whatever place of the earth the sun is vertical to at that time, the moon must be vertical to the antipodes of that place: so that the sun will be then visible to one half of the earth, and the moon to the other.

Find the place to which the sun is vertical at the given hour, (by Prob. 14;) elevate the pole to the latitude of that place, and bring the place to the upper part of the brasen meridian, as in the former problem: then, as the sun will be visible to all those parts of the globe which are above the horizon, the moon will be visible to all those parts which are below it, at the time of her greatest obscuration.

But with regard to an eclipse of the sun, there is no such thing as shewing to what places it will be visible, with any degree of certainty, by a common globe; because the moon's shadow covers but a small portion of the earth's surface, and her latitude, or declination from the ecliptic, throws her

ſhadow ſo variouſly upon the earth, that to determine the places on which it falls, recourſe muſt be had to long calculations.

PROBLEM XVII.

*To rectify the globe for the latitude, the * zenith, and the ſun's place.*

Find the latitude of the place (by Prob. I.) and if the place be in the northern hemiſphere, raiſe the north pole above the north point of the horizon, as many degrees (counted from the pole upon the braſen meridian) as are equal to the latitude of the place. If the place be in the ſouthern hemiſphere, raiſe the ſouth pole above the ſouth point of the horizon, as many degrees as are equal to the latitude. Then, having brought the place to its latitude on the braſen meridian, faſten the quadrant of altitude ſo, that the chamfered edge of its nut (which is even with the graduated edge) may be joined to the zenith, or point of latitude. This done, bring the ſun's place in the ecliptic for the given day (found by Prob. 10.) to the graduated ſide of the braſen meridian, and ſet the hour index to XII at noon, which is the uppermoſt XII on the hour circle; and the globe will be rectified.

* The *zenith*, in this ſenſe, is the higheſt point of the braſen meridian above the horizon; but in the proper ſenſe, it is that point of the heavens which is directly vertical to any given place.

The

The latitude of any place, is equal to the elevation of the nearest pole of the heaven above the horizon of that place; and the poles of the heaven are directly over the poles of the earth, each 90 degrees from the equinoctial line. Let us be upon what place of the earth we will, if the limits of our view be not intercepted by hills, we see one half of the heaven, or 90 degrees every way round, from that point which is over our heads. Therefore, if we were upon the equator, the poles of the heaven would lie in our horizon, or limit of view: if we go from the equator, towards either pole of the earth, we shall see the corresponding pole of the heaven rising gradually above our horizon, just as many degrees as we have gone from the equator: and if we were at either of our earth's poles, the corresponding pole of the heaven would be directly over our head. Consequently, the elevation or height of the pole in degrees above the horizon, is always equal to the number of degrees that the place is from the equator. Remark.

PROBLEM XVIII.

The latitude of any place, not exceeding * $66\frac{1}{2}$ *degrees, and the day of the month, being given; to find the time of sun-rising and setting, and consequently the length of the day and night.*

Having rectified the globe for the latitude, and the sun's place on the given day, (as directed in the

* All places whose latitude is more than $66\frac{1}{2}$ degrees, are in the frigid zones: and to those places the sun does not set

the preceding problem) bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon, and the hour index will ſhew the time of ſun-riſing; then turn the globe on its axis, until the ſun's place comes to the weſtern ſide of the horizon, and the index will ſhew the time of ſun-ſetting.

The hour of ſun-ſetting doubled, gives the length of the day; and the hour of ſun-riſing doubled, gives the length of the night.

PROBLEM XIX.

The latitude of any place, and the day of the month, being given; to find when the morning twilight begins, and the evening twilight ends, at that place.

This problem is often limited: for, when the ſun does not go 18 degrees below the horizon, the twilight continues the whole night; and for ſeveral nights together in ſummer, between 49 and $66\frac{1}{2}$ degrees of latitude: and the nearer to $66\frac{1}{2}$, the greater is the number of theſe nights. But when it does begin and end, the following method will ſhew the time for any given day.

Rectify the globe, and bring the ſun's place in the ecliptic to the eaſtern ſide of the horizon; then mark that point of the ecliptic with a chalk which is in the weſtern ſide of the horizon, it being the point oppoſite to the ſun's place: this done,

ſet in ſummer, for a certain number of diurnal revolutions; which occaſions this limitation of latitude.

lay

lay the quadrant of altitude over the said point, and turn the globe eastward, keeping the quadrant at the chalk-mark, until it be just 18 degrees high on the quadrant; and the index will point out the time that the morning twilight begins: for the sun's place will then be 18 degrees below the eastern side of the horizon. To find the time when the evening twilight ends, bring the sun's place to the western side of the horizon, and the point opposite to it, which was marked with the chalk, will be rising in the east: then, bring the quadrant over that point, and keeping it thereon, turn the globe westward, until the said point be 18 degrees above the horizon on the quadrant, and the index will shew the time when the evening twilight ends; the sun's place being then 18 degrees below the western side of the horizon.

PROBLEM XX.

To find on what day of the year the sun begins to shine constantly without setting, on any given place in the north frigid zone; and how long he continues to do so.

Rectify the globe to the latitude of the place, and turn it about until some point of the ecliptic, between *aries* and *cancer*, coincides with the north point of the horizon where the brasen meridian cuts it: then find, on the wooden horizon, what day of the year the sun is in that point of the ecliptic; which is the day that the sun begins to shine

ſhine conſtantly on the given place, without ſetting. This done, turn the globe until ſome point of the ecliptic, between *cancer* and *libra*, coincides with the north point of the horizon, where the braſs meridian cuts it; and find, on the wooden horizon, on what day the ſun is in that point of the ecliptic; which is the day that the ſun leaves off conſtantly ſhining on the ſaid place, and riſes and ſets to it as to other places on the globe. The number of natural days, or compleat revolutions of the ſun about the earth, between the two days above found, is the time that the ſun keeps conſtantly above the horizon without ſetting; for all that portion of the ecliptic, which lies between the two points that interſect the horizon in the very north, never ſets below it: and there is juſt as much of the oppoſite part of the ecliptic that never riſes; therefore, the ſun will keep as long conſtantly below the horizon in winter, as above it in ſummer.

Whoever conſiders the globe, will find, that all places of the earth do equally enjoy the benefit of the ſun, in reſpect of time, and are equally deprived of it. For, the days and nights are always equally long at the equator: and in all places that have latitude, the days at one time of the year, are exactly equal to the nights at the oppoſite ſeaſon.

PROBLEM

PROBLEM XXI.

To find in what latitude the ſun ſhines conſtantly without ſetting, for any length of time leſs than * $182\frac{1}{2}$ *of our days and nights.*

Find a point in the ecliptic half as many degrees from the beginning of *cancer*, (either toward *aries* or *libra*) as there are † natural days in the time given; and bring that point to the north ſide of the braſen meridian, on which the degrees are numbered from the pole towards the equator: then, keep the globe from turning on its axis, and ſlide the meridian up or down, until the foreſaid point of the ecliptic comes to the north point of the horizon, and then, the elevation of the pole will be equal to the latitude required.

PROBLEM XXII.

The latitude of a place, not exceeding $66\frac{1}{2}$ *degrees, and the day of the month being given; to find the ſun's amplitude, or point of the compaſs on which he riſes or ſets.*

Rectify the globe, and bring the ſun's place to the eaſtern ſide of the horizon; then obſerve what

* The reaſon of this limitation is, that $182\frac{1}{2}$ of our days and nights make half a year, which is the longeſt time that the ſun ſhines without ſetting, even at the poles of the earth.

† A natural day, contains the whole 24 hours: an artificial day, the time that the ſun is above the horizon.

point

point of the compaſs on the horizon ſtands right againſt the ſun's place, for that is his amplitude at riſing. This done, turn the globe weſtward, until the ſun's place comes to the weſtern ſide of the horizon, and it will cut the point of his amplitude at ſetting. Or, you may count the riſing amplitude in degrees, from the eaſt point of the horizon, to that point where the ſun's place cuts it; and the ſetting amplitude, from the weſt point of the horizon, to the ſun's place at ſetting.

PROBLEM XXIII.

*The latitude, the ſun's place, and his * altitude, being given; to find the hour of the day, and the ſun's azimuth, or number of degrees that he is diſtant from the meridian.*

Rectify the globe, and bring the ſun's place to the given height upon the quadrant of altitude; on the eaſtern ſide of the horizon, if the time be in the forenoon; or the weſtern ſide, if it be in the afternoon: then, the index will ſhew the hour, and the number of degrees in the horizon, intercepted between the quadrant of altitude and the ſouth point, will be the ſun's true azimuth at that time.

N. B. Always when the quadrant of altitude is mentioned in working any problem, the graduated edge of it is meant.

* The ſun's altitude, at any time, is his height above the horizon at that time.

If

If this be done at sea, and compared with the sun's azimuth, as shewn by the compass, if they agree, the compass has no variation in that place: but if they differ, the compass does vary; and the variation is equal to this difference.

PROBLEM XXIV.

The latitude, hour of the day, and the sun's place, being given; to find the sun's altitude and azimuth.

Rectify the globe, and turn it until the index points to the given hour; then lay the quadrant of altitude over the sun's place in the ecliptic, and the degree of the quadrant cut by the sun's place is his altitude at that time above the horizon; and the degree of the horizon cut by the quadrant is the sun's azimuth, reckoned from the south.

PROBLEM XXV.

The latitude, the sun's altitude, and his azimuth being given; to find his place in the ecliptic, the day of the month, and hour of the day, though they had all been lost.

Rectify the globe for the latitude and * zenith, and set the quadrant of altitude to the given azi-

* By rectifying the globe for the zenith, is meant screwing the quadrant of altitude to the given latitude on the brass meridian.

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muth in the horizon; keeping it there, turn the globe on its axis until the ecliptic cuts the quadrant in the given altitude: that point of the ecliptic which cuts the quadrant there, will be the sun's place; and the day of the month answering thereto, will be found over the like place of the sun on the wooden horizon. Keep the quadrant of altitude in that position, and having brought the sun's place to the brasen meridian, and the hour index to XII at noon, turn back the globe, until the sun's place cuts the quadrant of altitude again, and the index will shew the hour.

Any two points of the ecliptic which are equidistant from the beginning of *cancer* or of *capricorn*, will have the same altitude and azimuth at the same hour, though the months be different; and therefore it requires some care in this problem, not to mistake both the month and the day of the month; to avoid which, observe, that from the 20th of March to the 21st of June, that part of the ecliptic which is between the beginning of *aries* and beginning of *cancer* is to be used: from the 21st of June to the 23d of September, between the beginning of *cancer* and beginning of *libra:* from the 23d of September to the 21st of December, between the beginning of *libra* and the beginning of *capricorn:* and from the 21st of December to the 20th of March, between the beginning of *capricorn* and beginning of *aries*. And as one can never be at a loss to know in what quarter of the year he takes the sun's altitude and azimuth, the above caution, with regard to the quarters of the

the ecliptic, will keep him right as to the month, and day thereof.

PROBLEM XXVI.

To find the length of the longest day at any given place.

If the place be on the north side of the equator, find its latitude by Prob. 1.) and elevate the north pole to that latitude; then, bring the beginning of *cancer* ♋ to the brasen meridian, and set the hour index to XII at noon. But if the given place be on the south side of the equator, elevate the south pole to its latitude, and bring the beginning of *capricorn* ♑ to the brass meridian, and the hour index to XII. This done, turn the globe westward, until the beginning of *cancer* or *capricorn* (as the latitude is north or south) comes to the horizon; and the index will then point out the time of sun-setting, for it will have gone over all the afternoon hours, between mid-day and sun-set; which length of time being doubled, will give the whole length of the day, from sun-rising to sun-setting. For, in all latitudes, the sun rises as long before mid-day as he sets after it.

PROBLEM XXVII.

To find in what latitude the longest day is of any given length, less than 24 hours.

If the latitude be north, bring the beginning of *cancer* to the brasen meridian, and elevate the north pole

pole to about $66\frac{1}{2}$ degrees; but if the latitude be south, bring the beginning of *capricorn* to the meridian, and elevate the south pole to about $66\frac{1}{2}$ degrees; because the longest day in north latitude, is when the sun is in the first point of *cancer*; and in south latitude, when he is in the first point of *capricorn*. Then set the hour index to XII at noon, and turn the globe westward, until the index points at half the number of hours given; which done, keep the globe from turning on its axis, and slide the meridian down in the notches, until the aforesaid point of the ecliptic (viz. *cancer* or *capricorn*) comes to the horizon; then, the elevation of the pole will be equal to the latitude required.

PROBLEM XXVIII.

*The latitude of any place, not exceeding $66\frac{1}{2}$ degrees, being given; to find in what * climate the place is.*

Find the length of the longest day at the given place, by Prob. 26. and whatever be the number of hours whereby it exceedeth twelve, double that

* A *climate*, from the equator to either of the polar circles, is a tract of the earth's surface, included between two such parallels of latitude, that the length of the longest day in the one exceeds that in the other by half an hour: but from the polar circles to the poles, where the sun keeps long above the horizon without setting, each climate differs a whole month from the one next to it. There are 24 climates between the equator and each of the polar circles; and six from each polar circle to its respective pole.

number,

number, and the sum will give the climate in which the place is.

PROBLEM XXIX.

The latitude, and the day of the month, being given; to find the hour of the day when the sun shines.

Set the wooden horizon truly level, and the brasen meridian due north and south by a mariner's compass: then, having rectified the globe, stick a small sewing-needle into the sun's place in the ecliptic, perpendicular to that part of the surface of the globe: this done, turn the globe on its axis, until the needle comes to the brasen meridian, and set the hour index to XII at noon; then, turn the globe on its axis, until the needle points exactly towards the sun (which it will do when it casts no shadow on the globe) and the index will shew the hour of the day.

PROBLEM XXX.

A pleasant way of shewing all those places of the earth which are enlightened by the sun, and also the time of the day, when the sun shines.

Take the terrestrial ball out of the wooden horizon, and also out of the brasen meridian; then set it upon a pedestal in sun-shine, in such a manner that its north pole may point directly towards the north pole of the heaven, and the meridian of the place where you are be directly towards the

 south.

south. Then, the sun will shine upon all the like places of the globe that he does on the real earth, rising to some when he is setting to others; as you may perceive by that part where the enlightened half of the globe is divided from the half in the shade, by the boundary of the light and darkness: all those places on which the sun shines, at any time, having day; and all those on which he does not shine, having night.

If a narrow slip of paper be put round the equator, and divided into 24 equal parts, beginning at the meridian of your place, and the hours set to those divisions in such a manner, that one of the VI's may be upon your meridian; the sun being upon that meridian at noon, will then shine exactly to the two XII's; and at one o'clock to the two I's, &c. So that the place, where the enlightened half of the globe is parted from the shaded half, in this circle of hours, will shew the hour of the day.

The principles of dialing shall be explained in the next lecture, by the terrestrial globe. At present we shall only add the following observations upon it; and then proceed to the use of the celestial globe.

1. *The latitude of any place is always equal to the elevation of the pole above the horizon of that place; and the elevation of the equator is equal to the complement of the latitude, that is, to what the latitude wants of 90 degrees.*

2. *Those places which lie on the equator have no latitude, it being there that the latitude begins; and those places which lie on the first meridian have no longitude,*

gitude, it being there that the longitude begins. Consequently, that particular place of the earth where the first meridian intersects the equator, has neither longitude nor latitude.

3. *In all places of the earth except the poles, all the points of the compass may be distinguished in the horizon: but from the north pole, every place is south; and from the south pole, every place is north. Therefore, as the sun is constantly above the horizon of each pole for half a year in its turn, he cannot be said to depart from the meridian of either pole for half a year together. Consequently, at the north pole it may be said to be noon every moment for half a year; and let the winds blow from what part they will, they must always blow from the south; and at the south-pole, from the north.*

4. *Because one half of the ecliptic is above the horizon of the pole, and the sun, moon and planets, move in (or nearly in) the ecliptic; they will all rise and set to the poles. But, because the stars never change their declinations from the equator, (at least not sensibly in one age) those which are once above the horizon of either pole, never set below it; and those which are once below it never rise.*

5. *All places of the earth do equally enjoy the benefit of the sun in respect of time, and are equally deprived of it.*

6. *All places upon the equator have their days and nights equally long, that is,* 12 *hours each, at all times of the year. For, although the sun declines alternately, from the equator towards the north and towards the south, yet, as the horizon of the equator*

cuts all the parallels of latitude and declination in halves, the ſun muſt always continue above the horizon for one half a diurnal revolution about the earth, and for the other half below it.

7. When the ſun's declination is greater than the latitude of any place, upon either ſide of the equator, the ſun will come twice to the ſame azimuth or point of the compaſs in the forenoon, at that place; and twice to a like azimuth in the afternoon; that is, he will go twice back every day, whilſt his declination continues to be greater than the latitude. Thus, ſuppoſe the globe rectified to the latitude of Barbadoes, which is 13 degrees north; and the ſun to be any where in the ecliptic, between the middle of taurus and middle of leo; if the quadrant of altitude be ſet to about * *18 degrees north of the eaſt in the horizon, the ſun's place be marked with a chalk upon the ecliptic, and the globe be then turned weſtward on its axis, the ſaid mark will riſe in the horizon a little to the north of the quadrant, and thence aſcending, it will croſs the quadrant towards the ſouth; but before it arrives at the meridian it will croſs the quadrant again, and paſs over the meridian northward of Barbadoes. And if the quadrant be ſet about 18 degrees north of the weſt, the ſun's place will croſs it twice, as it deſcends from the meridian towards the horizon, in the afternoon.*

8. In all places of the earth between the equator and poles, the days and nights are equally long, viz. 12 hours each, when the ſun is in the equinoctial: for, in all elevations of the pole, ſhort of 90 degrees

* From the middle of gemini to the middle of cancer, the quadrant may be ſet 20 degrees,

(which

(which is the greatest) one half of the equator or equinoctial will be above the horizon, and the other half below it.

9. *The days and nights are never of an equal length at any place between the equator and polar circles, but when the sun enters the signs ♈ aries and ♎ libra. For in every other part of the ecliptic, the circle of the sun's daily motion is divided into two unequal parts by the horizon.*

10. *The nearer that any place is to the equator, the less is the difference between the length of the days and nights in that place; and the more remote, the contrary. The circles which the sun describes in the heaven every 24 hours, being cut more nearly equal in the former case, and more unequally in the latter.*

11. *In all places lying upon any given parallel of latitude, however long or short the day or night be at any one of these places, at any time of the year, it is then of equal length at all the rest; for, in turning the globe round its axis (when rectified according to the sun's declination) all these places will keep equally long above or below the horizon.*

12. *The sun is vertical twice a year to every place between the tropics; to those under the tropics, once a year, but never any where else. For, there can be no place between the tropics, but what there are two points in the ecliptic whose declination from the equator is equal to the latitude of that place; and but one point of the ecliptic which has a declination equal to the latitude of places on the tropic which that point of the ecliptic touches: and as the sun never goes*

without the tropics, he can never be vertical to any place that lies without them.

13. *To all places in the* * *torrid zone, the duration of the twilight is least, because the sun's daily motion is the most perpendicular to the horizon. In the frigid* † *zones, greatest; because the sun's daily motion is nearly parallel to the horizon; and therefore he is the longer of getting* 18 *degrees below it (till which time the twilight always continues). And in the* § *temperate zones it is at a medium between the two, because the obliquity of the sun's daily motion is so.*

14. *In all places lying exactly under the polar circles, the sun, when he is in the nearest tropic, continues* 24 *hours above the horizon without setting; because no part of that tropic is below their horizon. And when the sun is in the farthest tropic, he is for the same length of time without rising; because no part of that tropic is above their horizon. But, at all other times of the year, he rises and sets there, as in other places; because all the circles that can be drawn parallel to the equator, between the tropics, are more or less cut by the horizon, as they are farther from, or nearer to, that tropic which is all above the horizon: and when the sun is not in either of the tropics, his diurnal course must be in one or other of these circles.*

15. *To all places in the northern hemisphere, from the equator to the polar circle, the longest day and shortest night is when the sun is in the northern tropic;*

* Between the tropics. † Between the polar circles and poles. § Between the tropics and polar circles.

and

and the shortest day and longest night is when the sun is in the southern tropic; because no circle of the sun's daily motion is so much above the horizon, and so little below it, as the northern tropic; and none so little above it, and so much below it, as the southern. In the southern hemisphere, the contrary.

16. *In all places between the polar circles and poles, the sun appears for some number of days (or rather diurnal revolutions) without setting; and at the opposite time of the year without rising; because some part of the ecliptic never sets in the former case, and as much of the opposite part never rises in the latter. And the nearer unto, or the more remote from the pole, these places are, the longer or shorter is the sun's continual presence or absence.*

17. *If a ship sets out from any port, and sails round the earth eastward to the same port again, let her take what time she will to do it in, the people in that ship, in reckoning their time, will gain one compleat day at their return, or count one day more than those who reside at the same port; because, by going contrary to the sun's diurnal motion, and being forwarder every evening than they were in the morning, their horizon will get so much the sooner above the setting sun, than if they had kept for a whole day at any particular place. And thus, by cutting off a part proportionable to their own motion, from the length of every day, they will gain a compleat day of that sort at their return; without gaining one moment of absolute time more than is elapsed during their course, to the people at the port. If they sail westward, they will reckon one day less than the people do who reside*

 at

at the ſaid port, becauſe by gradually following the apparent diurnal motion of the ſun, they will keep him each particular day ſo much longer above their horizon, as anſwers to that day's courſe: and by that means they cut off a whole day in reckoning, at their return, without loſing one moment of abſolute time.

Hence, if two ſhips ſhould ſet out at the ſame time from any port, and ſail round the globe, one eaſtward and the other weſtward, ſo as to meet at the ſame port on any day whatever; they will differ two days in reckoning their time, at their return. If they ſail twice round the earth, they will differ four days; if thrice, then ſix, &c.

The celeſtial globe. Having done, for the preſent, with the terreſtrial globe, we ſhall proceed to the uſe of the celeſtial; firſt premiſing, that as the equator, ecliptic, tropics, polar circles, horizon, and brazen meridian, are exactly alike on both globes, all the former problems concerning the ſun are ſolved the ſame way by both, and therefore it is needleſs to repeat them. To rectify it. The method alſo of rectifying the celeſtial globe is the ſame as rectifying the terreſtrial, *viz.* Elevate the pole according to the latitude of your place, then ſcrew the quadrant of altitude to the zenith, on the braſs meridian; bring the ſun's place in the ecliptic to the graduated edge of the braſs meridian, on the ſide which is above the ſouth point of the wooden horizon, and ſet the hour index to the uppermoſt XII, which ſtands for noon.

N. B. The ſun's place for any day of the year ſtands directly over that day, on the horizon of the celeſtial globe, as it does on that of the terreſtrial.

The

The *latitude* and *longitude* of any *star*, or other celestial phenomenon, is reckoned in a very different manner from the latitude and longitude of any place on the earth: for all terrestrial latitudes are reckoned from the equator; and longitudes from the meridian of some remarkable place, as of London by the English, and Paris by the French; though most of the French maps begin their longitude at the meridian of the island *Ferro*.—But the astronomers of all nations agree in reckoning the *latitudes* of the stars, planets, and comets, from the *ecliptic*; and their *longitudes* from the * *equinoctial colure*, in that semicircle of it which cuts the ecliptic at the beginning of *aries* ♈; and thence eastward, quite round, to the same semicircle again. Consequently, all those stars which lie between the equinoctial and the northern half of the ecliptic, have north declination and south latitude; and all those which lie between the equinoctial and the southern half of the ecliptic, have south declination and north latitude; and all those which lie between the tropics and poles, have their declinations and latitudes of the same denomination.

Latitude and *longitude* of the *stars*.

* *The great circle* that passes through the *equinoctial points* at the beginning of ♈ and ♎, and through the *poles of the world*, (which are two opposite points, each 90 degrees from the equinoctial) is called the *equinoctial colure*: and the *great circle* that passes through the *beginning of* ♋ and ♑, and also through the *poles of the ecliptic*, and poles of the world, is called the *solsticial colure*.

Colures.

There

There are six great circles on the celestial globe, which cut the ecliptic perpendicularly, and meet in two opposite points in the polar circles; which points are each 90 degrees from the ecliptic, and are called its poles. These points divide the said circles into 12 semicircles; and they cut the ecliptic at the beginnings of the 12 signs. They resemble so many meridians on the terrestrial globe; and as all places which lie under any particular meridian semicircle on that globe, have the same longitude, so all those points of the heaven, through which any one of the above semicircles are drawn, have the same longitude.—And, as the greatest latitudes on the earth are at the north and south poles of the earth, so the greatest latitudes in the heaven, are at the north and south poles of the ecliptic.

Constellations. In order to distinguish the stars, with regard to their situations and positions in the heaven, the antients divided the whole visible firmament of stars into particular systems, which they called *constellations*; and digested them into the forms of such animals as are delineated upon the celestial globe. And those which lie between the figures of these imaginary animals, and could not be brought within the compass of any of them, were called *unformed stars*.

Because the moon and all the planets were observed to move in circles or orbits which cross the ecliptic (or line of the sun's path) at small angles, and to be on the north side of the ecliptic for one half of their course round the heaven of stars, and on

on the south side of it for the other half, but never to go quite 8 degrees from it on either side, the antients distinguished that space by two lesser circles, parallel to the ecliptic, (one on each side) at 8 degrees distance from it. And the space included between these circles, they called the *zodiac*, because most of the 12 constellations placed therein, resemble some living creature.—These constellations are, 1. *Aries* ♈, the ram; 2. *Taurus* ♉, the bull; 3. *Gemini* ♊, the twins; 4. *Cancer* ♋, the crab; 5. *Leo* ♌, the lion; 6. *Virgo* ♍, the virgin; 7. *Libra* ♎, the balance; 8. *Scorpio* ♏, the scorpion; 9. *Sagittarius* ♐, the archer; 10. *Capricornus* ♑, the goat; 11. *Aquarius* ♒, the water bearer; and 12. *Pisces* ♓, the fishes. *Zodiac.* Its signs, or divisions.

It is to be observed, that in the infancy of astronomy, these twelve constellations stood at or near the places of the ecliptic, where the above characteristics are marked upon the globe: but now, each constellation is a whole sign forwarder, on account of the recession of the equinoctial points from their former places. So that the constellation of *aries*, is now in the former place of *taurus*; that of *taurus*, in the former place of *gemini*; and so on. Remark.

The stars appear of different magnitudes to the eye; probably because they are at different distances from us. Those which appear brightest and largest, are called *stars of the first magnitude*; the next to them in size and lustre, *stars of the second magnitude*; and so on to the *sixth*, which are the smallest that can be discerned by the bare eye.

Some

Some of the most remarkable stars have names given them, as *castor* and *pollux* in the heads of the twins, *sirius* in the mouth of the *great dog*, *procyon* in the side of the *little dog*, *rigel* in the left foot of *orion*, *arcturus* near the right thigh of *bootes*, &c.

These things being premised, which I think are all that the young *Tyro* need be acquainted with, before he begins to work any problem by this globe, we shall now proceed to the most useful of those problems; omitting several which are of little or no consequence.

PROBLEM I.

*To find the * right ascension and † declination of the sun, or any fixed star.*

Bring the sun's place in the ecliptic to the brasen meridian, then that degree in the equinoctial which is cut by the meridian, is the sun's *right ascension*; and that degree of the meridian which is over the sun's place, is his *declination*. Bring any fixed star to the meridian, and its *right ascension* will be cut by the meridian in the equinoctial; and the degree of the meridian that stands over it, is its *declination*.

* The degree of the equinoctial, reckoned from the beginning of *aries*, that comes to the meridian with the sun or star, is its *right ascension*.

† The distance of the sun or star in degrees from the equinoctial, towards either of the poles, north or south, is its declination, which is north or south accordingly.

So

So that *right ascension* and *declination*, on the celestial globe, are found in the same manner as *longitude* and *latitude* on the terrestrial.

PROBLEM II.

To find the latitude and longitude of any star.

If the given star be on the north side of the ecliptic, place the 90th degree of the quadrant of altitude on the north pole of the ecliptic, where the twelve semicircles meet; which divide the ecliptic into the 12 signs: but if the star be on the south side of the ecliptic, place the 90th degree of the quadrant on the south pole of the ecliptic: keeping the 90th degree of the quadrant on the proper pole, turn the quadrant about, until its graduated edge cuts the star: then, the number of degrees in the quadrant, between the ecliptic and the star, is its latitude; and the degree of the ecliptic cut by the quadrant, is the star's longitude, reckoned according to the sign in which the quadrant then is.

PROBLEM III.

To represent the face of the starry firmament, as seen from any given place of the earth, at any hour of the night.

Rectify the celestial globe for the given latitude, the zenith, and sun's place, in every respect, as taught

taught by the 17th problem, for the terrestrial; and turn it about, until the index points to the given hour: then, the upper hemisphere of the globe will represent the visible half of the heaven for that time: all the stars upon the globe being then in such situations, as exactly correspond to those in the heaven. And if the globe be placed duly north and south, by means of a small sea-compass, every star in the globe will point toward the like star in the heaven: by which means, the constellations and remarkable stars may be easily known. All those stars which are in the eastern side of the horizon, are then rising in the eastern side of the heaven; all in the western, are setting in the western side; and all those under the upper part of the brasen meridian, between the south point of the horizon and the north pole, are at their greatest altitude, if the latitude of the place be north: but if the latitude be south, those stars which lie under the upper part of the meridian, between the north point of the horizon and the south pole, are at their greatest altitude.

PROBLEM IV.

The latitude of the place, and day of the month, being given; to find the time when any known star will rise, or be upon the meridian, or set.

Having rectified the globe, turn it about until the given star comes to the eastern side of the horizon, and the index will shew the time of the star's

ſtar's riſing; then turn the globe weſtward, and when the ſtar comes to the braſen meridian, the index will ſhew the time of the ſtar's coming to the meridian of your place; laſtly, turn on, until the ſtar comes to the weſtern ſide of the horizon, and the index will ſhew the time of the ſtar's ſetting.

N. B. In northern latitudes, thoſe ſtars which are leſs diſtant from the north pole, than the quantity of its elevation above the north point of the horizon, never ſet; and thoſe which are leſs diſtant from the ſouth pole, than the number of degrees that it is depreſſed below the horizon, never riſe: and *vice verſâ* in ſouthern latitudes.

PROBLEM V.

To find at what time of the year a given ſtar will be upon the meridian, at a given hour of the night.

Bring the given ſtar to the upper ſemicircle of the braſs meridian, and ſet the index to the given hour; then turn the globe, until the index points to XII at noon, and the upper ſemicircle of the meridian will then cut the ſun's place, anſwering to the day of the year ſought; which day may be eaſily found againſt the like place of the ſun, among the ſigns on the wooden horizon.

PROBLEM

PROBLEM VI.

*The latitude, day of the month, and * azimuth of any known ſtar being given; to find the hour of the night.*

Having rectified the globe for the latitude, zenith, and ſun's place; lay the quadrant of altitude to the given degree of azimuth, in the horizon; then, turn the globe on its axis, until the ſtar comes to the graduated edge of the quadrant; and when it does ſo, the index will point out the hour of the night.

PROBLEM VII.

The latitude of the place, the day of the month, and altitude † of any known ſtar, being given; to find the hour of the night.

Rectify the globe as in the former problem, gueſs at the hour of the night, and turn the globe until the index points at the ſuppoſed hour; then lay the graduated edge of the quadrant of altitude over the known ſtar, and if the degree of the ſtar's

* The number of degrees that the ſun, moon, or any ſtar, is from the meridian, either to the eaſt or weſt, is called its *azimuth*.

† The number of degrees that the ſtar is above the horizon, as obſerved by means of a common quadrant, is called its *altitude*.

height

height in the quadrant upon the globe, answers exactly to the degree of the star's observed altitude in the heaven, you have guessed exactly: but if the star on the globe is higher or lower than it was observed to be in the heaven, turn the globe backwards or forwards, keeping the edge of the quadrant upon the star, until its center comes to the observed altitude in the quadrant; and then, the index will shew the true time of the night.

PROBLEM VIII.

An easy method for finding the hour of the night by any two known stars, without knowing either their altitude or azimuth; and then, of finding both their altitude and azimuth, and thereby the true meridian.

Tie one end of a thread to a common musket bullet; and, having rectified the globe as above, hold the other end of the thread in your hand, and carry it slowly round betwixt your eye and the starry heaven, until you find it cuts any two known stars at once. Then, guessing at the hour of the night, turn the globe until the index points to that time in the hour circle; which done, lay the graduated edge of the quadrant over any one of these two stars on the globe, which the thread cut in the heaven. If the said edge of the quadrant cuts the other star also, you have guessed the time exactly; but if it does not, turn the globe slowly backwards or forwards, until the quadrant (kept

upon either star) cuts them both through their centers: and then, the index will point out the exact time of the night; the degree of the horizon, cut by the quadrant, will be the true azimuth of both these stars from the south; and the stars themselves will cut their true altitudes in the quadrant. At which moment, if a common azimuth compass be so set upon a floor or level pavement, that these stars in the heaven may have the same bearing upon it (allowing for the variation of the needle) as the quadrant of altitude has in the wooden horizon of the globe, a thread extended over the north and south points of that compass will be directly in the plane of the meridian: and if a line be drawn upon the floor or pavement, along the course of the thread, and an upright wire be placed in the southmost end of the line, the shadow of the wire will fall upon that line, when the sun is on the meridian, and shines upon the pavement.

PROBLEM IX.

To find the place of the moon, or of any planet; and thereby to shew the time of its rising, southing, and setting.

Seek in *Parker*'s or *Weaver*'s Ephemeris the * geocentric place of the moon or planet in the ecliptic, for the given day of the month; and, ac-

* The place of the moon or planet, as seen from the earth.

cording

cording to its longitude and latitude, as shewn by the Ephemeris, mark the same with a chalk upon the globe. Then, having rectified the globe, turn it round its axis westward; and, as the said mark comes to the eastern side of the horizon, to the brasen meridian, and to the western side of the horizon, the index will shew at what time the planet rises, comes to the meridian, and sets, in the same manner as for a fixed star.

PROBLEM X.

To explain the phenomena of the harvest-moon.

In order to do this, we must premise the following things. 1. That as the sun goes only once a year round the ecliptic, he can be but once a year in any particular point of it: and that his motion is almost a degree every 24 hours, at a mean rate. 2. That as the moon goes round the ecliptic once in 27 days and 8 hours, she advances $13\frac{1}{6}$ degrees in it, every day, at a mean rate. 3. That as the sun goes through part of the ecliptic in the time the moon goes round it, the moon cannot at any time, be either in conjunction with the sun, or opposite to him, in that part of the ecliptic where she was so the last time before; but must travel as much forwarder, as the sun has advanced in the said time; which being $29\frac{1}{2}$ days, makes almost a whole sign. Therefore, 4. The moon can be but once a year opposite to the sun, in any particular part of the ecliptic. 5. That the

moon is never full but when ſhe is oppoſite to the ſun, becauſe at no other time can we ſee all that half of her which the ſun enlightens. 6. That when any point of the ecliptic riſes, the oppoſite point ſets. Therefore, when the moon is oppoſite to the ſun, ſhe muſt riſe at * ſun-ſet. 7. That the different ſigns of the ecliptic, riſe at very different angles or degrees of obliquity with the horizon, eſpecially in conſiderable latitudes; and that the ſmaller this angle is, the greater is the portion of the ecliptic that riſes in any ſmall part of time; and *vice verſâ*. 8. That, in northern latitudes, no part of the ecliptic riſes at ſo ſmall an angle with the horizon, as *piſces* and *aries* do; and therefore, a greater portion of the ecliptic riſes in one hour, about theſe ſigns, than about any of the reſt. 9. That the moon can never be full in *piſces* and *aries* but in our autumnal months, for at no other time of the year is the ſun in the oppoſite ſigns *virgo* and *libra*.

Theſe things premiſed, take $13\frac{1}{6}$ degrees of the ecliptic in your compaſſes, and, beginning at *piſces*, carry that extent all round the ecliptic, marking the places with a chalk, where the points of the compaſſes ſucceſſively fall. So you will have the moon's daily motion marked out for one compleat revolution in the ecliptic (according to § 2 of the laſt paragraph.)

* This is not always ſtrictly true, becauſe the moon does not keep in the ecliptic, but croſſes it twice every month. However, the difference need not be regarded in a general explanation.

Rectify

Rectify the globe for any considerable northern latitude, (as suppose that of London) and then, turning it round its axis, observe how much of the hour-circle the index has gone over, at the rising of each particular mark on the ecliptic; and you will find that seven of the marks (which take in as much of the ecliptic as the moon moves through in a week) will all rise successively about *pisces* and *aries*, in the time that the index goes over two hours. Therefore, whilst the moon is in *pisces* and *aries*, she will not differ in general above two hours in her rising for a whole week. But if you take notice of the marks on the opposite signs, *virgo* and *libra*, you will find that seven of them take nine hours to rise; which shews, that when the moon is in these two signs, she differs nine hours in her rising within the compass of a week. And so much later as every mark is of rising than the one that rose next before it, so much later will the moon be of rising any day, than she was on the day before, in the corresponding part of the heaven. The marks about *cancer* and *capricorn*, rise with a mean difference of time between those about *aries* and *libra*.

Now, although the moon is in *pisces* and *aries* every month, and therefore must rise in those signs within the space of two hours later for a whole week, or only about 17 minutes later every day than she did on the former; yet she is never full in these signs, but in our autumnal months *August* and *September*, when the sun is in *virgo* and *libra*. Therefore, no full moon in the year will continue

to rise so near the time of sun-set for a week or so, as these two full moons do, which fall in the time of harvest.

In the winter months, the moon is in *pisces* and *aries* about her first quarter; and as these signs rise about noon in winter, the moon's rising in them passes unobserved. In the spring months, the moon changes in these signs, and consequently rises at the same time with the sun; so that it is impossible to see her at that time. In the summer months she falls in these signs about her third quarter, and rises not until midnight, when her rising is but very little taken notice of; especially as she is on the decrease. But in the harvest months she is at the full, when in these signs, and being opposite to the sun, she rises when the sun sets, (or soon after) and shines all the night.

In southern latitudes, *virgo* and *libra* rise at as small angles with the horizon, as *pisces* and *aries* do in the northern; and as our spring is at the time of their harvest, it is plain their harvest full moons must fall in *virgo* and *libra*; and will therefore rise with as little difference of time, as ours do in *pisces* and *aries*.

For a fuller account of this matter, I must refer the reader to my Astronomy, in which it is described at large.

PROBLEM XI.

To explain the equation of time, or difference of time between well regulated clocks and true sun-dials.

The earth's motion on its axis being perfectly equable, and thereby causing an apparent equable motion of the starry heaven, round the same axis produced to the poles of the heaven; it is plain that equal portions of the celestial equator pass over the meridian in equal parts of time, because the axis of the world is perpendicular to the plane of the equator. And therefore, if the sun kept his annual course in the celestial equator, he would always revolve from the meridian to the meridian again in 24 hours exactly, as shewn by a well regulated clock.

But as the sun moves in the ecliptic, which is oblique both to the plane of the equator and axis of the world, he cannot always revolve from the meridian to the meridian again in 24 equal hours; but sometimes a little sooner, and at other times a little later, because equal portions of the ecliptic pass over the meridian in unequal parts of time, on account of its obliquity. And this difference is the same in all latitudes.

To shew this by a globe, make chalk-marks all around the equator and ecliptic, at equal distances from one another (suppose 10 degrees) beginning at *aries* or at *libra*, where these two circles intersect each other. Then turn the globe round its

its axis, and you will see that all the marks in the first quadrant of the ecliptic, or from the beginning of *aries* to the beginning of *cancer*, come sooner to the brasen meridian than their corresponding marks do on the equator : those in the second quadrant, or from the beginning of *cancer* to the beginning of *libra*, come later: those in the third quadrant, from *libra* to *capricorn*, sooner ; and those in the south, from *capricorn* to *aries*, later. But those at the beginning of each quadrant come to the meridian at the same time with their corresponding marks on the equator.

Therefore, whilst the sun is in the first and third quadrants of the ecliptic, he comes sooner to the meridian every day than he would do if he kept in the equator ; and consequently he is faster than a well regulated clock, which always keeps equable or equatoreal time : and whilst he is in the second and fourth quadrants, he comes later to the meridian every day than he would do if he kept in the equator ; and is therefore slower than the clock. But at the beginning of each quadrant, the sun and clock are equal.

And thus, if the sun moved equably in the ecliptic, he would be equal with the clock on four days of the year, which would have equal intervals of time between them. But as he moves faster at some times than at others, (for he is eight days longer in the northern half of the ecliptic than in the southern) this will create a second inequality ; which combined with the former, arising from the obliquity of the ecliptic to the equator, makes up that

that difference, which is ſhewn by the common equation tables to be between good clocks and true ſun-dials.

The deſcription and uſe of the armillary ſphere.

Whoever has ſeen a common *armillary ſphere*, and underſtands how to uſe it, muſt be ſenſible that the machine here referred to, is of a very different, and much more advantageous conſtruction. And whoever has ſeen the curious glaſs ſphere invented by Dr. LONG, or the figure of it in his Aſtronomy, muſt know that the furniture of the terreſtrial globe in this machine, the form of the pedeſtal, and the manner of turning either the earthly globe, or the circles which ſurround it, are all copied from the Doctor's glaſs ſphere; and that the only difference is, a parcel of rings inſtead of a glaſs celeſtial globe; and all the additions are a moon within the ſphere, and a ſemicircle upon the pedeſtal. Plate XX. Fig. 1.

The exterior parts of this machine are a compages of braſs rings, which repreſent the principal circles of the heaven: *viz.* 1. The equinoctial *AA*, which is divided into 360 degrees (beginning at its interſection with the ecliptic in *aries)* for ſhewing the ſun's right aſcenſion in degrees; and alſo into 24 hours, for ſhewing his right aſcenſion in time. 2. The ecliptic *BB*, which is divided into 12 ſigns, and each ſign into 30 degrees, and alſo into the months and days of the year; in ſuch a manner, that the degree or point of the ecliptic in The *armillary ſphere.*

in which the ſun is, on any given day, ſtands over that day in the circle of months. 3. The tropic of *cancer CC*, touching the ecliptic at the beginning of *cancer* in *e*, and the tropic of *capricorn DD*, touching the ecliptic at the beginning of *capricorn* in *f*; each $23\frac{1}{2}$ degrees from the equinoctial circle. 4. The arctic circle *E*, and the antarctic circle *F*, each $23\frac{1}{2}$ degrees from its reſpective pole at *N* and *S*. 5. The equinoctial colure *GG*, paſſing through the north and ſouth poles of the heaven at *N* and *S*, and through the equinoctial points in the ecliptic *aries* and *libra*. 6. The ſolſtitial colure *HH*, paſſing through the poles of the heaven, and through the ſolſtitial points *cancer* and *capricorn*, in the ecliptic: each quarter of the former of theſe colures is divided into 90 degrees, from the equinoctial to the poles of the world, for ſhewing the declination of the ſun, moon, and ſtars; and each quarter of the latter, from the ecliptic at *e* and *f*, to its poles *b* and *d*, for ſhewing the latitudes of the ſtars.

In the north pole of the ecliptic is a nut *b*, to which is fixed one end of a quadrantal wire, and to the other end a ſmall ſun *Y*, which is carried round the ecliptic *BB*, by turning the nut: and in the ſouth pole of the ecliptic is a pin *d*, on which is another quadrantal wire, with a ſmall moon *Z* upon it, which may be moved round by hand: but there is a particular contrivance for cauſing the moon to move in an orbit which croſſes the ecliptic at an angle of $5\frac{1}{3}$ degrees, in two oppoſite points called the *moon's nodes*; and alſo for ſhifting theſe

points

points backward in the ecliptic, as the *moon's nodes* ſhift in the heaven.

Within theſe circular rings is a ſmall terreſtrial globe *I*, fixt on an axis *KK*, which extends from the north and ſouth poles of the globe at *n* and *s*, to thoſe of the celeſtial ſphere at *N* and *S*. On this axis is fixt the flat celeſtial meridian *LL*, which may be ſet directly over the meridian of any place on the globe, and then turned round with the globe, ſo as to keep over the ſame meridian upon it. This flat meridian is graduated the ſame way as the braſs meridian of a common globe, and its uſe is much the ſame. To this globe is fitted the moveable horizon *MM*, ſo as to turn upon two ſtrong wires proceeding from its eaſt and weſt points to the globe, and entering the globe at oppoſite points of its equator, which is a moveable braſs ring let into the globe in a groove all around its equator. The globe may be turned by hand within this ring, ſo as to place any given meridian upon it, directly under the celeſtial meridian *LL*. The horizon is divided into 360 degrees all around its outermoſt edge, within which are the points of the compaſs, for ſhewing the amplitude of the ſun and moon, both in degrees and points. The celeſtial meridian *LL* paſſes through two notches in the north and ſouth points of the horizon, as in a common globe: but here, if the globe be turned round, the horizon and meridian turn with it. At the ſouth pole of the ſphere is a circle of 24 hours, fixt to the rings, and

and on the axis is an index which goes round that circle, if the globe be turned round its axis.

The whole fabric is ſupported on a pedeſtal *N*, and may be elevated or depreſſed, upon the joint *O*, to any number of degrees from 0 to 90, by means of the arc *P*, which is fixed into the ſtrong braſs arm *Q*, and ſlides in the upright piece *R*, in which is the ſcrew *r*, to fix it at any proper elevation.

In the box *T* are two wheels, and two pinions whoſe axes come out at *V* and *U*; either of which may be turned by the ſmall winch *W*. When the winch is put upon the axis *V*, and turned backward, the terreſtrial globe, with its horizon and celeſtial meridian, keep at reſt; and the whole ſphere of circles turns round from eaſt, by ſouth, to weſt, carrying the ſun *Y*, and moon *Z*, round the ſame way, and cauſing them to riſe above and ſet below the horizon. But when the winch is put upon the axis *U*, and turned forward, the ſphere with the ſun and moon keep at reſt; and the earth with its horizon and meridian turn round from weſt, by ſouth, to eaſt; and bring the ſame points of the horizon to the ſun and moon, to which theſe bodies came when the earth kept at reſt, and they were carried round it; ſhewing that they riſe and ſet in the ſame points of the horizon, and at the ſame times in the hour circle, whether the motion be in the earth or in the heaven. If the earth be turned, the hour index goes round its circle; but if the ſphere be turned, the hour circle goes round below the index.

And

And ſo, by this conſtruction, the machine is equally fitted to ſhew either the real motion of the earth, or the apparent motion of the heaven.

To rectify the ſphere for uſe, firſt ſlacken the ſcrew *r* in the upright ſtem *R*, and taking hold of the arm *Q*, move it up or down until the given degree of latitude for any place be cut by the ſtem *R*; and then the axis of the ſphere will be properly elevated, ſo as to ſtand parallel to the axis of the world, if the machine be ſet north and ſouth by a ſmall compaſs: this done, count the latitude from the north pole, upon the celeſtial meridian *LL*, down towards the north notch of the horizon, and ſet the horizon to that latitude; then, turn the nut *b* until the ſun *Y* comes to the given day of the year in the ecliptic, and the ſun will be at its proper place for that day: find the place of the moon's aſcending node, and alſo the place of the moon, by an Ephemeris, and ſet them right accordingly: laſtly, turn the winch *W*, until either the ſun comes to the meridian *LL*, or until the meridian comes to the ſun, (according as you want the ſphere or earth to move) and ſet the hour index to the XII, marked noon, and the whole machine will be rectified.—Then turn the winch, and obſerve when the ſun or moon riſe and ſet in the horizon, and the hour index will ſhew the times thereof for the given day.

As thoſe who underſtand the uſe of the globes will be at no loſs to work many other problems by this ſphere, it ſeems needleſs to enlarge any farther upon it.

LECT. IX.

The principles of dialing.

Preliminaries. A Dial is a plane, upon which lines are defcribed in fuch a manner, that the fhadow of a wire, or of the upper edge of another plane, erected perpendicularly on the former, may fhew the true time of the day.

The edge of the plane by which the time of the day is found, is called the ftile of the dial, which is always parallel to the earth's axis; and the line on which the faid plane is erected, is called the fubftile.

The angle included between the fubftile and ftile, is called the elevation, or height of the ftile.

Thofe dials whofe planes are parallel to the plane of the horizon, are called horizontal dials; and thofe dials whofe planes are perpendicular to the plane of the horizon, are called vertical, or erect dials.

Thofe erect dials, whofe planes directly front the north or fouth, are called direct north or fouth dials; and all other erect dials are called decliners, becaufe their planes are turned away from the north or fouth.

Thofe dials whofe planes are neither parallel nor perpendicular to the plane of the horizon, are called inclining, or reclining dials, according as their planes make acute or obtufe angles with the horizon; and if their planes are alfo turned afide from

from facing the ſouth or north, they are called declining-inclining, or declining-reclining dials.

The interſection of the plane of the dial, with that of the meridian, paſſing through the ſtile, is called the meridian of the dial, or the hour-line of XII.

Thoſe meridians, whoſe planes paſs through the ſtile, and make angles of 15, 30, 45, 60, 75, and 90 degrees with the meridian of the place, (which marks the hour-line of XII) are called hour-circles: and their interſections with the plane of the dial, are called hour-lines.

In all declining dials, the ſubſtile makes an angle with the hour-line of XII; and this angle is called the diſtance of the ſubſtile from the meridian.

The declining plane's difference of longitude, is the angle formed at the interſection of the ſtile and plane of the dial, by two meridians; one of which paſſes through the hour-line of XII, and the other through the ſubſtile.

This much being premiſed concerning dials in general, we ſhall now proceed to explain the different methods of their conſtruction.

If the whole earth *aPcp* were tranſparent, and hollow, like a ſphere of glaſs, and had its equator divided into 24 equal parts by ſo many meridian ſemicircles, *a*, *b*, *c*, *d*, *e*, *f*, *g*, &c. one of which is the geographical meridian of any given place, as London, (which is ſuppoſed to be at the point *a*) and if the hours of XII were marked at the equator, both upon that meridian and the oppoſite one,

Plate XX. Fig. 2.

The univerſal principle on which *dialing* depends.

one, and all the reſt of the hours in order on the reſt of the meridians, theſe meridians would be the hour-circles of London: then, if the ſphere had an opake axis as *PEp*, terminating in the poles *P* and *p*, the ſhadow of the axis would fall upon every particular meridian and hour, when the ſun came to the plane of the oppoſite meridian, and would conſequently ſhew the time at London, and all other places on the meridian of London.

Horizontal dial. If this ſphere was cut through the middle by a ſolid plane *ABCD*, in the rational horizon of London, one half of the axis *EP* will be above the plane, and the other half below it; and if ſtraight lines were drawn from the center of the plane, to thoſe points where its circumference is cut by the hour-circles of the ſphere, theſe lines would be the hour-lines of a horizontal dial for London: for the ſhadow of the axis would fall upon each particular hour-line of the dial, when it fell upon the like hour-circle of the ſphere.

Fig. 3. If the plane which cuts the ſphere be upright, as *AFCG*, touching the given place (London) at *F*, and directly facing the meridian of London, it will then become the plane of an erect direct ſouth dial: and if right lines be drawn from its center *E*, to thoſe points of its circumference where the hour-circles of the ſphere cut it, theſe will be the hour-lines of a vertical or direct ſouth dial for London, to which the hours are to be ſet as in the figure, (contrary to thoſe on a horizontal dial) and the lower half *Ep* of the axis will caſt a ſhadow on the hour of the day in this dial, at the ſame time

Vertical dial.

[ti]me that it would fall upon the like hour-circle [o]f the sphere, if the dial-plane was not in the way.

If the plane (still facing the meridian) be made [t]o incline, or recline, any given number of degrees; [t]he hour-circles of the sphere will still cut the edge [o]f the plane in those points to which the hour-lines [m]ust be drawn straight from the center; and the [a]xis of the sphere will cast a shadow on these lines [a]t the respective hours. The like will still hold, [if] the plane be made to decline any given number [o]f degrees from the meridian, towards the east or [w]est: provided the declination be less than 90 de-[g]rees, or the reclination be less than the co-latitude [o]f the place: and the axis of the sphere will be [a] gnomon, or stile, for the dial. But it cannot, [w]hen the declination is quite 90 degrees, nor when [t]he reclination is equal to the co-latitude; because [i]n these two cases, the axis has no elevation above [t]he plane of the dial.

Inclining and reclining dials.

Declining dials.

And thus it appears, that the plane of every dial [r]epresents the plane of some great circle upon the [e]arth; and the gnomon the earth's axis, whether [i]t be a small wire, as in the above figures, or the [e]dge of a thin plate, as in the common horizontal [d]ials.

The whole earth, as to its bulk, is but a point [i]f compared to its distance from the sun: and [t]herefore, if a small sphere of glass be placed [u]pon any part of the earth's surface, so that its [a]xis be parallel to the axis of the earth, and the [s]phere have such lines upon it, and such planes [w]ithin it, as above described; it will shew the

 hours

hours of the day as truly as if it were placed in the earth's center, and the ſhell of the earth were as tranſparent as glaſs.

But becauſe it is impoſſible to have a hollow ſphere of glaſs perfectly true, blown round a ſolid plane; or if it was, we could not get at the plane within the glaſs to ſet it in any given poſition; we make uſe of a wire-ſphere to explain the principles of dialing, by joining 24 ſemicircles together at the poles, and putting a thin flat plate of braſs within it.

Dialing by the common *terreſtrial globe*

A common globe, of twelve inches diameter, has generally 24 meridian ſemicircles drawn upon it. If ſuch a globe be elevated to the latitude of any given place, and turned about until any one of theſe meridians cuts the horizon in the north point, where the hour of XII is ſuppoſed to be marked, the reſt of the meridians will cut the horizon at the reſpective diſtances of all the other hours from XII. Then, if the globe be taken out of the horizon, and a flat board or plate be put into its place, even with the ſurface of the horizon; and if ſtraight lines be drawn from the center of the board, to thoſe points of the horizon which were cut by the 24 meridian ſemicircles, they will be the hour-lines of a horizontal dial for that latitude, the edge of whoſe gnomon muſt be in the very ſame ſituation that the axis of the globe was, before it was taken out of the horizon: that is, the gnomon muſt make an angle with the plane of the dial, equal to the latitude of the place for which the dial is made.

If

If the pole of the globe be elevated to the * co-ıtitude of the given place, and any meridian be rought to the north point of the horizon, the reſt f the meridians will cut the horizon in the re-ſpective diſtances of all the hours from XII, for direct ſouth dial; whoſe gnomon muſt make an ngle with the plane of the dial, equal to the co-ıtitude of the place: and the hours muſt be ſet ne contrary way on this dial, to what they are on ne horizontal.

But if your globe have more than 24 meridian emicircles upon it, you muſt take the following nethod for making *horizontal* and *ſouth dials.*

To conſtruct a *horizontal dial.*

Elevate the pole to the latitude of your place, nd turn the globe until any particular meridian ſuppoſe the firſt) comes to the north point of the orizon, and the oppoſite meridian will cut the orizon in the ſouth. Then, ſet the hour-index o the uppermoſt XII on its circle; which done, urn the globe weſtward until 15 degrees of the quator paſs under the braſen meridian, and then he hour-index will be at I (for the ſun moves 15 egrees every hour) and the firſt meridian will cut he horizon in the number of degrees from the orth point, that I is diſtant from XII. Turn on, ıntil other 15 degrees of the equator paſs under he braſen meridian, and the hour-index will then e at II, and the firſt meridian will cut the ho-

* If the latitude be ſubtracted from 90 degrees, the re-mainder is called the co-latitude, or complement of the la-itude.

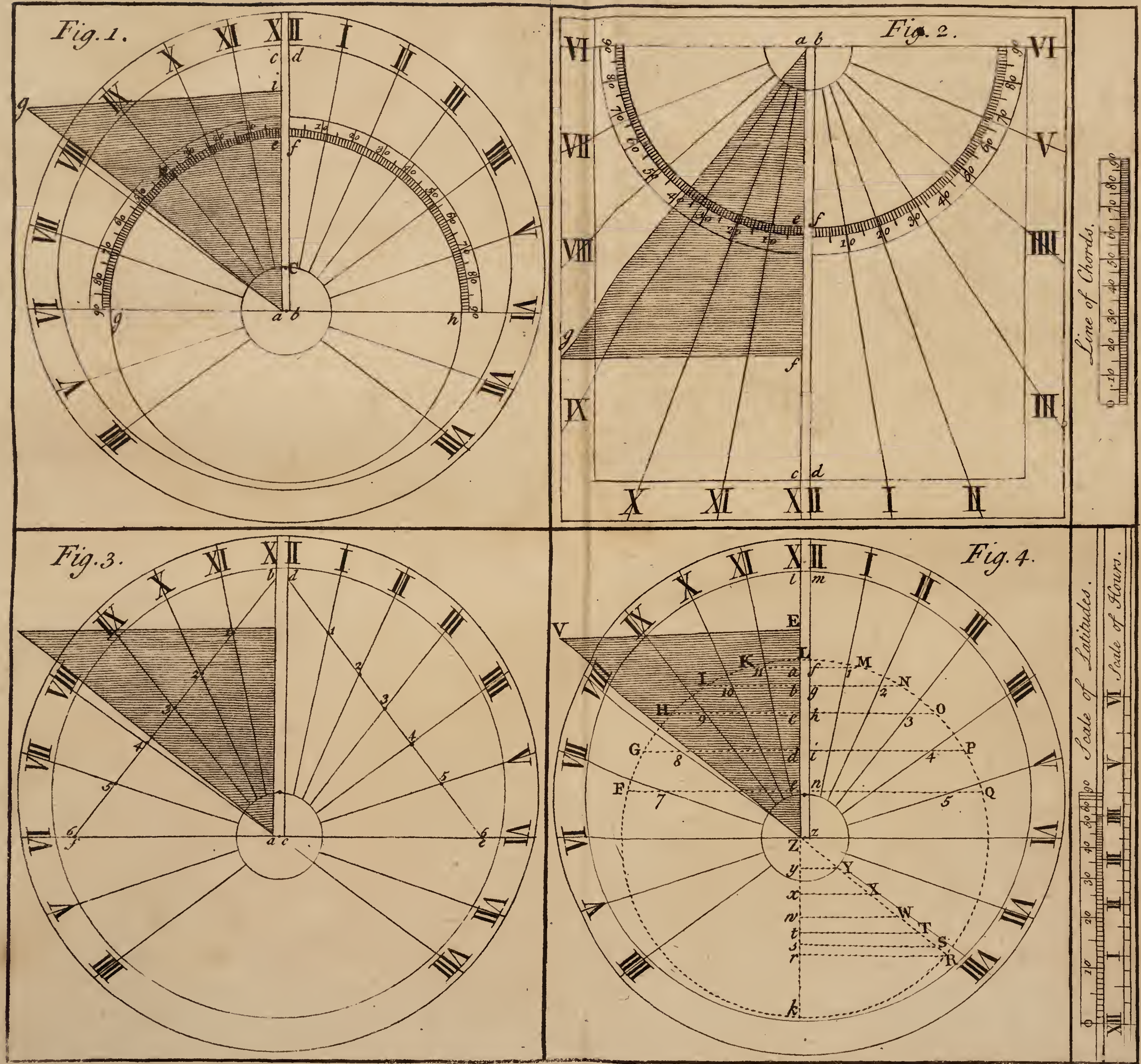
rizon in the number of degrees that II is diſtant from XII: and ſo, by making 15 degrees of the equator paſs under the braſen meridian for every hour, the firſt meridian of the globe will cut the horizon in the diſtances of all the hours from XII to VI, which is juſt 90 degrees; and then you need go no farther, for the diſtances of XI, X, IX, VIII, VII, and VI, in the forenoon, are the ſame from XII, as the diſtances of I, II, III, IV, V, and VI, in the afternoon: and theſe hour-lines continued through the center, will give the oppoſite hour-lines on the other half of the dial: but no more of theſe need be drawn, than what anſwer to the ſun's continuance above the horizon of your place on the longeſt day, which may be eaſily found by working the 26th problem of the foregoing lecture.

Thus, to make a horizontal dial for the latitude of London, which is $51\frac{1}{2}$ degrees north, I elevate the north pole of the globe $51\frac{1}{2}$ degrees above the north point of the horizon, and then, turn the globe until the firſt meridian (which is that of London on the Engliſh terreſtrial globe) cuts the north point of the horizon, and ſet the hour-index to XII at noon.

Then, turning the globe weſtward until the index points ſucceſſively to I, II, III, IIII, V, and VI, in the afternoon; or until 15, 30, 45, 60, 75, and 90 degrees of the equator paſs under the braſen meridian; I find the firſt meridian of the globe cuts the horizon in the following numbers of degrees from the north towards the eaſt, *viz.* $11\frac{2}{3}$,

$24\frac{1}{4}$,

PLATE XXI.



J. Ferguson delin.

J. Mynde sc.

$4\frac{1}{4}$, $38\frac{1}{12}$, $53\frac{1}{2}$, $71\frac{1}{15}$, and 90; which are the espective diſtances of the above hours from XII pon the plane of the horizon.

To transfer theſe, and the reſt of the hours, to horizontal plane, draw the right lines *ac* and *bd* pon that plane, as far from each other as is equal o the intended thickneſs of the gnomon or ſtile f the dial, and the ſpace included between them vill be the meridian or twelve o'clock line on the ial. Croſs this meridian at right angles with the ix o'clock line *gh*, and ſetting one foot of your ompaſſes in the interſection *a*, as a center, deſcribe he quadrant *ge* with any convenient radius or pening of the compaſſes: then, ſetting one foot n the interſection *b*, as a center, with the ſame adius deſcribe the quadrant *fh*, and divide each uadrant into 90 equal parts or degrees, as in the gure. Plate XXI. Fig. 1.

Becauſe the hour-lines are leſs diſtant from each ther about noon, than in any other part of the ial, it is beſt to have the centers of theſe quarants at a little diſtance from the center of the ial-plane, on the ſide oppoſite to XII, in order o enlarge the hour diſtances thereabouts, under he ſame angles on the plane. Thus, the center f the plane is at *C*, but the centers of the quarants are at *a* and *b*.

Laying a ruler over the point *b*, (and keeping it here for the center of all the afternoon hours in he quadrant *fh*) draw the hour-line of I through $1\frac{2}{3}$ degrees in the quadrant; the hour-line of II, hrough $24\frac{1}{4}$ degrees; of III, through $38\frac{1}{12}$ de-

grees; IIII, through $53\frac{1}{2}$, and V through $71\frac{1}{15}$: and becauſe the ſun riſes about four in the morning, on the longeſt days at London; continue the hour-lines of IIII and V in the afternoon, through the center *b* to the oppoſite ſide of the dial.—This done, lay the ruler to the center *a* of the quadrant *eg*, and through the like diviſions or degrees of that quadrant, *viz.* $11\frac{2}{3}$, $24\frac{1}{4}$, $38\frac{1}{12}$, $53\frac{1}{2}$, and $71\frac{1}{15}$, draw the forenoon hour-lines of XI, X, IX, VIII, and VII; and becauſe the ſun ſets not before eight in the evening on the longeſt days, continue the hour-lines of VII and VIII in the forenoon, through the center *a*, to VII and VIII in the afternoon; and all the hour-lines will be finiſhed on this dial; to which the hours may be ſet, as in the figure.

Laſtly, through $51\frac{1}{2}$ degrees of either quadrant, and from its center, draw the right line *ag* for the hypothenuſe or axis of the gnomon *agi*; and from *g*, let fall the perpendicular *gi*, upon the meridian line *ai*, (called alſo the ſubſtile) and there will be a triangle made, whoſe ſides are *ag*, *gi*, and *ia*. If this triangle be made a plate as thick as the diſtance between the lines *ac* and *bd*, and ſet upright between them, touching at *a* and *b*, its hypothenuſe *ag* will be parallel to the axis of the world, when the dial is truly ſet; and will caſt a ſhadow on the hour of the day.

N. B. The trouble of dividing the two quadrants may be ſaved, if you have a ſcale with a line of chords upon it, ſuch as that on the right hand of the plate: for if you extend the compaſſes

paſſes from 0 to 60 degrees on the line of chords, and with that extent as a radius, deſcribe the two quadrants upon their reſpective centers, the above diſtances may be taken with the compaſſes upon the line, and ſet off upon the quadrants.

To make an erect direct ſouth dial. Elevate the pole to the co-latitude of your place, and proceed in all reſpects as above taught for the horizontal dial, from VI in the morning to VI in the afternoon; only the hours muſt be reverſed, as in the figure; and the hypothenuſe *ag*, of the gnomon *agf*, muſt make an angle with the dial-plane equal to the co-latitude of the place. As the ſun can ſhine no longer on this dial, than from ſix in the morning until ſix in the evening, there is no occaſion for having any more than twelve hours upon it. Fig. 2. To conſtruct an *erect ſouth dial.*

To make an erect dial, declining from the ſouth towards the eaſt or weſt. Elevate the pole to the latitude of your place, and ſcrew the quadrant of altitude to the zenith. Then, if your dial declines toward the eaſt (which we ſhall ſuppoſe it to do at preſent) count in the horizon the degrees of declination, from the eaſt point towards the north, and bring the lower end of the quadrant to that degree of declination at which the reckoning ends. This done, bring any particular meridian of your globe (as ſuppoſe the firſt meridian) directly under the graduated edge of the upper part of the braſen meridian, and ſet the hour-index to XII at noon. Then, keeping the quadrant of altitude at the degree of declination in the horizon, turn the globe To conſtruct an *erect declining dial.*

eaſtward on its axis, and obſerve the degrees cut by the firſt meridian in the quadrant of altitude (counted from the zenith) as the hour-index comes to XI, X, IX, &c. in the forenoon, or as 15, 30, 45, &c. degrees of the equator paſs under the braſen meridian at theſe hours reſpectively; and the degrees then cut in the quadrant by the firſt meridian, are the reſpective diſtances of the forenoon hours from XII on the plane of the dial.—Then, for the afternoon hours, turn the quadrant of altitude round the zenith until it comes to the degree in the horizon oppoſite to that where it was placed before; namely, as far from the weſt point of the horizon toward the ſouth, as it was ſet at firſt from the eaſt point toward the north; and turn the globe weſtward on its axis, until the firſt meridian comes to the braſen meridian again, and the hour-index to XII: then, continue to turn the globe weſtward, and as the index points to the afternoon hours I, II, III, &c. or as 15, 30, 45, &c. degrees of the equator paſs under the braſen meridian, the firſt meridian will cut the quadrant of altitude in the reſpective number of degrees from the zenith, that each of theſe hours is from XII on the dial.—And note, that when the firſt meridian goes off the quadrant at the horizon, in the forenoon, the hour-index ſhews the time when the ſun will come upon this dial: and when it goes off the quadrant in the afternoon, the index will point to the time when the ſun goes off the dial.

Having thus found all the hour-diſtances from XII, lay them down upon your dial-plane, either by

by dividing a ſemicircle into two quadrants of 90 degrees each, (beginning at the hour-line of XII) or by the line of chords, as above directed.

In all declining dials, the line on which the ſtile or gnomon ſtands (commonly called the *ſubſtile-line)* makes an angle with the twelve o'clock line, and falls among the forenoon hour-lines, if the dial declines towards the eaſt; and among the afternoon hour-lines, when the dial declines towards the weſt: that is, to the left hand from the twelve o'clock line in the former caſe, and to the right hand from it in the latter.

To find the diſtance of the ſubſtile from the twelve o'clock line; if your dial declines from the ſouth toward the eaſt, count the degrees of that declination in the horizon from the eaſt point toward the north, and bring the lower end of the quadrant of altitude to that degree of declination where the reckoning ends: then, turn the globe until the firſt meridian cuts the horizon in the like number of degrees, counted from the ſouth point toward the eaſt; and the quadrant and firſt meridian will then croſs one another at right angles, and the number of degrees of the quadrant, which are intercepted between the firſt meridian and the zenith, is equal to the diſtance of the ſubſtile-line from the twelve o'clock line; and the number of degrees of the firſt meridian, which are intercepted between the quadrant and the north pole, is equal to the elevation of the ſtile above the plane of the dial.

If the dial declines weſtward from the ſouth, count that declination from the eaſt point of the horizon towards the ſouth, and bring the quadrant of altitude to the degree in the horizon at which the reckoning ends; both for finding the forenoon hours, and diſtance of the ſubſtile from the meridian: and for the afternoon hours, bring the quadrant to the oppoſite degree in the horizon, namely, as far from the weſt towards the north, and then proceed in all reſpects as above.

Thus, we have finiſhed our declining dial; and in ſo doing, we made four dials, *viz.*

1. A north dial, declining eaſtward the ſame number of degrees. 2. A north dial, declining weſt the ſame number. 3. A ſouth dial, declining eaſt. And, 4, a ſouth dial declining weſt.

Only, placing the proper number of hours, and the ſtile or gnomon reſpectively, upon each plane. For (as above-mentioned) in the ſouth-weſt plane, the ſubſtilar-line falls among the afternoon hours; and in the ſouth-eaſt, of the ſame declination, among the forenoon hours, at equal diſtances from XII. And ſo, all the morning hours on the weſt decliner will be like the afternoon hours on the eaſt decliner: the ſouth-eaſt decliner will produce the north-weſt decliner; and the ſouth-weſt decliner, the north-eaſt decliner, by only extending the hour-lines, ſtile and ſubſtile, quite through the center: the axis of the ſtile, (or edge that caſts the ſhadow on the hour of the day) being in all dials whatever parallel to the axis of the world, and conſequently pointing towards the north pole

pole of the heaven in north latitudes, and towards the ſouth pole, in ſouth latitudes. *See more of this in the following lecture.*

An eaſy method for conſtructing of *dials*.

But becauſe every one who would like to make a dial, may perhaps not be provided with a globe to aſſiſt him, and may probably not underſtand the method of doing it by logarithmic calculation; we ſhall ſhew how to perform it by the plain dialing lines, or ſcale of latitudes and hours; ſuch as thoſe on the right hand of Fig. 4. in Plate XXI, or at the top of Plate XXII, and which may be had on ſcales commonly ſold by the mathematical inſtrument makers.

This is the eaſieſt of all mechanical methods, and by much the beſt, when the lines are truly divided: and not only the half hours and quarters may be laid down by all of them, but every fifth minute by moſt, and every ſingle minute by thoſe where the line of hours is a foot in length.

Fig. 3.

Having drawn your double meridian line *ab*, *cd*, on the plane intended for a horizontal dial, and croſſed it at right angles by the ſix o'clock line *fe*, (as in Fig. 1.) take the latitude of your place with the compaſſes, in the ſcale of latitudes, and ſet that extent from *c* to *e*, and from *a* to *f*, on the ſix o'clock line: then, taking the whole ſix hours between the points of the compaſſes in the ſcale of hours, with that extent ſet one foot in the point *c*, and let the other foot fall where it will upon the meridian line *cd*, as at *d*. Do the ſame from *f* to *b*, and draw the right lines *ed* and *fb*, each of which will be equal in length to the

whole

whole ſcale of hours. This done, ſetting one foot of the compaſſes in the beginning of the ſcale at XII, and extending the other to each hour on the ſcale, lay off theſe extents from *d* to *e* for the afternoon hours, and from *b* to *f* for thoſe of the forenoon: this will divide the lines *de* and *bf* in the ſame manner as the hour-ſcale is divided, at 1, 2, 3, 4, and 6; on which the quarters may alſo be laid down, if required. Then, laying a ruler on the point *c*, draw the firſt five hours in the afternoon, from that point, through the dots at the numeral figures 1, 2, 3, 4, 5, on the line *de*; and continue the lines of IIII and V through the center *c* to the other ſide of the dial, for the like hours of the morning: which done, lay the ruler on the point *a*, and draw the laſt five hours in the forenoon through the dots 5, 4, 3, 2, 1, on the line *fb*; continuing the hour-lines of VII and VIII through the center *a* to the other ſide of the dial, for the like hours of the evening; and ſet the hours to their reſpective lines, as in the figure. Laſtly, make the gnomon the ſame way as taught above for the horizontal dial; and the whole will be finiſhed.

To make an erect ſouth dial, take the co-latitude of your place from the ſcale of latitudes, and then proceed in all reſpects for the hour-lines, as in the horizontal dial; only reverſing the hours, as in Fig. 2; and making the angle of the ſtile's height equal to the co-latitude.

I have drawn out a ſet of dialing lines upon the top of the 22d Plate, large enough for making a dial

dial of 9 inches diameter, or more inches if required; and have drawn them tolerably exact for common practice, to every quarter of an hour. This scale may be cut off from the plate, and pasted on wood, or upon the inside of one of the boards of this book; and then it will be somewhat more exact than as it is on the plate, for, being rightly divided upon the copper-plate, and printed off on wet paper, it shrinks as the paper dries: but when it is wetted again, it stretches to the same size as when newly printed; and if pasted on while wet, it will remain of that size afterward.

But lest the young *tyro* should have neither globe nor wooden scale, and should tear or otherwise spoil the paper one in pasting, we shall now shew him how he may make a dial without any of these helps. Only, if he has not a line of chords, he must divide a quadrant into 90 equal parts or degrees for taking the proper angle of the stile's elevation; which is easily done.

With any opening of the compasses, as ZL, describe the two semicircles LFk and LQk, upon the centers Z and z, where the six o'clock line crosses the double meridian line; and divide each semicircle into 12 equal parts, beginning at L (though, strictly speaking, only the quadrants from L to the six o'clock line need be divided:) then connect the divisions which are equidistant from L, by the parallel lines KM, IN, HO, GP, and FQ. Draw VZ for the hypothenuse of the stile, making the angle VZE equal to the latitude of your place; and continue the line VZ to R. Draw the line Rr parallel

Fig. 4.

Horizontal dial.

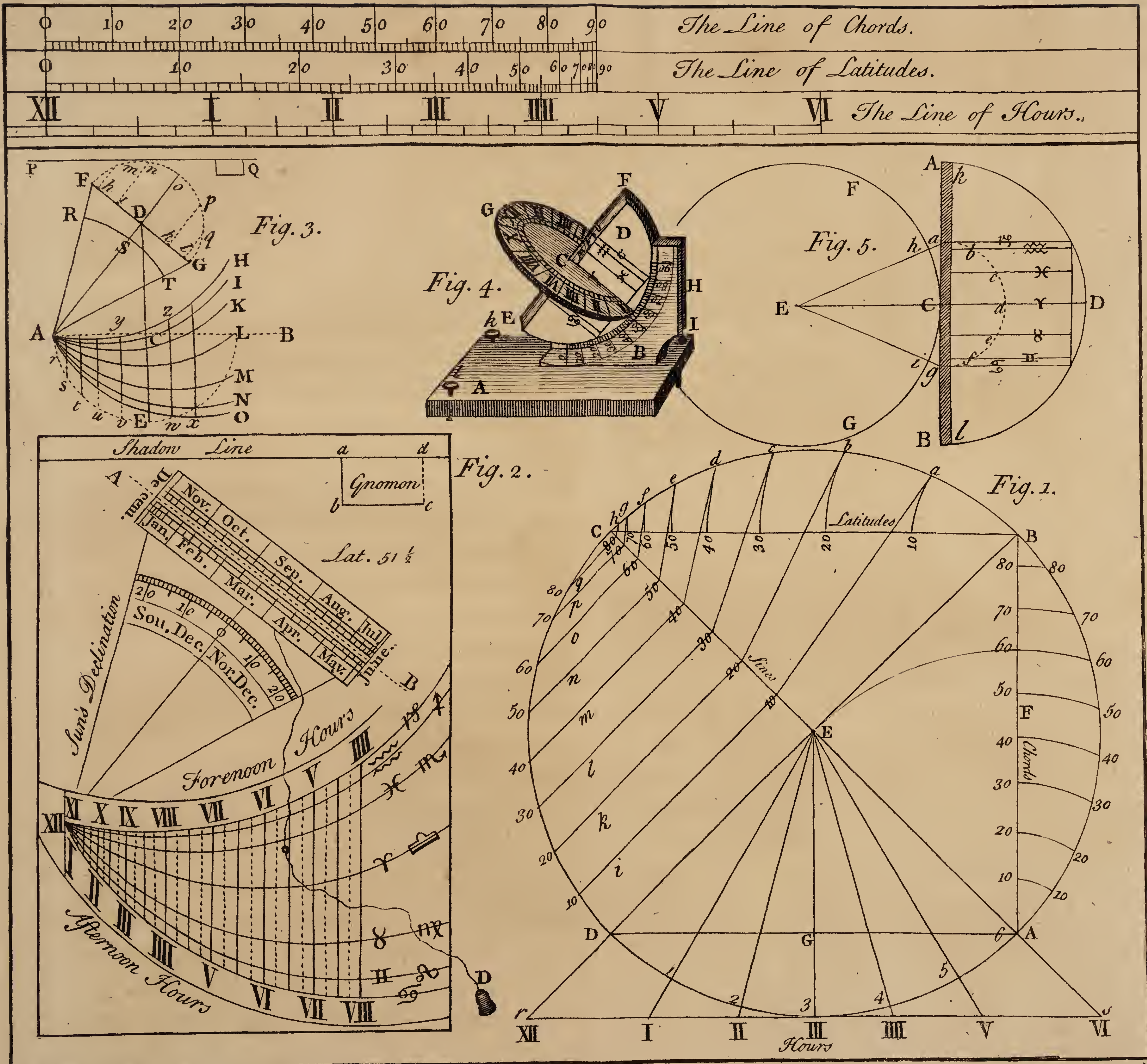
parallel to the ſix o'clock line, and ſet off the diſtance at *aK* from *Z* to *Y*, the diſtance *bI* from *Z* to *X*, *cH* from *Z* to *W*, *dG* from *Z* to *T*, and *eF* from *Z* to *S*. Then draw the lines *Ss*, *Tt*, *Ww*, *Xx*, and *Yy*, each parallel to *Rr*. Set off the diſtance *yY* from *a* to 11, and from *f* to 1; the diſtance *xX* from *b* to 10, and from *g* to 2; *wW* from *c* to 9, and from *h* to 3; *tT* from *d* to 8, and from *i* to 4; *sS* from *e* to 7, and from *n* to 5. Then laying a ruler to the center *Z*, draw the forenoon hour-lines through the points 11, 10, 9, 8, 7; and laying it to the center *z*, draw the afternoon lines through the points 1, 2, 3, 4, 5; continuing the forenoon lines of VII and VIII through the center *Z*, to the oppoſite ſide of the dial, for the like afternoon hours; and the afternoon lines IIII and V through the center *z* to the oppoſite ſide for the like morning hours. Set the hours to theſe lines as in the figure, and then erect the ſtile or gnomon, and the horizontal dial will be finiſhed.

South dial. To conſtruct a ſouth dial, draw the line *VZ*, making an angle with the meridian *ZL* equal to the co-latitude of your place; and proceed in all reſpects as in the above horizontal dial for the ſame latitude, reverſing the hours as in Fig. 2, and making the elevation of the gnomon equal to the co-latitude.

Perhaps it may not be unacceptable to explain the method of conſtructing the dialing lines, and ſome others; which is as follows.

With

PLATE XXII.



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With any opening of the compaſſes, as *EA*, according to the intended length of the ſcale, deſcribe the circle *ABCD*, and croſs it at right angles by the diameters *AEC* and *BED*. Divide the quadrant *AB* firſt into 9 equal parts, and then each part into 10; ſo ſhall the quadrant be divided into 90 equal parts or degrees. Draw the right line *AFB* for the chord of this quadrant, and ſetting one foot of the compaſſes in the point *A*, extend the other to the ſeveral diviſions of the quadrant, and transfer theſe diviſions to the line *AFB* by the arcs 10 10, 20 20, &c. and this will be a line of chords, divided into 90 unequal parts: which, if transferred from the line back again to the quadrant, will divide it equally. It is plain by the figure that the diſtance from *A* to 60 in the line of chords, is juſt equal to *AE*, the radius of the circle from which that line is made; for if the arc 60 60 be continued, of which *A* is the center, it goes exactly through the center *E* of the arc *AB*.

Pl. XXII. Fig. 1.

Dialing lines how conſtructed.

And therefore, in laying down any number of degrees on a circle, by the line of chords, you muſt firſt open the compaſſes ſo, as to take in juſt 60 degrees upon that line, as from *A* to 60; and then, with that extent as a radius, deſcribe a circle, which will be exactly of the ſame ſize with that from which the line was divided: which done, ſet one foot of the compaſſes in the beginning of the chord-line, as at *A*, and extend the other to the number of degrees you want upon the line; which extent, applied to the circle, will include the like number of degrees upon it.

Divide

Divide the quadrant *C D* into 90 equal parts, and from each point of divifion draw right lines, as *i*, *k*, *l*, &c. to the line *CE*, all perpendicular to that line, and parallel to *DE*, which will divide *EC* into a line of fines; and although thefe are feldom put among the dialing lines on a fcale, yet they affift in drawing the line of latitudes. For, if a ruler be laid upon the point *D*, and over each divifion in the line of fines, it will divide the quadrant *BC* into 90 unequal parts, as *Ba*, *Bb*, &c. fhewn by the right lines 10*a*, 20*b*, 30*c*, &c. drawn along the edge of the ruler. If the right line *BC* be drawn, fubtending this quadrant, and the neareft diftances *Ba*, *Bb*, *Bc*, &c. be taken in the compaffes from *B*, and fet upon this line in the fame manner as directed for the line of chords; we have a line of latitudes *BC*, equal in length to the line of chords *AB*, and of an equal number of divifions, but very unequal as to their lengths.

Draw the right line *DGA*, fubtending the quadrant *DA*, and parallel to it draw the right line *rs*, touching the quadrant *DA* at the numeral figure 3. Divide this quadrant into fix equal parts, as 1, 2, 3, &c. and through thefe points of divifion draw right lines from the center *E* to the line *rs*, which will divide it at the points where the fix hours are to be placed, as in the figure. If every fixth part of the quadrant be fub-divided into four equal parts, right lines drawn from the center through thefe points of divifion, and continued to the line *rs*, will divide each hour upon it into quarters.

In

In Fig. 2. we have the reprefentation of a portable dial, which may be eafily drawn on a card, and carried in a pocket-book. The lines *ad*, *ab*, and *ac* of the gnomon are cut quite through the card; and as the end *ab* of the gnomon is raifed occafionally above the plane of the dial, it turns, as it were, upon the uncut line *cd*. The line *AB* is flit quite through the card, and the thread *C* is put through the flit, and has a knot tied behind, to keep it from being eafily drawn out. On the other end of this thread is a fmall plummet *D*, and on the middle of it a fmall bead for fhewing the hour of the day.

Fig. 2. A *dial* on a *card*.

To rectify this dial, fet the thread in the flit, right againft the day of the month, and ftretch the thread from the day of the month over the angular point where the curve lines meet at XII, then fhift the bead to that point on the thread, and the dial will be rectified.

To find the hour of the day, raife the gnomon (no matter how much or how little) and hold the edge of the dial next the gnomon towards the fun, fo as the uppermoft edge of the fhadow of the gnomon may juft cover the *fhadow-line*; and the bead then playing freely on the face of the dial, by the weight of the plummet, will fhew the time of the day among the hour-lines, as it is forenoon or afternoon.

To find the time of fun-rifing and fetting, move the thread among the hour-lines, until it either covers fome one of them, or lies parallel betwixt any two; and then it will cut the time of fun-

riſing among the forenoon hours, and of ſun-ſetting among the afternoon-hours, for that day of the year to which the thread is ſet in the ſcale of months.

To find the ſun's declination, ſtretch the thread from the day of the month, over the angular point at twelve, and it will cut the ſun's declination, as it is north or ſouth, for that day, in the proper ſcale.

To find on what days the ſun enters the ſigns: when the bead, as above rectified, moves along any of the curve lines which have the ſigns of the zodiac marked upon them, the ſun enters thoſe ſigns on the days pointed out by the thread in the ſcale of months.

Fig. 3. The conſtruction of this dial is very eaſy, eſpecially if the reader compares it all along with Fig. 3, as he reads the following explanation of that figure.

Draw the occult line *AB* parallel to the top of the card, and croſs it at right angles with the ſix o'clock line *ECD*. Then, upon *C*, as a center with the radius *CA*, deſcribe the ſemicircle *AEL*, and divide it into 12 equal parts (beginning at *A*) as *Ar*, *As*, &c. and from theſe points of diviſion, draw the hour-lines *r*, *s*, *t*, *u*, *v*, *E*, *w*, and *x*, all parallel to the ſix o'clock line *EC*. If each part of the ſemicircle be ſubdivided into four equal parts, they will give the half hour lines and quarters (as in Fig. 2.) Draw the right line *ASDo*, making the angle *SAB* equal to the latitude of your place. Upon the center *A*, deſcribe the arch *RST*, and ſet

ſet off upon it the arcs *SR* and *ST*, each equal to $23\frac{1}{2}$ degrees, for the ſun's greateſt declination; and divide them accordingly, as in Fig. 2. Through the interſection *D* of the lines *ECD* and *ADo*, draw *FDG* at right angles to *ADo*. Lay a ruler on the points *A* and *R*, and draw the line *ARF* through $23\frac{1}{2}$ degrees of ſouth declination in the arc *SR*; and then laying the ruler over the points *A* and *T*, draw the line *ATG* through $23\frac{1}{2}$ degrees of north declination in the arc *ST*: ſo ſhall the lines *ARF* and *ATG* cut the line *FDG* in the proper length for the ſcale of months. Upon the center *D*, with the radius *DF*, deſcribe the ſemicircle *FoG*; which divide into ſix equal parts, *Fm*, *mn*, *no*, &c. and from theſe points of diviſion, draw the right lines *mh*, *ni*, *pk*, and *ql*, each parallel to *oD*. Then ſetting one foot of the compaſſes in the point *E*, extend the other to *A*, and deſcribe the arc *AzH* for the tropic of ♑: with the ſame extent, ſetting one foot in *G*, deſcribe the arc *AEO* for the tropic of ♋. Next, ſetting one foot in the point *h*, and extending the other to *A*, deſcribe the arc *ACI* for the beginning of ♒ and ♐; and with the ſame extent, ſetting one foot in the point *l*, deſcribe the arc *AN* for the beginning of ♊ and ♌. Set one foot in the point *i*, and having extended the other to *A*, deſcribe the arc *AK* for the beginning of ♓ and ♏; and with the ſame extent, ſet one foot in *k*, and deſcribe the arc *AM* for the beginning of ♉ and ♍. Then, ſetting one foot in the point *D*, and extending the other to *A*, deſcribe the curve *AL* for the beginning of ♈

 and

and ♎; and the ſigns will be finiſhed. This done, lay a ruler from the point *A* over the ſun's declination in the arch *RST* (found by the following table) for every fifth day of the year; and where the ruler cuts the line *FDG*, make marks; and place the days of the months right againſt theſe marks, in the manner ſhewn by Fig. 2. Laſtly, draw the ſhadow-line *PQ* parallel to the occult line *AB*; make the gnomon, and ſet the hours to their reſpective lines, as in Fig. 2. and the dial will be finiſhed.

There are ſeveral kinds of dials which are called *univerſal*, becauſe they ſerve for all latitudes. Of Fig. 4. theſe, the beſt one that I know, is Mr. *Pardie*'s, which conſiſts of three principal parts; the firſt An *univer-ſal dial.* whereof is called the *horizontal plane (A)*, becauſe in practice it muſt be parallel to the horizon. In this plane is fixed an upright pin, which enters into the edge of the ſecond part *BD*, called the *meridional plane*; which is made of two pieces, the loweſt whereof *(B)* is called the *quadrant*, becauſe it contains a quarter of a circle, divided into 90 degrees; and it is only into this part, near *B*, that the pin enters. The other piece is a *ſemicircle (D)* adjuſted to the quadrant, and turning in it by a groove, for raiſing or depreſſing the diameter *(EF)* of the ſemicircle, which diameter is called the *axis* of the inſtrument. The third piece is a *circle (G)*, divided on both ſides into 24 equal parts, which are the hours. This circle is put upon the meridional plane ſo, that the axis *(EF)* may be perpendicular to the circle; and the point

point *C* be the common center of the circle, semicircle, and quadrant. The ſtraight edge of the ſemicircle is chamfered on both ſides to a ſharp edge, which paſſes through the center of the circle. On one ſide of the chamfered part, the firſt ſix months of the year are laid down, according to the ſun's declination for their reſpective days; and on the other ſide, the laſt ſix months. And againſt the days on which the ſun enters the ſigns, there are ſtraight lines drawn upon the ſemicircle, with the characters of the ſigns marked upon them. There is a black line drawn along the middle of the upright edge of the quadrant, over which hangs a thread *(H)*, with its plummet *(I)*, for leveling the inſtrument. *N. B.* From the 23d of September to the 20th of March, the upper ſurface of the circle muſt touch both the center *C* of the ſemicircle, and the line of ♈ and ♎; and from the 20th of March to the 23d of September, the lower ſurface of the circle muſt touch that center and line.

To find the time of the day by this dial. Having ſet it on a level place in ſun-ſhine, and adjuſted it by the leveling ſcrews *k* and *l*, until the plumb line hangs over the black line upon the edge of the quadrant, and parallel to the ſaid edge; move the ſemicircle in the quadrant, until the line of ♈ and ♎ (where the circle touches) comes to the latitude of your place in the quadrant: then, turn the whole meridional plane *B D*, with its circle *G*, upon the horizontal plane *A*, until the edge of the ſhadow of the circle falls

 preciſely

precisely on the day of the month in the semicircle; and then, the meridional plane will be due north and south, the axis *EF* will be parallel to the axis of the world, and will cast a shadow upon the true time of the day, among the hours on the circle.

N. B. As, when the instrument is thus rectified, the quadrant and semicircle are in the plane of the meridian, so the circle is then in the plane of the equinoctial. Therefore, as the sun is above the equinoctial in summer, (in northern latitudes) and below it in winter; the axis of the semicircle will cast a shadow on the hour of the day, on the upper surface of the circle, from the 20th of March to the 23d of September: and from the 23d of September, to the 20th of March, the hour of the day will be determined by the shadow of the semicircle, upon the lower surface of the circle. In the former case, the shadow of the circle falls upon the day of the month, on the lower part of the diameter of the semicircle; and in the latter case, on the upper part.

Fig. 5. The method of laying down the months and signs upon the semicircle, is as follows. Draw the right line *ACB*, equal to the diameter of the semicircle *ADB*, and cross it in the middle at right angles with the line *ECD*, equal in length to *ADB*; then *EC* will be the radius of the circle *FCG*, which is the same as that of the semicircle. Upon *E*, as a center, describe the circle *FCG*, on which, set off the arcs *Ch* and *Ci*, each equal to $23\frac{1}{2}$ degrees, and divide them accordingly into that number, for the sun's declination. Then, laying

laying the edge of a ruler over the center *E*, and alſo over the ſun's declination for every * fifth day of each month, (as in the card dial) mark the points on the diameter *AB* of the ſemicircle, from *a* to *g*, which are cut by the ruler; and *there*, place the days of the months accordingly, anſwering to the ſun's declination. This done, ſetting one foot of the compaſſes in *C*, and extending the other to *a* or *g*, deſcribe the ſemicircle *abcdefg*; which divide into ſix equal parts, and through the points of diviſion draw right lines, parallel to *CD*, for the beginning of the ſines, (of which one half are on one ſide of the ſemicircle, and the other half on the other) and ſet the characters of the ſigns to their proper lines, as in the figure.

The following table ſhews the ſun's place and declination, in degrees and minutes, at the noon of every day of the ſecond year after leap-year; which is a mean between thoſe of leap-year itſelf, and the firſt and third years after. It is uſeful for inſcribing the months and their days on ſun-dials; and alſo for finding the latitudes of places, according to the methods preſcribed at the end of the table.

* The intermediate days may be drawn in by hand, if the ſpace be large enough to contain them.

A Table shewing the sun's place and declination.

January.					February.				
Days	Sun's Pl.		Sun's Dec.		Days	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	11 ♑	5	23 S	1	1	12 ♒	38	17 S	2
2	12	6	22	55	2	13	39	16	45
3	13	8	22	49	3	14	40	16	27
4	14	9	22	43	4	15	41	16	10
5	15	10	22	37	5	16	41	15	51
6	16	11	22	29	6	17	42	15	33
7	17	12	22	22	7	18	43	15	14
8	18	13	22	14	8	19	43	14	55
9	19	14	22	5	9	20	44	14	36
10	20	16	21	56	10	21	45	14	17
11	21	17	21	47	11	22	45	13	57
12	22	18	21	37	12	23	46	13	37
13	23	19	21	27	13	24	46	13	17
14	24	20	21	17	14	25	47	12	57
15	25	21	21	6	15	26	47	12	36
16	26	22	20	54	16	27	48	12	15
17	27	24	20	43	17	28	48	11	54
18	28	25	20	30	18	29	48	11	33
19	29	26	20	18	19	0 ♓	49	11	12
20	0 ♒	27	20	5	20	1	49	10	50
21	1	28	19	52	21	2	50	10	29
22	2	29	19	38	22	3	50	10	7
23	3	30	19	24	23	4	50	9	45
24	4	31	19	10	24	5	51	9	23
25	5	32	18	55	25	6	51	9	0
26	6	33	18	40	26	7	51	8	38
27	7	34	18	24	27	8	51	8	16
28	8	35	18	9	28	9	51	7	53
29	9	35	17	53					
30	10	36	17	36					
31	11	37	17	19					

In these Tables, N signifies north declination, and S south.

A Table ſhewing the ſun's place and declination.

March.					April.				
Days	Sun's Pl.		Sun's Dec.		Days	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	10♓	52	7 S	30	1	11♈	38	4 N	36
2	11	52	7	7	2	12	37	4	59
3	12	52	6	44	3	13	36	5	22
4	13	52	6	21	4	14	35	5	45
5	14	52	5	58	5	15	34	6	8
6	15	52	5	35	6	16	33	6	31
7	16	51	5	12	7	17	31	6	53
8	17	51	4	48	8	18	30	7	16
9	18	51	4	25	9	19	29	7	38
10	19	51	4	2	10	20	28	8	0
11	20	51	3	38	11	21	27	8	22
12	21	50	3	14	12	22	25	8	44
13	22	50	2	51	13	23	24	9	6
14	23	50	2	27	14	24	23	9	28
15	24	49	2	4	15	25	21	9	49
16	25	49	1	40	16	26	20	10	11
17	26	48	1	16	17	27	18	10	32
18	27	48	0	53	18	28	17	10	53
19	28	48	0	29	19	29	15	11	14
20	29	47	0	5	20	0♉	14	11	34
21	0♈	47	0 N	19	21	1	12	11	55
22	1	46	0	42	22	2	11	12	15
23	2	45	1	6	23	3	9	12	35
24	3	45	1	29	24	4	7	12	55
25	4	44	1	53	25	5	6	13	14
26	5	43	2	17	26	6	4	13	34
27	6	42	2	40	27	7	2	13	53
28	7	42	3	3	28	8	0	14	12
29	8	41	3	27	29	8	59	14	30
30	9	40	3	50	30	9	57	14	49
31	10	39	4	13					

A Table shewing the sun's place and declination.

May.					June.				
Days	Sun's Pl.		Sun's Dec.		Days	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	10 ♉	55	15 N	7	1	10 ♊	44	22 N	5
2	11	53	15	25	2	11	41	22	13
3	12	51	15	43	3	12	39	22	21
4	13	49	16	0	4	13	36	22	28
5	14	47	16	18	5	14	34	22	35
6	15	45	16	35	6	15	31	22	41
7	16	43	16	51	7	16	28	22	47
8	17	41	17	8	8	17	26	22	53
9	18	39	17	24	9	18	23	22	58
10	19	36	17	40	10	19	20	23	3
11	20	34	17	55	11	20	18	23	7
12	21	32	18	10	12	21	15	23	11
13	22	30	18	25	13	22	12	23	15
14	23	28	18	40	14	23	9	23	18
15	24	25	18	54	15	24	7	23	20
16	25	23	19	8	16	25	4	23	22
17	26	21	19	22	17	26	1	23	24
18	27	19	19	35	18	26	58	23	26
19	28	16	19	48	19	27	56	23	27
20	29	14	20	1	20	28	53	23	28
21	0 ♊	11	20	13	21	29	50	23	28
22	1	9	20	25	22	0 ♋	47	23	28
23	2	7	20	37	23	1	45	23	28
24	3	4	20	48	24	2	42	23	27
25	4	2	20	59	25	3	39	23	26
26	4	59	21	10	26	4	36	23	24
27	5	57	21	20	27	5	33	23	21
28	6	54	21	30	28	6	31	23	19
29	7	52	21	39	29	7	28	23	16
30	8	49	21	48	30	8	25	23	12
31	9	47	21	57					

A Table shewing the sun's place and declination.

July.					August.				
Days	Sun's Pl.		Sun's Dec.		Days	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	9♋	22	23 N	8	1	8♌	58	18 N	2
2	10	19	23	4	2	9	55	17	47
3	11	16	23	0	3	10	53	17	32
4	12	14	22	55	4	11	50	17	16
5	13	11	22	49	5	12	48	17	0
6	14	8	22	43	6	13	45	16	43
7	15	5	22	37	7	14	43	16	26
8	16	0	22	30	8	15	41	16	9
9	17	2	22	23	9	16	38	15	52
10	17	57	22	16	10	17	36	15	35
11	18	54	22	8	11	18	33	15	17
12	19	51	22	0	12	19	31	14	59
13	20	49	21	52	13	20	29	14	41
14	21	46	21	43	14	21	26	14	23
15	22	43	21	33	15	22	24	14	4
16	23	40	21	22	16	23	22	13	45
17	24	38	21	14	17	24	20	13	26
18	25	35	21	3	18	25	17	13	7
19	26	32	20	52	19	26	15	12	47
20	27	29	20	41	20	27	13	12	27
21	28	27	20	30	21	28	11	12	7
22	29	24	20	18	22	29	9	11	47
23	0♌	21	20	6	23	0♍	7	11	27
24	1	19	19	54	24	1	5	11	6
25	2	16	19	41	25	2	3	10	46
26	3	13	19	28	26	3	1	10	25
27	4	11	19	14	27	3	59	10	4
28	5	8	19	1	28	4	57	9	43
29	6	6	18	46	29	5	55	9	21
30	7	3	18	32	30	6	53	9	0
31	8	0	18	17	31	7	51	8	38

A Table shewing the sun's place and declination.

September.					October.				
Days	Sun's Pl.		Sun's Dec.		Days	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	8 ♍	49	8 N	16	1	8 ♎	8	3 S	14
2	9	47	7	55	2	9	7	3	37
3	10	46	7	33	3	10	7	4	1
4	11	44	7	10	4	11	6	4	24
5	12	42	6	48	5	12	5	4	47
6	13	40	6	26	6	13	4	5	10
7	14	39	6	3	7	14	4	5	33
8	15	37	5	41	8	15	3	5	56
9	16	35	5	18	9	16	3	6	19
10	17	34	4	55	10	17	2	6	42
11	18	32	4	32	11	18	1	7	5
12	19	31	4	9	12	19	1	7	27
13	20	29	3	46	13	20	0	7	50
14	21	28	3	23	14	21	0	8	12
15	22	26	3	0	15	22	0	8	35
16	23	25	2	37	16	23	0	8	57
17	24	24	2	14	17	23	59	9	19
18	25	22	1	50	18	24	59	9	41
19	26	21	1	27	19	25	58	10	3
20	27	20	1	4	20	26	58	10	24
21	28	19	0	40	21	27	58	10	46
22	29	17	0	17	22	28	58	11	7
23	0 ♎	16	0 S	6	23	29	58	11	28
24	1	15	0	30	24	0 ♏	58	11	49
25	2	14	0	53	25	1	58	12	10
26	3	13	1	17	26	2	58	12	31
27	4	12	1	40	27	3	58	12	51
28	5	11	2	4	28	4	58	13	12
29	6	10	2	27	29	5	58	13	32
30	7	9	2	50	30	6	58	13	51
					31	7	58	14	11

A Table shewing the sun's place and declination.

November.					December.				
Days	Sun's Pl.		Sun's Dec.		Days	Sun's Pl.		Sun's Dec.	
	D.	M.	D.	M.		D.	M.	D.	M.
1	8 ♏	58	14 S	30	1	9 ♐	16	21 S	52
2	9	58	14	50	2	10	17	22	1
3	10	59	15	8	3	11	18	22	10
4	11	59	15	27	4	12	19	22	18
5	12	59	15	45	5	13	20	22	26
6	13	59	16	3	6	14	21	22	33
7	15	0	16	21	7	15	22	22	40
8	16	0	16	39	8	16	23	22	46
9	17	0	16	56	9	17	24	22	52
10	18	1	17	13	10	18	25	22	58
11	19	1	17	30	11	19	26	23	3
12	20	2	17	46	12	20	27	23	8
13	21	2	18	2	13	21	28	23	12
14	22	3	18	18	14	22	29	23	15
15	23	4	18	34	15	23	30	23	18
16	24	4	18	49	16	24	31	23	21
17	25	5	19	4	17	25	33	23	24
18	26	5	19	18	18	26	34	23	26
19	27	6	19	32	19	27	35	23	27
20	28	7	19	46	20	28	36	23	28
21	29	7	19	59	21	29	37	23	28
22	0 ♐	8	20	12	22	0 ♑	38	23	28
23	1	9	20	25	23	1	40	23	28
24	2	10	20	37	24	2	41	23	27
25	3	11	20	49	25	3	42	23	25
26	4	11	21	1	26	4	43	23	23
27	5	12	21	12	27	5	44	23	21
28	6	13	21	23	28	6	46	23	18
29	7	14	21	33	29	7	47	23	15
30	8	15	21	43	30	8	48	23	11
					31	9	49	23	6

To find the latitude of any place by obſervation.

The latitude of any place, is equal to the elevation of the pole above the horizon of that place. Therefore it is plain, that if there was a ſtar fixt in the pole, there would be nothing required to find the latitude, but to take the height of that ſtar with a good inſtrument. But although there is no ſtar in the pole, yet the latitude may be found by taking the greateſt and leaſt height of any ſtar that never ſets: for if half the difference between theſe heights be added to the leaſt height, or ſubtracted from the greateſt, the ſum or remainder, will be equal to the height of the pole at the place of obſervation.

But becauſe the night muſt be more than 12 hours in length, in order to have two ſuch obſervations; the ſun's meridian altitude and declination are generally made uſe of for finding the latitude, by means of its complement, which is equal to the elevation of the equinoctial above the horizon; and if this complement be ſubtracted from 90 degrees, the remainder is the latitude: concerning which, I think the following rules take in all the various caſes.

1. If the ſun has north declination, and is on the meridian to the ſouth of your place, ſubtract the declination from the meridian altitude, (taken by a good quadrant) and the remainder is the height of the equinoctial or complement of the latitude north.

EX-

EXAMPLE.

Suppose { The ſun's meridian altitude	42°	20′	South
And his declination, (ſubtr.)	10	15	North
Rem. the comp. of the latitude	32	5	
Which ſubtract from — —	90	0	
And the rem. is the latitude	57	55	North.

2. If the ſun has ſouth declination, and is ſouthward of your place at noon, add the declination to the meridian altitude; the ſum, if leſs than 90 degrees, is the complement of the latitude north: but if the ſum exceeds 90 degrees, the latitude is ſouth, and if 90 be taken from that ſum, the remainder will be the latitude.

EXAMPLES.

The ſun's meridian altitude	65°	10′	South
The ſun's declination, (add)	15	30	South
Comp. of the latitude —	80	40	
Subtract from — —	90	0	
Rem. the latitude —	9	20	North.

The ſun's meridian altitude	80°	40′	South
The ſun's declination, (add)	20	10	South
The ſum is - - - -	100	50	
From which ſubtract -	90	0	
Remains the latitude —	10	50	South.

3. If

3. If the ſun has north declination, and is on the meridian north of your place, add the declination to the north meridian altitude; the ſum, if leſs than 90 degrees, is the complement of the latitude ſouth: but if the ſum is more than 90 degrees, ſubtract 90 from it, and the remainder is the latitude north.

EXAMPLES.

Sun's meridian altitude -	60°	30′	North
Sun's declination, add - -	20	10	North
Compl. of the latitude - -	80	40	
Subtract from - - - -	90	0	
Remains the latitude - -	9	20	South.

Sun's meridian altitude -	70°	20′	North
Sun's declination, add - -	23	20	North
The ſum is - - - -	93	40	
From which ſubtract - -	90	0	
Remains the latitude - -	3	40	North.

4. If the ſun has ſouth declination, and is north of your place at noon, ſubtract the declination from the north meridian altitude, and the remainder is the complement of the latitude ſouth.

EXAM-

EXAMPLE.

Sun's meridian altitude - -	52° 30′	North
Sun's declination, ſubtract -	20 10	South
Compl. of the latitude - -	32 20	
Subtract this from - - -	90 0	
And the rem. is the latitude	57 40	South.

5. If the ſun has no declination, and is ſouth f your place at noon, the meridian altitude is the omplement of the latitude north: but if the ſun e then north of your place, his meridian altitude s the complement of the latitude ſouth.

EXAMPLES.

Sun's meridian altitude - -	38° 30′	South
Subtract from - - - -	90 0	
Remains the latitude -	51 30	North.

Sun's meridian altitude - -	38° 30′	North
Subtract from - - - -	90 0	
Remains the latitude -	51 30	South.

6. If you obſerve the ſun beneath the pole, btract his declination from 90 degrees, and add e remainder to his altitude; and the ſum is the titude.

EXAMPLE.

Sun's declination - - -	20°	30′	
Subtract from - - - -	90	0	
Remains - - - - - -	69	30	add
Sun's altitude below the pole	10	20	
The ſum is the latitude -	79	50.	

Which is north or ſouth, according as the ſun's declination is north or ſouth: for when the ſun has ſouth declination, he is never ſeen below the north pole; nor is he ever ſeen below the ſouth pole, when his declination is north.

7. If the ſun be in the zenith at noon, and at the ſame time has no declination, you are then under the equinoctial, and ſo have no latitude.

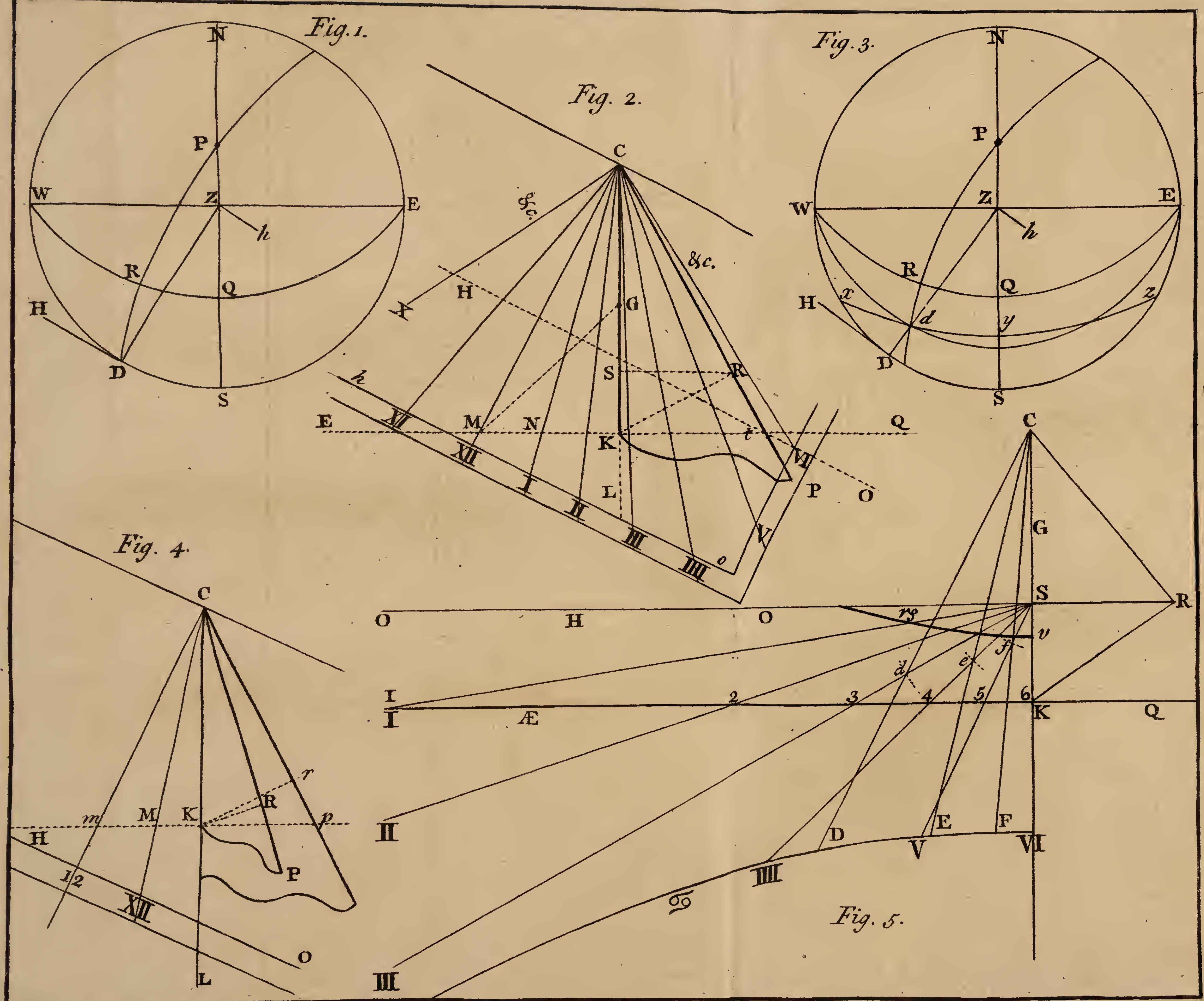
8. If the ſun be in the zenith at noon, and has declination, the declination is equal to the latitude, north or ſouth. Theſe two caſes require no examples.

LECT. X.

Of dialing.

HAVING ſhewn in the preceding Lecture how to make ſun-dials by the aſſiſtance of a good globe, or of a dialing ſcale, we ſhall now proceed to the method of conſtructing them arithmetically; which will be more agreeable to thoſe who

PLATE XXIII.



J. Ferguson del.

J. Mynde sc.

who have learnt the elements of trigonometry, because globes and scales can never be so accurate as the logarithms, in finding the angular distances of the hours. Yet, as a globe may be found exact enough for some other requisites in dialing, we shall take it in occasionally.

The construction of sun-dials on all planes whatever, may be included in one general rule: intelligible, if that of a horizontal dial for any given latitude be well understood. For there is no plane, however obliquely situated at any given place, but what is parallel to the horizon of some other place; and therefore, if we can find that other place by a problem on the terrestrial globe, or by a trigonometrical calculation, and construct a horizontal dial for it; that dial, applied to the plane where it is to serve, will be a true dial for that place.—Thus, an erect direct south dial in $51\frac{1}{2}$ degrees north latitude, would be a horizontal dial in the same meridian, 90 degrees southward of $51\frac{1}{2}$ degrees north latitude; which falls in with $38\frac{1}{2}$ degrees of south latitude. But if the upright plane declines from facing the south at the given place, it would still be a horizontal plane 90 degrees from that place; but for a different longitude: which would alter the reckoning of the hours accordingly.

CASE I.

1. Let us suppose that an upright plane at London, declines 36 degrees westward from facing

the ſouth; and that it is required to find a place on the globe, to whoſe horizon the ſaid plane is parallel; and alſo the difference of longitude between London and that place.

Rectify the globe to the latitude of London, and bring London to the zenith under the braſs meridian, then, that point of the globe which lies in the horizon at the given degree of declination, (counted weſtward from the ſouth point of the horizon) is the place at which the abovementioned plane would be horizontal.—Now, to find the latitude and longitude of that place, keep your eye upon the place, and turn the globe eaſtward, until it comes under the graduated edge of the braſs meridian; then, the degree of the braſs meridian that ſtands directly over the place, is its latitude; and the number of degrees in the equator, that are intercepted between the meridian of London and braſs meridian, is the place's difference of longitude.

Thus, as the latitude of London is $51\frac{1}{2}$ degrees north, and the declination of the place is 36 degrees weſt; I elevate the north pole $51\frac{1}{2}$ degrees above the horizon, and turn the globe until London comes to the zenith, or under the graduated edge of the meridian; then, I count 36 degrees on the horizon, weſtward from the ſouth point, and make a mark on that place of the globe over which the reckoning ends, and bringing the mark under the graduated edge of the braſs meridian, I find it to be under $30\frac{1}{4}$ degrees in ſouth latitude; keeping it there, I count in the equator

equator the number of degrees between the meridian of London and the braſen meridian, (which now becomes the meridian of the required place) and find it to be $42\frac{3}{4}$. Therefore, an upright plane at London, declining 36 degrees weſtward from the ſouth, would be a horizontal plane at that place, whoſe latitude is $30\frac{1}{4}$ degrees ſouth of the equator, and longitude $42\frac{3}{4}$ degrees weſt of the meridian of London.

Which difference of longitude being converted into time, is 2 hours 51 minutes.

The vertical dial, declining weſtward 36 degrees at London, is therefore to be drawn in all reſpects as a horizontal dial for ſouth latitude $30\frac{1}{4}$ degrees; ſave only, that the reckoning of the hours is to anticipate the reckoning on the horizontal dial, by 2 hours 51 minutes: for ſo much ſooner will the ſun come to the meridian of London, than to the meridian of any place whoſe longitude is $42\frac{3}{4}$ degrees weſt from London.

2. But, to be more exact than the globe will ſhew us, we ſhall uſe a little trigonometry.

Let $NESW$ be the horizon of London, whoſe zenith is Z, and P the north pole of the ſphere: and let Zh be the poſition of a vertical plane at Z, declining weſtward from S, (the ſouth) at an angle of 36 degrees; on which plane an erect dial for London at Z is to be deſcribed. Make the ſemidiameter ZD perpendicular to Zh, and it will cut the horizon in D, 36 degrees weſt of the ſouth S. Then, a plane in the tangent HD, touching the ſphere in D, will be parallel to the plane Zh; Pl. XXIII. Fig. 1.

 and

and the axis of the ſphere will be equally inclined to both theſe planes.

Let WQE be the equinoctial, whoſe elevation above the horizon of Z (London) is $38\frac{1}{2}$ degrees; and PRD be the meridian of the place D, cutting the equinoctial in R. Then, it is evident, that the arc RD is the latitude of the place D, (where the plane Zb would be horizontal) and the arc RQ is the difference of longitude of the planes Zb and DH.

In the ſpherical triangle WDR, the arc WD is given, for it is the complement of the plane's declination from S the ſouth; which complement is $54°$ (*viz.* $90°—36°$): the angle at R, in which the meridian of the place D cuts the equator, is a right angle; and the angle RWD meaſures the elevation of the equinoctial above the horizon of Z, namely $38\frac{1}{2}$ degrees. Say therefore, as radius is to the co-ſine of the plane's declination from the ſouth, ſo is the co-ſine of the latitude of Z to the ſine of RD the latitude of D: which is of a different denomination from the latitude of Z, becauſe Z and D are on different ſides of the equator.

As radius	—	—	10.00000
To co-ſine	$36°\ 0' = RQ$		9.90796
So co-ſine	$51°\ 30' = QZ$		9.79415

To ſine $30°\ 14' = DR$ (9.70211) = the lat. of D, whoſe horizon is parallel to the vertical plane Zb at Z.

N. B.

N. B. When radius is made the first term, it may be omitted, and then, by subtracting it mentally from the sum of the other two, the operation will be shortened. Thus, in the present case,

To the logarithmic sine of *WR* = *54° 0′ 9.90796
Add the logarithmic sine of *RD* = †38° 30′ 9.79415

Their sum — radius — — — 9.70211
gives the same solution as above. And we shall keep to this method in the following part of the work.

To find the difference of longitude of the places *D* and *Z*, say, as radius is to the co-sine of $38\frac{1}{2}$ degrees, the height of the equinoctial at *Z*, so is the co-tangent of 36 degrees, the plane's declination, to the co-tangent of the difference of longitudes. Thus,

To the log. sine of ** 51° 30′ — 9.89354
Add the log. tang. of ‡ 54° 0′ — 10.13874

Their sum — radius — — 10.03228 is the nearest tangent of 47° 8′, = *WR*; which is the co-tangent of 42° 52′, = *RQ* the difference of longitude sought. Which difference, being reduced to time, is 2 h. $51\frac{1}{2}$ m.

* The co-sine of 36° 0′, or of *RQ*.
† The co-sine of 51° 30′, or of *QZ*.
** The co-sine of 38° 30″, or of *WDR*.
‡ The co-tangent of 36°, or of *DW*.

3. And thus having found the exact latitude and longitude of the place *D*, to whose horizon the vertical plane at *Z* is parallel, we shall proceed to the construction of a horizontal dial for the place *D*, whose latitude is 30° 14′ south; but anticipating the time at *D* by 2 hours 51 minutes, (neglecting the ½ minute in practice) because *D* is so far westward in longitude from the meridian of London; and this will be a true vertical dial at London, declining westward 36 degrees.

Fig. 2. Assume any right line *CSL* for the substile of the dial, and make the angle *KCP* equal to the latitude of the place (*viz.* 30° 14′) to whose horizon the plane of the dial is parallel; then *CRP* will be the axis of the stile, or edge that casts the shadow on the hours of the day in the dial. This done, draw the contingent line *EQ*, cutting the substilar line at right angles in *K*; and from *K* make *KR* perpendicular to the axis *CRP*. Then *KG* (= *KR*) being made radius, that is, equal to the chord of 60° or tangent of 45° on a good sector, take 42° 52′ (the difference of longitude of the places *Z* and *D*) from the tangents, and having set it from *K* to *M*, draw *CM* for the hour-line of XII. Take *KN*, equal to the tangent of an angle less by 15 degrees than *KM*; that is, the tangent 27° 52′; and through the point *N* draw *CN* for the hour-line of *I*. The tangent of 12° 52′, (which is 15° less than 27° 52′) set off the same way, will give a point between *K* and *N*, through which the hour-line of II is to be drawn. The tangent of 2° 8′ (the difference between 45° and

and 42° $52'$) placed on the other ſide of *CL*, will determine the point through which the hour-line of III is to be drawn: to which 2° $8'$, if the tangent of 15° be added, it will make 17° $8'$; and this ſet off from *K* towards *Q*, on the line *EQ*, will give a point for the hour-line of IIII: and ſo of the reſt.—The forenoon hour-lines are drawn the ſame way, by the continual addition of the tangents 15°, 30°, 45°, *&c.* to 42° $52'$ (= the tangent of *KM*) for the hours of XI, X, IX, *&c.* as far as neceſſary; that is, until there be five hours on each ſide of the ſubſtile. The ſixth hour, accounted from that hour or part of the hour on which the ſubſtile falls, will be always in a line perpendicular to the ſubſtile, and drawn through the center *C*.

4. In all erect dials, *CM*, the hour-line of XII, is perpendicular to the horizon of the place for which the dial is to ſerve: for that line is the interſection of a vertical plane with the plane of the meridian of the place, both which are perpendicular to the plane of the horizon: and any line *HO*, or *ho*, perpendicular to *CM*, will be a horizontal line on the plane of the dial, along which line the hours may be numbered; and *CM* being ſet perpendicular to the horizon, the dial will have its true poſition.

5. If the plane of the dial had declined by an equal angle toward the eaſt, its deſcription would have differed only in this, that the hour-line of XII would have fallen on the other ſide of the ſubſtile

ſubſtile *CL*, and the line *HO* would have a ſub-contrary poſition to what it has in this figure.

6. And theſe two dials, with the upper points of their ſtiles turned toward the north pole, will ſerve for other two planes parallel to them; the one declining from the north toward the eaſt, and the other from the north toward the weſt, by the ſame quantity of angle. The like holds true of all dials in general, whatever be their declination, and obliquity of their planes to the horizon.

CASE II.

Fig. 3. 7. If the plane of the dial not only *declines*, but alſo *reclines*, or *inclines*. Suppoſe its declination from fronting the ſouth *S*, be equal to the arc *SD* on the horizon; and its reclination be equal to the arc *Dd*, of the vertical circle *DZ*: then it is plain, that if the quadrant of altitude *ZdD*, on the globe, cuts the point *D* in the horizon, and the reclination is counted upon the quadrant from *D* to *d*; the interſection of the hour-circle *PRd*, with the equinoctial *WQE*, will determine *Rd*, the latitude of the place *d*, whoſe horizon is parallel to the given plane at *Z*; and *RQ* will be the difference in longitude of the planes at *d* and *Z*.

Trigonometrically thus: let a great circle paſs through the three points, *W*, *d*, *E*; and in the triangle *WDd*, right angled at *D*, the ſides *WD* and *Dd* are given; and thence the angle *DWd* is found, and ſo is the hypothenuſe *Wd*. Again, the difference, or the ſum, of *DWd*, and *DWR* the

the elevation of the equinoctial above the horizon of *Z*, gives the angle *dWR*; and the hypothenuſe of the triangle *WRd*, was juſt now found: whence the ſides *Rd* and *WR* are found, the former being the latitude of the place *d*, and the latter the complement of *RQ* the difference of longitude ſought.

Thus, if the latitude of the place *Z* be 52° 10′ north, the declination *SD* of the plane *Zh* (which would be horizontal at *d*) be 36°, and the reclination be 15°, or equal to the arc *Dd*; the ſouth latitude of the place *d*, that is, the arc *Rd*, will be 15° 9′; and *RQ* the difference of longitude 36° 2′. From theſe data therefore, let the dial (Fig. 4.) be deſcribed; as in the former example.

8. Only it is to be obſerved, that in reclining or inclining dials, the horizontal line will not ſtand at right angles to the hour-line of XII, as in erect dials; but its poſition may be found as follows.

To the common ſubſtilar line *CKL*, on which Fig. 4. the dial for the place *d* was deſcribed, draw the dial *Crpm* 12 for the place *D*, whoſe declination is the ſame as that of *d* (*viz.* the arc *SD*): and *HO*, perpendicular to *Cm* the hour-line of XII on this dial, will be a horizontal line on the dial *CPRM* XII. For, the declination of both dials being the ſame, the horizontal line remains parallel to itſelf, while the erect poſition of one dial is reclined, or inclined, with reſpect to the poſition of the other.

Or, the poſition of the dial may be found by applying it to its plane, ſo as to mark the true hour

of

of the day by the ſun, as ſhewn by another dial; or by a clock, regulated by a true meridian line and equation table.

9. There are ſeveral other things neceſſary in the practice of dialing; the chief of which, I ſhall give in the form of arithmetical rules, ſimple and eaſy to thoſe who have learnt the elements of trigonometry. For, in practical arts of this kind, arithmetic ſhould be uſed as far as it can go; and ſcales never truſted to, except in the final conſtruction, where they are abſolutely neceſſary in laying down the calculated hour-diſtances, on the plane of the dial. And although the inimitable artiſts of this metropolis have no occaſion for ſuch inſtructions, yet they may be of ſome uſe to ſtudents, and to private gentlemen who amuſe themſelves this way.

RULE I.

To find the angles which the hour-lines on any dial make with the ſubſtile.

To the logarithmic ſine of the given latitude, or of the ſtile's elevation above the plane of the dial, add the logarithmic tangent of the * hour-diſtance from the meridian, or from the † ſubſtile;

* That is, of 15, 30, 45, 60, 75°, for the hours of I, II, III, IIII, V, in the afternoon; and XI, X, IX, VIII, VII, in the forenoon.

† In all horizontal dials, and erect north or ſouth dials, the ſubſtile and meridian are the ſame: but in all declining dials, the ſubſtile line makes an angle with the meridian.

and

and the ſum *minus* radius will be the logarithmic tangent of the angle ſought.

For, in Fig. 2. *KC* is to *KM* in the ratio compounded of the ratio of *KC* to *KG*, (= *KR*) and of *KG* to *KM*; which, making *CK* the radius 10,000000, or 10,0000, or 10, or 1, are the ratio of 10,000000, or of 10,0000, or of 10, or of 1, to $KG \times KM$.

Thus, in a horizontal dial, for lat. 51° 30′, to find the angular diſtance of XI in the forenoon, or I in the afternoon, from XII.

To the log. ſine of 51° 30′	9.89354*
Add the log. tang. of 15° 0′	9.42805

The ſum — radius is 9.32159 = the logarithmic tangent of 11° 50′, or of the angle which the hour-line of XI or I makes with the hour of XII.

And by computing in this manner, with the ſine of the latitude, and the tangents of 30, 45, 60, and 75°, for the hours of II, III, IIII, and V in the afternoon; or of X, IX, VIII, and VII in the forenoon; you will find their angular diſtances from XII to be 24° 18′, 38° 3′, 53° 35′, and 71° 6′; which are all that there is occaſion to compute for.——And theſe diſtances may be ſet off from XII by a line of chords; or rather, by taking 1000 from a ſcale of equal parts, and ſetting that

* In which caſe, the radius *CK* is ſuppoſed to be divided into 1000000 equal parts.

extent

Fig. 2. extent as a radius from *C* to XII; and then, taking 209 of the ſame parts, (which, in the tables, are the natural tangent of 11° 50′) and ſetting them from XII to XI and to I, on the line *ho*, which is perpendicular to *C* XII: and ſo for the reſt of the hour-lines, which, in the table of natural tangents, againſt the above diſtances, are 451, 782, 1355, and 2920, of ſuch equal parts from XII, as the radius *C* XII contains 1000. And laſtly, ſet off 1257 (the natural tangent of 51° 30′) for the angle of the ſtile's height, which is equal to the latitude of the place.

The reaſon why I prefer the uſe of the tabular numbers, and of a ſcale decimally divided, to that of the line of chords, is becauſe there is the leaſt chance of miſtake and error in this way; and likewiſe, becauſe in ſome caſes it gives us the advantage of a *nonius'* diviſion.

In the univerſal ring-dial, for inſtance, the diviſions on the axis are the tangents of the angles of the ſun's declination placed on either ſide of the center. But inſtead of laying them down from a line of tangents, I would make a ſcale of equal parts, whereof 1000 ſhould anſwer exactly to the length of the ſemi-axis, from the center to the inſide of the equinoctial ring; and then lay down 434 of theſe parts toward each end from the center, which would limit all the diviſions on the axis, becauſe 434 are the natural tangent of 23° 29′. And thus, by a *nonius* affixed to the ſliding piece, and taking the ſun's declination from an ephemeris, and the tangent of that declination from

from the table of natural tangents, the ſlider might be always ſet true to within two minutes of a degree.

And this ſcale of 434 equal parts might be placed right againſt the $23\frac{1}{2}$ degrees of the ſun's declination, on the axis, inſtead of the ſun's place, which is there of very little uſe. For then, the ſlider might be ſet in the uſual way, to the day of the month, for common uſe; but to the natural tangent of the declination, when great accuracy is required.

The like may be done wherever a ſcale of ſines or tangents is required on any inſtrument.

RULE II.

The latitude of the place, the ſun's declination, and his hour-diſtance from the meridian, being given; to find (1.) *his altitude*, (2.) *his azimuth.*

1. Let d be the ſun's place, dR his declination; Fig. 3.
and in the triangle PZd, Pd the ſum, or the difference, of dR and the quadrant PR, being given by the ſuppoſition, as alſo the complement of the latitude PZ, and the angle dPZ which meaſures the horary diſtance of d from the meridian; we ſhall (by Caſe 4. of *Keill*'s oblique ſpheric. Tr.) find the baſe Zd, which is the ſun's diſtance from the zenith, or the complement of his altitude.

And (2.) As ſin. Zd : ſin. Pd :: ſin. dPZ : dZP, or of its ſupplement DZS, the azimuthal diſtance from the ſouth.

Or,

Or, the practical rule may be as follows.

Write A for the sine of the sun's altitude, L and l for the sine and co-sine of the latitude, D and d for the sine and co-sine of the sun's declination, and H for the sine of the horary distance from VI.

Then the relation of H to A will have three varieties.

1. When the declination is toward the elevated pole, and the hour of the day is between XII and VI; it is $A = LD + Hld$, and $H = \frac{A - LD}{ld}$.

2. When the hour is after VI, it is $A = LD - Hld$, and $H = \frac{LD - A}{ld}$.

3. When the declination is toward the depressed pole; we have $A = Hld - LD$, and $H = \frac{A + LD}{ld}$.

Which theorems will be found useful, and expeditious enough, for solving those problems in geography and dialing, which depend on the relation of the sun's altitude to the hour of the day.

EXAMPLE I.

Suppose the latitude of the place to be $51\frac{1}{2}$ degrees north; the time five hours distant from XII, that is, an hour after six in the morning, or before VI in the evening; and the sun's declination 20° north. *Required the sun's altitude.*

Then,

Then, to log. L, = log. sin. 51° 30′—1.89354*
add log. D, = log. sin. 20° 00′—1.53405

Their sum - - —1.42759

gives LD, = log. 0.267664, in the natural sines.

And, to log. H, = log. sin. † 15° 00′—1.41300

add { log. l, = log. sin. ‡ 38° 00′—1.79415
log. d, = log. sin. || 70° 00′—1.97300 }

Their sum - - - - —1.18015

gives Hld, = log. 0.151408, in the natural sines.

And these two numbers (0.267664 and 0.151408) make 0.419072 = A; which, in the table, is the nearest natural sine of 24° 47′, the sun's altitude sought.

The same hour-distance being assumed on the other side of VI, then LD — Hld is 0.116256, the sine of 6° $40\frac{1}{2}$′; which is the sun's altitude at V in the morning, or VII in the evening, when his north declination is 20°.

But when the declination is 20° south (or towards the depressed pole) the difference Hld — LD becomes negative, and thereby shews that, an hour before VI in the morning, or past VI in the evening, the sun's center is 6° $40\frac{1}{2}$′ below the horizon.

* Here we consider the radius as unity, and not 10.00000; by which, instead of the index 9 we have — 1, as above: which is of no farther use, than making the work a little easier.

† The distance of one hour from VI.

‡ The co-latitude of the place.

|| The co-declination of the sun.

EXAMPLE II.

In the ſame latitude and north declination, from the given altitude to find the hour.

Let the altitude be 48°; and becauſe, in this caſe, $H = \frac{A - LD}{ld,}$ and A (the natural ſine of 48°) = .743145, and LD = .267664, $A - LD$ will be 0.475481, whoſe logarithmic ſine is — 1.6771331 from which taking the log. ſine of $l \times d$, = } — 1.7671354

Remains — 1.9099977 the log. ſine of the hour-diſtance ſought, viz. of 54° 22′; which, reduced to time, is 3 hours $37\frac{1}{2}$ min. that is, IX h. $37\frac{1}{2}$ m. in the forenoon, or II h. $22\frac{1}{2}$ m. in the afternoon.

Put the altitude = 18°, whoſe natural ſine is .3090170; and thence $A - LD$ will be = .0491953; which divided by $l \times d$, gives .0717179, the ſine of 4° $6\frac{1}{2}'$; in time $16\frac{1}{2}$ minutes nearly, before VI in the morning, or after VI in the evening, when the ſun's altitude is 18°.

And, if the declination 20° had been towards the ſouth pole, the ſun would have been depreſſed 18° below the horizon at $16\frac{1}{2}$ minutes after VI in the evening; at which time, the twilight would end: which happens about the 22d of November, and 19th of January, in the latitude of $51\frac{1}{2}$° north. The ſame way may the end of twilight, or beginning of dawn, be found for any time of the year.

NOTE 1. If in theorem 2 and 3 (pag. 384) A is put $=$ o, and the value of H is computed, we have the hour of ſun-riſing and ſetting, for any latitude, and time of the year. And if we put $H = 0$, and compute A, we have the ſun's altitude or depreſſion at the hour of VI. And laſtly, if H, A, and D, are given, the latitude may be found by the reſolution of a quadratic equation; for $l = \sqrt{1 - L^2}$.

NOTE 2. When A is equal o, H is equal $\frac{LD}{ld} = T, L \times T, D$, the tangent of the latitude multiplied by the tangent of the declination.

As, if it was required, *what is the greateſt length of day in lat.* $51^\circ\ 30'$?

To the log. tangent of $51^\circ\ 30'$ - - 0.0993948
Add the log. tangent of $23^\circ\ 29'$ — 1.6379563

Their ſum — 1.7373511

is the log. ſine of the hour-diſtance $33^\circ\ 7'$; in time 2 h. $12\frac{1}{2}$ m. The longeſt day therefore is 12 h. + 4 h. 25 m. $=$ 16 h. 25 m. And the ſhorteſt day is 12 h. — 4 h. 25 m. $=$ 7 h. 35 m.

And if the longeſt day is given, the latitude of the place is found; $\frac{H}{T, D}$ being equal to T, L. Thus, if the longeſt day is $13\frac{1}{2}$ hours $= 2 \times 6$ h. + 45 m. and 45 minutes in time being equal to $11\frac{1}{4}$ degrees.

From the log. ſine of $11^\circ\ 15'$ — 1.2902357
Take the log. tang. of $23^\circ\ 29'$ — 1.6379562

Remains — 1.6522795 $=$ the log. tangent of lat $24^\circ\ 11'$.

 And

And the ſame way, the latitudes where the ſeveral geographical *climates* and parallels begin, may be found; and the latitudes of places that are aſſigned in authors from the length of their days, may be examined and corrected.

NOTE 3. The ſame rule for finding the longeſt day in a given latitude, diſtinguiſhes the hour-lines that are neceſſary to be drawn on any dial, from thoſe which would be ſuperfluous.

In lat. 52° 10′, the longeſt day is 16 h. 32 m. and the hour-lines are to be marked from 44 m. after III in the morning, to 16 m. after VIII in the evening.

In the ſame latitude, let the dial of Art. 7. Fig. 4. be propoſed; and the elevation of its ſtile (or the latitude of the place *d*, whoſe horizon is parallel to the plane of the dial) being 15° 9′; the longeſt day at *d*, that is, the longeſt time that the ſun can illuminate the plane of the dial, will (by the rule $H = T, L \times T, D$) be twice 6 hours 27 minutes, = 12 h. 54 m. The difference of longitude of the planes *d* and *Z* was found in the ſame example to be 36° 2′; in time, 2 hours 24 minutes: and the declination of the plane was from the ſouth towards the weſt. Adding therefore 2 h. 24 m. to 5 h. 33 m. the earlieſt ſun-riſing on a horizontal dial at *d*, the ſum 7 h. 57 m. ſhews that the morning hours, or the parallel dial at *Z*, ought to begin at 3 min. before VIII. And to the lateſt ſun-ſetting at *d*, which is 6 h. 27 m. adding the ſame 2 h. 24 m. the ſum 8 h. 51 m. exceeding 6 h. 16 m. the lateſt ſun-ſetting at *Z*, by 35 m. ſhews that

Fig. 3.

that none of the afternoon hour-lines are ſuperfluous. And the 4 h. 13 m. from III h. 44 m. the ſun-riſing at Z to VII h. 57 m. the ſun-riſing at d, belong to the other face of the dial: that is, to a dial declining 36° from north to eaſt, and inclining 15°.

EXAMPLE III.

From the ſame *data* to find the ſun's *azimuth.*

If H, L, and D are given, then (by Art. 2. of Rule II.) from H having found the altitude, and its complement Zd; and the arc Pd (the diſtance from the pole) being given; ſay, As the co-ſine of the altitude is to the ſine of the diſtance from the pole, ſo is the ſine of the hour-diſtance from the meridian to the ſine of the azimuth-diſtance from the meridian.

Let the latitude be 51° 30′ north, the declination 15° 9′ ſouth, and the time II h. 24 m. afternoon, when the ſun begins to illuminate a vertical wall: and it is required to find the poſition of the wall.

Then, by the foregoing theorems, the complement of the altitude will be 81° $32\frac{1}{2}$′, and Pd the diſtance from the pole being 109° 5′, and the horary diſtance from the meridian, or the angle dPZ 36°.

To log. ſin. 74° 51′ - - —1.98464
Add log. ſin. 36° 0′ - - —1.76922

And from the ſum - - - —1.75386
Take the log. ſin. 81° $32\frac{1}{2}$′ —1.99525

Remains —1.75861 = log. ſin. 35°, the azimuth diſtance ſouth.

 When

When the altitude is given, find from thence the hour, and proceed as above.

This praxis is of ſingular uſe on many occaſions; in finding the declination of vertical planes more exactly than in the common way, eſpecially if the tranſits of the ſun's center is obſerved by applying a ruler with ſights, either plain or teleſcopical, to the wall or plane, whoſe declination is required.—In drawing a meridian line, and finding the magnetic variation.—In finding the bearings of places in terreſtrial ſurveys; the tranſits of the ſun over any place, or his horizontal diſtance from it being obſerved, together with the altitude and hour.—And thence determining ſmall differences of longitude.—In obſerving the variation at ſea, &c.

The learned Mr. *Andrew Reid* invented an inſtrument ſeveral years ago, for finding the latitude at ſea from two altitudes of the ſun, obſerved on the ſame day, and the interval of the obſervations, meaſured by a common watch. And this inſtrument, whoſe only fault was that of its being ſomewhat expenſive, was made by Mr. *Jackſon*. Tables have been lately computed for that purpoſe.

But we may often, from the foregoing rules, reſolve the ſame problem without much trouble; eſpecially if we ſuppoſe the maſter of the ſhip to know within 2 or 3 degrees what his latitude is. Thus,

Aſſume the two neareſt probable limits of the latitude, and by the theorem $H = \frac{A+LD}{ld}$, compute the hours of obſervation for both ſuppoſitions. If one interval of thoſe computed hours coincides

with

with the interval obſerved, the queſtion is ſolved. If not, the two diſtances of the intervals computed, from the true interval, will give a proportional part to be added to, or ſubtracted from, one of the latitudes aſſumed. And if more exactneſs is required, the operation may be repeated with the latitude already found.

But which ever way the queſtion is ſolved, a proper allowance is to be made for the difference of latitude ariſing from the ſhip's courſe in the time between the two obſervations.

Of the double horizontal dial; and the Babylonian and Italian dials.

To the *gnomonic* projection, there is ſometimes added a *ſtereographic* projection of the hour-circles, and the parallels of the ſun's declination, on the ſame horizontal plane; the upright ſide of the gnomon being ſloped into an edge, ſtanding perpendicularly over the center of the projection: ſo that the dial, being in its due poſition, the ſhadow of *that* perpendicular edge is a vertical circle paſſing through the ſun, in the ſtereographic projection.

The months being duly marked on this dial, the ſun's declination, and the length of the day at any time, are had by inſpection; (as alſo his altitude, by means of a ſcale of tangents). But its chief property is, that it may be placed true, whenever the ſun ſhines, without the help of any other inſtrument.

Let *d* be the ſun's place in the ſtereographic projection, *xdyz* the parallel of the ſun's declination, Fig. 3.

 Zd

Zd a vertical circle through the ſun's center, *Pd* the hour-circle; and it is evident, that the diameter *NS* of this projection being placed duly north and ſouth, theſe three circles will paſs through the point *d*. And therefore, to give the dial its due poſition, we have only to turn its gnomon toward the ſun, on a horizontal plane, until the hour on the common gnomonic projection coincides with that marked by the hour-circle *Pd*, which paſſes through the interſection of the ſhadow *Z d* with the circle of the ſun's preſent declination.

The *Babylonian* and *Italian* dials reckon the hours not from the meridian, as with us, but from the ſun's riſing and ſetting. Thus, in *Italy*, an hour before ſun-ſet is reckoned the 23d hour; two hours before ſun-ſet, the 22d hour; and ſo of the reſt. And the ſhadow that marks them on the hour-lines, is *that* of the point of a ſtile. This occaſions a perpetual variation between their dials and clocks, which they have to correct from time to time, before it ariſes to any ſenſible quantity; by ſetting their clocks ſo much faſter or ſlower. And in *Italy*, they begin their day, and regulate their clocks, not from ſun-ſet, but from about mid-twilight, when the *ave Maria* is ſaid; which corrects the difference that would otherwiſe be between the clock and the dial.

The improvements which have been made in all ſorts of inſtruments and machines for meaſuring time, have rendered ſuch dials of little account. Yet, as the theory of them is ingenious, and they are really, in ſome reſpects, the beſt contrived of any

any for vulgar uſe, a general idea of their deſcription may not be unacceptable.

Let Fig. 5. repreſent an erect direct ſouth wall, on which a *Babylonian dial* is to be drawn, ſhewing the hours from ſun-riſing; the latitude of the place, whoſe horizon is parallel to the wall, being equal to the angle KCR. Make, as for a common dial, $KG = KR$ (which is perpendicular to CR) the radius of the equinoctial *Æ Q*, and draw RS perpendicular to CK for the ſtile of the dial; the ſhadow of whoſe point R is to mark the hours, when SR is ſet upright on the plane of the dial. Fig. 5.

Then it is evident that, in the contingent line *Æ Q*, the ſpaces $K1$, $K2$, $K3$, &c. being taken equal to the tangents of the hour-diſtances from the meridian, to the radius KG, one, two, three, &c. hours after ſun-riſing, on the equinoctial day; the ſhadow of the point R will be found, at theſe times, reſpectively in the points 1, 2, 3, &c.

Draw, for the like hours after ſun-riſing, when the ſun is in the tropic of capricorn ♑ V, the like common lines CD, CE, CF, &c. and at thoſe hours the ſhadow of the point R will be found in thoſe lines reſpectively. Find the ſun's altitudes above the plane of the dial at theſe hours, and with their co-tangents Sd, Se, Sf, &c. to radius SR, deſcribe arcs interſecting the hour-lines in the points d, e, f, &c. So ſhall the right lines $1d$, $2e$, $3f$, &c. be the lines of I, II, III, &c. hours after ſun-riſing.

The conſtruction is the ſame in every other caſe, due regard being had to the difference of longitude

of

of the place at which the dial would be horizontal, and the place for which it is to ſerve. And likewiſe, taking care to draw no lines but what are neceſſary; which may be done partly by the rules already given for determining the time that the ſun ſhines on any plane; and partly from this, that on the tropical days, the hyperbola deſcribed by the ſhadow of the point *R*, limits the extent of all the hour-lines.

The moſt uſeful however, as well as the ſimpleſt of ſuch dials, is that which is deſcribed on the two ſides of a meridian plane.

That the *Babylonian* and *Italic* hours are truly enough marked by right lines, is eaſily ſhewn. Mark the three points on a globe, where the horizon cuts the equinoctial, and the two tropics, toward the eaſt, or weſt; and turn the globe on its axis 15°, or 1 hour; and it is plain, that the three points which were in a great circle (viz. the horizon) will be in a great circle ſtill; which will be projected geometrically into a ſtraight line. But theſe three points are univerſally the ſun's places, one hour after ſun-ſet (or one hour before ſun-riſe) on the equinoctial and ſolſtitial days. The like is true of all other circles of declination, beſides the tropics; and therefore, the hours on ſuch dials are truly marked by ſtraight lines limited by the projections of the tropics; and which are rightly drawn, as in the foregoing example.

Note 1. The ſame dials may be delineated without the hour-lines *CD*, *CE*, *CF*, &c. by ſetting off the ſun's azimuths on the plane of the dial,

dial, from the center *S*, on either ſide of the ſubſtile *C S K*, and the correſponding co-tangents of altitude from the ſame center *S*, for I, II, III, &c. hours before or after the ſun is in the horizon of the place for which the dial is to ſerve, on the equinoctial and ſolſtitial days.

2. One of theſe dials has its name from the hours being reckoned from ſun-riſing, the beginning of the *Babylonian* day. But we are not thence to imagine that the *equal* hours, which it ſhews, to be thoſe in which the aſtronomers of that country marked their obſervations. Theſe, we know with certainty, were unequal, like the *Jewiſh*, as being twelfth parts of the natural day: and an hour of the night was, in like manner, a twelfth part of the night; longer or ſhorter according to the ſeaſon of the year. So that an hour of the day, and an hour of the night, at the ſame place, would always make $\frac{1}{12}$ of 24, or 2 equinoctial hours. In Paleſtine, among the Romans, and in ſeveral other countries, 3 of theſe unequal nocturnal hours were a *vigilia* or *watch*. And the reduction of equal and unequal hours into one another, is extremely eaſy. If, for inſtance, it is found, by a foregoing rule, that in a certain latitude, at a given time of the year, the length of a day is 14 equinoctial hours, the unequal hour is then $\frac{14}{12}$ or $\frac{7}{6}$ of an hour, that is, 70 minutes; and the nocturnal hour is 50 minutes. The firſt watch begins at VII (ſun-ſet), the ſecond at three times 50 minutes after, *viz.* IX h. 30 m. the third always at midnight; the morning watch at $\frac{1}{2}$ hour paſt II.

If

If it were required to draw a dial for ſhewing theſe unequal hours, or 12th parts of the day; we muſt take as many declinations of the ſun as are thought neceſſary, from the equator towards each tropic: and having computed the ſun's altitude and azimuth for $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$th parts, *&c.* of each of the diurnal arcs belonging to the declinations aſſumed: by theſe, the ſeveral points in the circles of declination, where the ſhadow of the ſtile's point falls, are determined: and curve lines drawn through the points of an homologous diviſion will be the hour-lines required.

Of the right placing of dials, and having a true meridian line for the regulating of clocks and watches.

The plane on which the dial is to reſt; being duly prepared, and every thing neceſſary for fixing it; you may find the hour tolerably exact by a large equinoctial ring-dial, and ſet your watch to it. And then the dial may be fixed by the watch at your leiſure.

If you would be more exact, take the ſun's altitude by a good quadrant, noting the preciſe time of obſervation by a clock or watch. Then, compute the time for the altitude obſerved, (by the rule, page 386) and ſet the watch to agree with that time, according to the ſun. A *Hadley*'s quadrant is very convenient for this purpoſe; for, by it you may take the angle between the ſun and his image, reflected from a baſon of water; the half of

of which angle, ſubtracting the refraction, is the altitude required. This is beſt done in ſummer, and the nearer the ſun is to the prime vertical (the eaſt or weſt azimuth) when the obſervation is made, ſo much the better.

Or, in ſummer, take two equal altitudes of the ſun in the ſame day; one any time between 7 and 10 o'clock in the morning, the other between 2 and 5 in the afternoon; noting the moments of theſe two obſervations by a clock or watch: and if the watch ſhews the obſervations to be at equal diſtances from noon, it agrees exactly with the ſun: if not, the watch muſt be corrected by half the difference of the forenoon and afternoon intervals; and then the dial may be ſet true by the watch.

Thus, for example, ſuppoſe you had taken the ſun's altitude when it was 20 minutes paſt VIII in the morning by the watch; and found by obſerving in the afternoon, that the ſun had the ſame altitude 10 minutes before IIII; then it is plain, that the watch was 5 minutes too faſt for the ſun: for 5 minutes after XII is the middle time between VIII h. 20 m. in the morning, and III h. 50 m. in the afternoon; and therefore, to make the watch agree with the ſun, it muſt be ſet back five minutes.

A good *meridian line*, for regulating clocks or watches, may be had by the following method. *A meridian line.*

Make a round hole, almoſt a quarter of an inch diameter, in a thin plate of metal; and fix the plate in the top of a ſouth window, in ſuch a man-

ner,

ner, that it may recline from the zenith at an angle equal to the co-latitude of your place, as nearly as you can guefs: for then, the plate will face the fun directly at noon on the equinoctial days. Let the fun fhine freely through the hole into the room; and hang a plumb-line to the cieling of the room, at leaft five or fix feet from the window, in fuch a place as that the fun's rays, tranfmitted through the hole, may fall upon the line when it is noon by the clock; and having marked the faid place on the cieling, take away the line.

Having adjufted a fliding bar to a dove-tail groove, in a piece of wood about 18 inches long, and fixed a hook into the middle of the bar, nail the wood to the abovementioned place on the cieling, parallel to the fide of the room in which the window is: the groove and bar being towards the floor. Then, hang the plumb-line upon the hook in the bar, the weight or plummet reaching almoft to the floor; and the whole will be prepared for farther and proper adjuftment.

This done, find the true folar time by either of the two laft methods, and thereby regulate your clock. Then, at the moment of next noon by the clock, when the fun fhines, move the fliding bar in the groove until the fhadow of the plumb-line bifects the image of the fun (made by his rays tranfmitted through the hole) on the floor, wall, or on a white fcreen placed on the north fide of the line; the plummet or weight at the end of the line hanging freely in a pale of water placed below it on the floor.—But becaufe this may not be quite correct

correct for the firft time, becaufe the plummet will not fettle immediately, even in water; it may be farther corrected on the following days, by the above method, with the fun and clock; and fo brought to a very great exactnefs.

N. B. The rays tranfmitted through the hole will caft but a faint image of the fun, even on a white fcreen, unlefs the room be fo darkened that no fun-fhine may be allowed to enter, but what comes through the fmall hole in the plate. And always, for fome time before the obfervation is made, the plummet ought to be immerfed in a jar of water, where it may hang freely; by which means the line will foon become fteady, which otherwife would be apt to continue fwinging.

As this meridian line will not only be fufficient for regulating of clocks and watches to the true mean time by equation tables, but alfo for moft aftronomical purpofes, I fhall fay nothing of the magnificent and expenfive meridian lines at *Bologna* and *Rome*, nor of the better methods by which aftronomers obferve precifely the tranfits of the heavenly bodies on the meridian.

LECT. XI.

Shewing how to calculate the mean time of any new or full moon, or eclipfe, from the creation of the world to the year of CHRIST, 5800.

IN computing the following tables for this purpofe, I have kept to the mean motions in Mr. *Meyer*'s celebrated tables, publifhed in the *Gottingen* Tranf-

Transactions; but have altered the form, in order to bring them into as short a compass as possible.

PRECEPTS.

To find the mean time of any new or full moon in any given year and month after the Christian Æra.

1. If the given year be found in the second column of the *Table of the moon's mean motion from the sun*, under the title *years before and after* CHRIST; write out that year, with the mean motions belonging to it, and thereto join the given month with its mean motions. But, if the given year be not in the table, take out the next lesser one to it that you find, in the same column; and thereto add as many *compleat years*, as will make up the given year: then, join the given month, and all the respective mean motions.

2. Collect these mean motions into one sum of signs, degrees, minutes, and seconds; remembering, that 60 seconds (″) make a minute, 60 minutes (′) a degree, 30 degrees (°) a sign, and 12 signs (^{s}) a circle. When the signs exceed 12, or 24, or 36, (which are whole circles) reject them, and set down only the remainder; which, together with the odd degrees, minutes, and seconds already set down, must be reckoned the whole sum of the collection.

3. Subtract the result or sum of this collection, from 12 signs; and write down the remainder, Then,

Then, look in the table, under *days*, for the next leſs mean motions to this remainder, and ſubtract them from it, writing down their remainder.

This done, look in the table under *hours*, (marked H.) for the next leſs mean motions to this laſt remainder, and ſubtract them from it, writing down their remainder.

Then, look in the table under *minutes*, (marked M.) for the next leſs mean motions to this remainder, and ſubtract them from it, writing down their remainder.

Laſtly, look in the table under *ſeconds*, (marked S.) for the next leſs mean motions to this remainder, either greater or leſs; and againſt it you have the ſeconds anſwering thereto.

4. And theſe times collected, will give the mean time of the *required new moon*; which will be right in common years, and alſo in January and February in leap-years; but always one day too late in leap-years after February.

EXAMPLE I.

Required the mean time of new moon in September 1764? (a year not inserted in the table.)

		Moon from sun.			
		s	°	′	″
To the year after *Christ*'s birth	1753	10	9	24	56
Add compleat years —	11	0	10	14	20
	(sum 1764)				
And join September —	—	2	22	21	7
The sum of these mean motions is		1	12	0	23
Which, being sub. from a circle, or		12	0	0	0
Leaves remaining — — —		10	17	59	37
Next less mean mot. for 26 days, sub.		10	16	57	34
And there remains — — —			1	2	3
Next less mean mot. for 2 hours, sub.			1	0	57
And the remainder will be —				1	6
Next less mean mot. for 2 min. sub.				1	1
Remains the mean mot. of 12 sec.					5

These times, being collected, would shew the mean time of the required new moon in September 1764, to be on the 26th day, at 2 hours 2 min. 12 sec. past noon. But, as it is in a leap-year, and after February, the time is one day too late. So, the true mean time is Sept. the 25th, at 2 m. 12 sec. past II in the afternoon.

N. B. The tables always begin the day at noon, and reckon thence forward, to the noon of the day following.

To find the mean time of full moon in any given year and month after the Christian Æra.

Having collected the moon's mean motion from the fun for the beginning of the given year and month, and fubtracted their fum from 12 figns, (as in the former example) add 6 figns to the remainder, and then proceed in all refpects as above.

EXAMPLE II.

Required the mean time of full moon in September 1764?

		Moon from fun.			
		s	°	′	″
To the year after *Chrift*'s birth	1753	10	9	24	56
Add compleat years ——	11	0	10	14	20
	(fum 1764)				
And join September — —		2	22	21	7
The fum of thefe mean motions is		1	12	0	23
Which, being fubtr. from a circle, or		12	0	0	0
Leaves remaining — — —		10	17	59	37
To which remainder add —		6	0	0	0
And the fum will be — —		4	17	59	37
Next lefs mean mot. for 11 days, fubt.		4	14	5	54
And there remains — — —			3	53	43
Next lefs mean mot. for 7 hours, fubtr.			3	33	20
And the remainder will be —				20	23
Next lefs mean mot. for 40 min. fubt.				20	19
Remains the mean mot. for 8 fec.					4

 So,

So, the mean time, according to the tables, is the 11th of September, at 7 hours, 40 min. 8 sec. paſt noon. One day too late, being after February in a leap-year.

And thus may the mean time of any new or full moon be found, in any year after the Chriſtian Æra.

To find the mean time of new or full moon in any given year and month before the Chriſtian Æra.

If the given year before the year of CHRIST 1 be found in the ſecond column of the table, under the title *years before and after* CHRIST, write it out, together with the given month, and join the mean motions. But, if the given year be not in the table, take out the next greater one to it that you find; which being ſtill farther back than the given year, add as many compleat years to it as will bring the time forward to the given year: then join the month, and proceed in all reſpects as above.

EXAMPLE III.

Required the mean time of new moon in May, the year before Chriſt 585?

The next greater year in the table is 600; which being 15 years before the given year, add the mean motions for 15 years to thoſe of 600, together with thoſe for the beginning of May.

To

	Moon from ſun.			
	s	°	′	″
To the year before *Chriſt* 600	5	11	6	16
Add compleat years mot. 15	6	0	55	24
And the mean motions for May	0	22	53	23
The whole ſum is — — —	0	4	55	3
Which, being ſubtr. from a circle, or	12	0	0	0
Leaves remaining — — —	11	25	4	57
Next leſs mean mot. for 29 days, ſub.	11	23	31	54
And there remains — — —		1	33	3
Next leſs mean mot. for 3 hours, ſub.		1	31	26
And the remainder will be —			1	37
Next leſs mean mot. for 3 min. ſub.			1	31
Rem. the mean mot. of 14 ſeconds				6

So, the mean time, by the tables, was the 29th of May, at 3 hours, 3 min. 14 ſec. paſt noon. A day later than the truth, on account of its being in a leap-year. For, as the year of CHRIST 1 was the firſt after a leap-year, the year 585 before the year 1 was a leap-year, of courſe.

If the given year be after the Chriſtian Æra, divide its date by 4, and if nothing remains, it is a leap-year in the old ſtyle. But if the given year was before the Chriſtian Æra, (or year of CHRIST 1) ſubtract one from its date, and divide the remainder by 4; then, if nothing remains, it was a leap-year; otherwiſe, not.

To find whether the ſun is eclipſed at the time of any given change, or the moon at any given full.

Of eclipſes. From the *Table of the ſun's mean motion*, (or diſtance) *from the moon's aſcending node*, collect the mean motions anſwering to the given time; and if the reſult ſhews the ſun to be within 18 degrees of either of the nodes at the time of new moon, the ſun will be eclipſed at that time. Or, if the reſult ſhews the ſun to be within 12 degrees of either of the nodes at the time of full moon, the moon will be eclipſed at that time, near the contrary node: otherwiſe not.

EXAMPLE IV.

The moon changes on the 26th of September 1764, *at* 2 *h.* 2 *m.* *(neglecting the ſeconds) after noon.* (See Example I.) *Qu. Whether the ſun will be eclipſed at that time?*

	Sun from node.			
	s	°	′	″
To the year after *Chriſt*'s birth 1753	1	28	0	19
Add compleat years — 11	7	2	3	56
(ſum 1764)				
And September — — —	8	12	22	49
And 26 days — — — —		27	0	13
And 2 hours — — — —			5	12
And 2 minutes — — —				5
Sun's diſtance from the aſcend. node	6	9	32	34

Now,

Now, as the descending node is just opposite to the ascending, (*viz.* 6 signs distant from it) and the tables shew only how far the sun has gone from the ascending node, which, by this example, appears to be 6 signs, 9 deg. 32 min. 34 sec. it is plain that he must be eclipsed; being then only 9° 32′ 34″ short of the descending node.

The times thereof.

EXAMPLE V.

The moon will be full on the 11*th of September* 1764, *at* 7 *hours* 40 *min. past noon.* (See Example II.) *Qu. Whether she will be eclipsed at that time?*

		Sun from node.			
		s	°	′	″
To the year after *Christ*'s birth	1753	1	28	0	19
Add compleat years	11	7	2	3	56
	(sum 1764)				
And September		8	12	22	49
And 11 days			11	25	29
And 7 hours				18	11
And 40 minutes				1	44
Sun's distance from the asc. node		5	24	12	28

Which being subtracted from 6 signs, leaves only 5° 47′ 32″ remaining: and this being all the space that the sun is short of the descending node, it is plain that the moon must then be eclipsed, because she is no farther from the contrary node.

EXAMPLE VI.

Qu. Whether the ſun was eclipſed in May, the year before CHRIST 585? (See Example III.)

	Sun from node.			
	s	°	′	″
To the year before *Chriſt* 600 —	9	9	23	51
Add the mean mot. of 15 compl. years	9	19	27	49
And May — — — —	4	4	37	57
And 29 days — — — —	1	0	7	10
And 3 hours — — —			7	48
And 3 min. (neglecting the ſeconds)				8
Sun's diſtance from the aſc. node	0	3	44	43

Which being leſs than 18 degrees, ſhews that the ſun was eclipſed at that time.

Thales's eclipſe. This eclipſe was foretold by *Thales*, and appears to be the eclipſe which put an end to the war between the Medes and Lydians.

When *eclipſes* muſt happen. The times of the *ſun's conjunction with the nodes*, and conſequently the *eclipſe-months* of any given year, are eaſily found by the *Table of the ſun's mean motion from the moon's aſcending node*; and much in the ſame way as the mean conjunctions of the ſun and moon are found by the table of the moon's mean motion from the ſun. For, collect the ſun's mean motion (which is the ſame as his diſtance gone) from the moon's aſcending node, for the beginning of any given *year*, and ſubtract it from 12

ſigns;

signs; then, from the remainder, subtract the next less mean motions belonging to whatever *month* you find them in the table; and from their remainder subtract the next less mean motions for *days*, and so on for *hours* and *minutes*: the result of all which will shew the time of the sun's mean conjunction with the *ascending node* of the moon's orbit.

EXAMPLE VII.

Required the time of the sun's conjunction with the ascending node in the year 1764?

		Sun from node.			
		s	°	′	″
To the year after *Christ*'s birth	1753	1	28	0	19
Add compleat years	11	7	2	3	56
Mean dist. at begin. of A. D. 1764		9	0	4	15
Subtr. this distance from a circle, or		12	0	0	0
And there remains — — —		2	29	55	45
Next less mean motion for March subt.		2	1	16	39
And the remainder will be —		0	28	39	6
Next less mean mot. for 27 days, subt.		0	28	2	32
And there remains — — —				36	34
Next less mean mot. for 14 hours, subt.				36	21
Rem. (nearly) the mean mot. of 5 min.					13

Hence it appears, that the sun will pass by the moon's *ascending node* on the 27th of March, at 14 hours

hours 5 minutes paſt noon; *viz.* on the 28th day, at 5 minutes after II in the morning, according to the tables: but this being in a leap-year, and after February, the time is one day too late. Conſequently, the true time is at 5 minutes paſt II in the morning on the 27th day; at which time, the deſcending node will be directly oppoſite to the ſun.

If 6 ſigns be added to the remainder ariſing from the firſt ſubtraction, (*viz.* from 12 ſigns) and then the work carried on as in the laſt example, the reſult will give the mean time of the ſun's conjunction with the deſcending node. Thus in

EXAMPLE VIII.

Let it be required to find when the ſun will be in conjunction with the deſcending node *in the year* 1764?

		Sun from node.			
		s	°	′	″
To the year after *Chriſt*'s birth	1753	1	28	0	19
Add compleat years — —	11	7	2	3	56
Mean diſt. fr. aſc. nod. at beg. of	1764	9	0	4	15
Subtr. this diſtance from a circle, or		12	0	0	0
And the remainder will be —		2	29	55	45
To which add half a circle, or —		6	0	0	0
And the ſum will be — —		8	29	55	45
Next leſs mean mot. for Septemb. ſubt.		8	12	22	49
And there remains — — —		0	17	32	56
Next leſs mean mot. for 16 days, ſubt.		0	16	37	4
And the remainder will be —				55	52
Next leſs mean mot. for 21 hours, ſubt.				54	32
Rem. (nearly) the mean mot. of 31 min.				1	20

So

So that, according to the tables, the sun will be in conjunction with the *descending node* on the 16th of September, at 21 hours 31 minutes past noon: one day later than the truth, on account of the leap-year.

When the moon changes within 18 days before or after the sun's conjunction with either of the nodes, the sun will be eclipsed at that change: and when the moon is full within 12 days before or after the time of the sun's conjunction with either of the nodes, she will be eclipsed at that full: otherwise not. The limits of *eclipses.*

If to the mean time of any eclipse, either of the sun or moon, we add 557 Julian years, 21 days, 18 hours, 11 minutes, and 51 seconds, (in which there are exactly 6890 mean lunations) we shall have the mean time of the return of that eclipse. For, at the end of that time, the moon will be either new or full, according as we add it to the time of new or full moon; and the sun will be only 45″ farther from the same node, at the end of the said time, than he was at the beginning of it; as appears by the following example *. Their period and restitution.

The

* Dr. HALLEY's period of eclipses contains only 18 years, 11 days, 7 hours, 43 minutes, 15 seconds; in which time, according to his tables, there are just 223 mean lunations: But, as in that time, the sun's mean motion from the node is no more than $11^s\ 29^\circ\ 31'\ 49''$, which wants $28'\ 11''$ of being as nearly in conjunction with the same node at the end of the period as it was at the beginning; this period can be of no long duration for finding eclipses, because it will in time fall quite without their limits. The following tables make this period 31 seconds

The period.		Moon fr. sun.					Sun fr. node.			
		s	°	′	″		s	°	′	″
Compl. years	500 —	3	5	32	47	—	10	14	45	8
	40 —	8	26	50	37	—	1	23	58	49
	17 —	3	2	21	39	—	10	28	40	55
days	21 —	8	16	0	21	—		21	48	38
hours	18 —		9	8	35	—			46	44
minutes	11 —			5	35	—				29
seconds	51 —				26	—				2
Mean motions		0	0	0	0	—	0	0	0	45

And this period is so very near, that in 6000 years it will vary no more from the truth, as to the restitution of eclipses, than $8\frac{1}{4}$ minutes of a degree; which may be reckoned next to nothing. It is the shortest in which, after many trials, I can find so near a co-incidence of the sun, moon, and the same node.

31 seconds shorter, as appears by the following calculation.

The period.		Moon from sun.					Sun from node.			
		s	°	′	″		s	°	′	″
Compl. years	18 ——	7	11	59	4	——	11	17	46	18
days	11 ——	4	14	5	54	——		11	25	29
hours	7 ——		3	33	20	——			18	11
min.	42 ——			21	20	——			1	49
sec.	44 ——				22	——				2
Mean motions		0	0	0	0	——	11	29	31	49

This

This table is made by the continual addition of a mean lunation, viz. $29^d\ 12^h\ 44^m\ 3^s\ 6'''\ 21^{iv}\ 14^{v}\ 24^{vi}\ 0^{vii}$.

Lun.		Days.	H.	M.	S.	Th.
1		29	12	44	3	6
2		59	1	28	6	13
3	Contain	88	14	12	9	19
4		118	2	56	12	25
5		147	15	40	15	32
6		177	4	24	18	38
7		206	17	8	21	44
8		236	5	52	24	51
9		265	18	36	27	57
10		295	7	20	31	3
20		590	14	41	2	7
30		885	22	1	33	11
40		1181	5	22	4	14
50		1476	12	42	35	18
100		2953	1	25	10	35
200		5906	2	50	21	11
300		8859	4	15	31	46
400		11812	5	40	42	22
500		14765	7	5	52	57
1000		29530	14	11	45	54
2000		59061	4	23	31	48
3000		88591	18	35	17	42
4000		118122	8	47	3	36
5000		147652	22	58	49	30
10000		295305	21	57	39	0
20000		590611	19	55	18	0
30000		885917	17	52	57	0
40000		1181223	15	50	36	0
50000		1476529	13	48	15	0
100000		2953059	3	36	30	0

In 100000 mean lunations, there are 8085 Julian years, 12 days, 21 hours, 36 minutes, 30 seconds = 2953059 days, 3 hours, 36 minutes, 30 seconds.

Proof of the Table.

In		Moon fr. sun s	°	′	″
Jul. years	4000	1	14	22	12
	4000	1	14	22	12
	80	5	23	41	15
	5	10	0	18	28
Days	12	4	26	17	20
Hours	21		10	40	1
Min.	36			18	17
Sec.	30				15
M. fr. sun		0	0	0	0

Having by the former precepts computed the mean time of new moon in January, for any given year, it is easy by this Table to find the mean time of new moon in January for any number of years afterward; and by means of a small table of lunations for 12 or 13 months, to make a general table for finding the mean time of new or full moon in any given year and month whatever.

	D.	H.	M.	S.	Th.
In 11 lunations there are	324	20	4	34	10.
In 12 lunations ——	354	8	48	37	16.
In 13 lunations ——	383	21	32	40	23.

But then it would be best to begin the year with March, to avoid the inconvenience of losing a day by mistake in leap-year.

Julian period.		Years bef. and after CHRIST.	Moon from ſun. s	°	′	″
706	Before the year of CHRIST 1.	4008	5	28	1	17
714		4000	5	9	23	24
1714		3000	11	20	28	57
2714		2000	6	1	34	36
3714		1000	0	12	40	3
3814		900	10	19	46	36
3914		800	8	26	53	9
4014		700	7	3	59	43
4114		600	5	11	6	16
4214		500	3	18	12	49
4314		400	1	25	19	23
4414		300	0	2	25	56
4514		200	10	9	32	29
4614		100	8	16	39	3
4714	After CHRIST.	1	6	23	45	36
4814		101	5	0	52	9
4914		201	3	7	58	43
5014		301	1	15	5	16
5114		401	11	22	11	49
5214		501	9	29	18	23
5714		1001	1	4	51	9
6414		1701	0	24	37	2
6466		1753	10	9	24	56
6514		1801	6	5	26	15

	Compleat years.	Moon from ſun. s	°	′	″
All theſe are Julian years, conſiſting of 365 d. 6 h.	1	4	9	37	24
	2	8	19	14	48
	3	0	28	52	13
	4	5	20	41	4
	5	10	0	18	28
	6	2	9	55	52
	7	6	19	33	17
	8	11	11	22	7
	9	3	20	59	32
	10	8	0	36	55

Compleat years.	Moon from ſun. s	°	′	″
11	0	10	14	20
12	5	2	3	11
13	9	11	40	35
14	1	21	18	0
15	6	0	55	24
16	10	22	44	15
17	3	2	21	39
18	7	11	59	4
19	11	21	36	27
20	4	13	25	19
40	8	26	50	37
60	1	10	15	56
80	5	23	41	15
100	10	7	6	33
200	8	14	13	7
300	6	21	19	40
400	4	28	26	13
500	3	5	32	47
1000	6	11	5	33
2000	0	22	11	6
3000	7	3	16	39
4000	1	14	22	12

Months	Moon from ſun. s	°	′	″
Jan.	0	0	0	0
Feb.	0	17	54	48
Mar.	11	29	15	16
April	0	17	10	3
May	0	22	53	23
June	1	10	48	11
July	1	16	31	32
Aug.	2	4	26	20
Sept.	2	22	21	8
Oct.	2	28	4	29
Nov.	3	15	59	17
Dec.	3	21	42	37

The year of the Julian period 706 is ſuppoſed to be the year of the creation.

This table agrees with the *old ſtyle*, until the year 1753; and after that, with the *new*.

Days.	Moon from ſun.				Moon from ſun.				Moon from ſun.			
	s	°	'	"	H. M. S.	° ' "	' " '''	" ''' ''''	M. S. Th.	' " '''	" ''' ''''	" '''' v
1	0	12	11	27								
2	0	24	22	53								
3	1	6	34	20	1	0	30	29	31	15	44	47
4	1	18	45	47	2	1	0	57	32	16	15	16
5	2	0	57	13	3	1	31	26	33	16	45	44
6	2	13	8	40	4	2	1	54	34	17	16	13
7	2	25	20	7	5	2	32	23	35	17	46	42
8	3	7	31	34	6	3	2	52	36	18	17	10
9	3	19	43	0	7	3	33	20	37	18	47	39
10	4	1	54	27	8	4	3	49	38	19	18	7
11	4	14	5	54	9	4	34	18	39	19	48	36
12	4	26	17	20	10	5	4	46	40	20	19	5
13	5	8	28	47	11	5	35	15	41	20	49	33
14	5	20	40	14	12	6	5	43	42	21	20	2
15	6	2	51	40	13	6	36	12	43	21	50	31
16	6	15	3	7	14	7	6	41	44	22	20	59
17	6	27	14	34	15	7	37	9	45	22	51	28
18	7	9	26	0	16	8	7	38	46	23	21	56
19	7	21	37	27	17	8	38	6	47	23	52	25
20	8	3	48	54	18	9	8	35	48	24	22	54
21	8	16	0	21	19	9	39	4	49	24	53	22
22	8	28	11	47	20	10	9	32	50	25	23	51
23	9	10	23	14	21	10	40	1	51	25	54	19
24	9	22	34	41	22	11	10	30	52	26	24	48
25	10	4	46	7	23	11	40	58	53	26	55	17
26	10	16	57	34	24	12	11	27	54	27	25	45
27	10	29	9	1	25	12	41	55	55	27	56	14
28	11	11	20	27	26	13	12	24	56	28	26	43
29	11	23	31	54	27	13	42	53	57	28	57	11
30	0	5	43	21	28	14	13	21	58	29	27	40
31	0	17	54	47	29	14	43	50	59	29	58	8
32	1	0	6	15	30	15	14	18	60	30	28	37

1 Lunation $= 29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 6^{th}\ 21^{iv}\ 14^{v}\ 24^{vi}\ 0^{vii}$.

In leap-years, after February, a day and its motion muſt be added to the time for which the moon's mean diſtance from the ſun is given. But, when the mean time of any new or full moon is required in leap-year after February, a day muſt be ſubtracted from the mean time thereof, as found by the tables. In common years they give the day right.

Years of the world.		Years bef. and after Christ.	Sun from node. s	°	′	″
0	Before the year of Christ 1.	4008	7	6	17	9
8		4000	0	11	4	55
1008		3000	9	10	35	11
2008		2000	6	10	5	28
3008		1000	3	9	35	44
3108		900	7	24	32	46
3208		800	0	9	29	48
3308		700	4	24	26	49
3408		600	9	9	23	51
3508		500	1	24	20	53
3608		400	6	9	17	54
3708		300	10	24	14	56
3808		200	3	9	11	58
3908		100	7	24	8	59
4008	After Christ.	1	0	9	6	1
4108		101	4	24	3	3
4208		201	9	9	0	4
4308		301	1	23	57	6
4408		401	6	8	54	8
4508		501	10	23	51	9
5008		1001	9	8	36	18
5708		1701	4	23	15	30
5760		1753	1	28	0	19
5808		1801	8	25	44	44

Compleat years.	Sun from node. s	°	′	″
11	7	2	3	56
12	7	22	11	39
13	8	11	17	2
14	9	0	22	25
15	9	19	27	49
16	10	9	35	31
17	10	28	40	55
18	11	17	46	18
19	0	6	51	43
20	0	26	59	24
40	1	23	58	49
60	2	20	58	13
80	3	17	57	37
100	4	14	57	2
200	8	29	54	3
300	1	14	51	5
400	5	29	48	7
500	10	14	45	8
1000	8	29	30	17
2000	5	29	0	33
3000	2	28	30	50
4000	11	28	1	6

The 4008th year bef. the y. of Christ 1 is ſup. to be the year of the creation.

	Compleat years.	Sun from node. s	°	′	″
All theſe are Julian years, as in the preceding table.	1	0	19	5	23
	2	1	8	10	47
	3	1	27	16	10
	4	2	17	23	53
	5	3	6	29	16
	6	3	25	34	40
	7	4	14	40	3
	8	5	4	47	46
	9	5	23	53	9
	10	6	12	58	33

Months	Sun from node. s	°	′	″
Jan.	0	0	0	0
Feb.	1	2	11	48
Mar.	2	1	16	39
April	3	3	28	27
May	4	4	37	57
June	5	6	49	45
July	6	7	59	14
Aug.	7	9	11	1
Sept.	8	12	22	49
Oct.	9	13	32	18
Nov.	10	15	44	5
Dec.	11	16	53	34

This table agrees with the *old ſtyle*, until the year 1753; and after that, with the *new*.

Days.	Sun from node.				Sun from node.				Sun from node.			
	s	°	'	''	H. M. S.	° ' ''	' '' '''	'' ''' ''''	M. S. Th.	' '' '''	'' ''' ''''	''' '''' v
1	0	1	2	19								
2	0	2	4	38								
3	0	3	6	57	1	0	2	36	31	1	20	31
4	0	4	9	16	2	0	5	12	32	1	23	7
5	0	5	11	36	3	0	7	48	33	1	25	43
6	0	6	13	54	4	0	10	23	34	1	28	9
7	0	7	16	13	5	0	12	59	35	1	31	55
8	0	8	18	32	6	0	15	35	36	1	33	31
9	0	9	20	51	7	0	18	11	37	1	36	6
10	0	10	23	10	8	0	20	47	38	1	38	42
11	0	11	25	29	9	0	23	23	39	1	41	18
12	0	12	27	48	10	0	25	58	40	1	43	54
13	0	13	30	7	11	0	28	33	41	1	46	36
14	0	14	32	26	12	0	31	9	42	1	49	5
15	0	15	34	15	13	0	33	45	43	1	51	41
16	0	16	37	4	14	0	36	21	44	1	54	17
17	0	17	39	23	15	0	38	57	45	1	56	53
18	0	18	41	41	16	0	41	32	46	1	59	29
19	0	19	44	0	17	0	44	8	47	2	2	5
20	0	20	46	19	18	0	46	44	48	2	4	41
21	0	21	48	38	19	0	49	20	49	2	7	17
22	0	22	50	57	20	0	51	56	50	2	9	53
23	0	23	53	16	21	0	54	32	51	2	12	29
24	0	24	55	35	22	0	57	8	52	2	15	5
25	0	25	57	54	23	0	59	43	53	2	17	41
26	0	27	0	13	24	1	2	19	54	2	20	17
27	0	28	2	32	25	1	4	55	55	2	22	53
28	0	29	4	51	26	1	7	31	56	2	25	29
29	1	0	7	10	27	1	10	7	57	2	28	4
30	1	1	9	29	28	1	12	43	58	2	30	40
31	1	2	11	48	29	1	15	9	59	2	33	16
32	1	3	14	7	30	1	17	55	60	2	35	52

In leap-years, after February add one day and one day's motion to the time at which the ſun's mean diſtance from the aſcending node is required.

FINIS.

INDEX.

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I N D E X.

DIRECTIONS to the BOOK-BINDER.

The plates must open to the left hand, fronting the right hand pages in the following order.

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